

# *Mechanics of Materials*

(<http://bernoulli.iam.ntu.edu.tw/>)



## Chapter 4

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### Material Properties and Stress-Strain Relationships

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### Contents

#### Material Properties and Stress-Strain Relationships

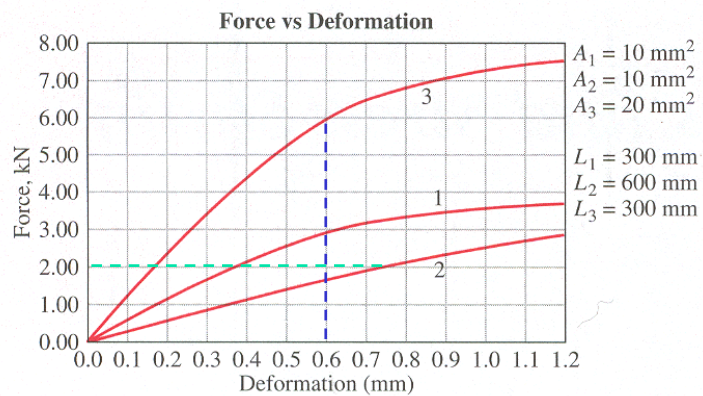
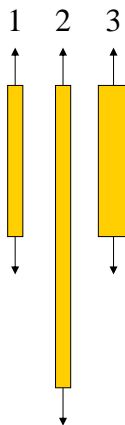
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- 4-1 Introduction
- **4-2 Stress-Strain Diagrams**
- **4-3 Generalized Hooke's Law**
- **4-4 Thermal Strain**
- 4-5 Stress-Strain Equation for Orthotropic Materials (Self read)

## 4-1 Introduction

- Deformation or Strain
  - due to applied loads or reactions
  - temperature changes, i.e., thermal expansions
- Decouple the effects caused by loads and temperature changes
  - Superposition for linear problem
  - Compute separately and add together for total deformation

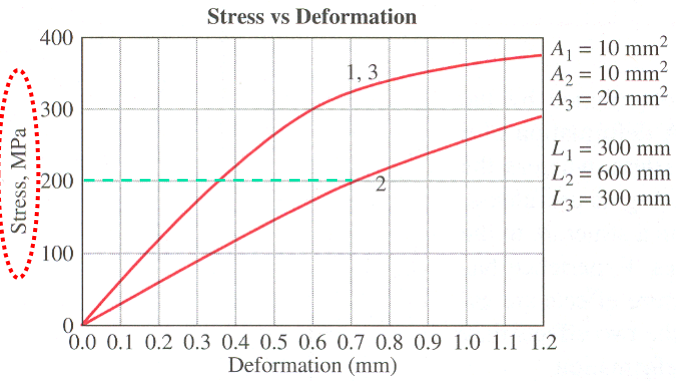
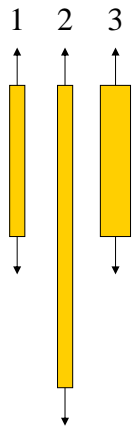
## 4-2 Stress-Strain Diagrams



p. 154



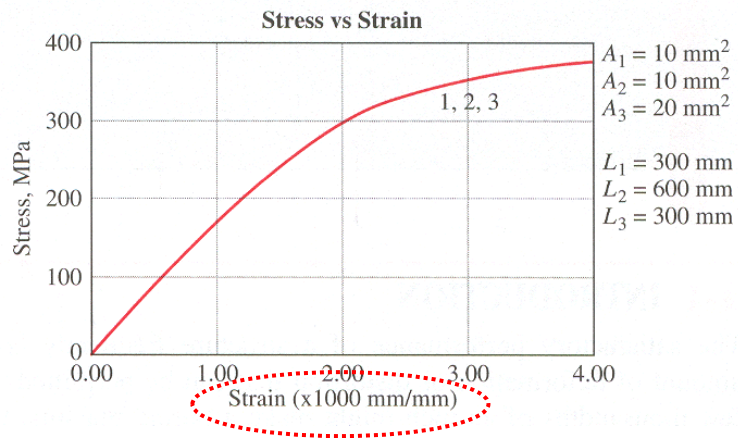
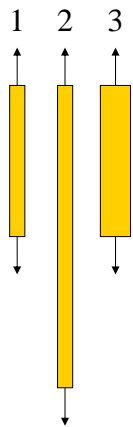
## 4-2 Stress-Strain Diagrams



p. 154



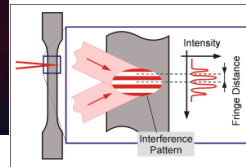
## 4-2 Stress-Strain Diagrams



p. 154

## Tensile Test

- Stress = load / **initial** area  
(most commonly used)
- True stress = load / **actual** area
- Strain Measurement
  - measured by strain gages or extensometers (or compressometers)
  - In engineering structures, usually  $\epsilon < 0.001$

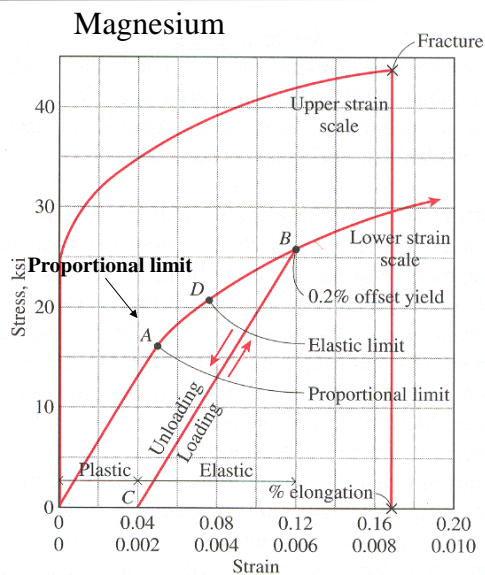
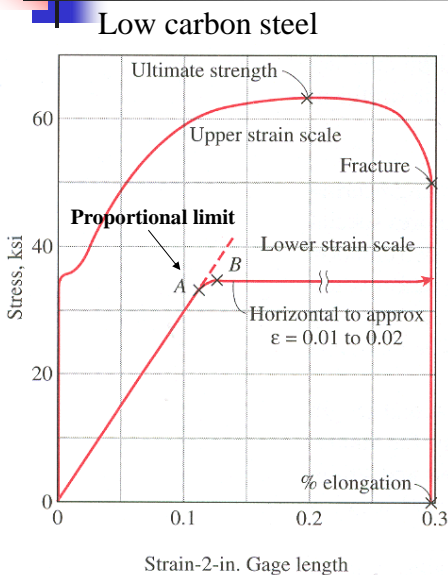


Laser Doppler Extensometer

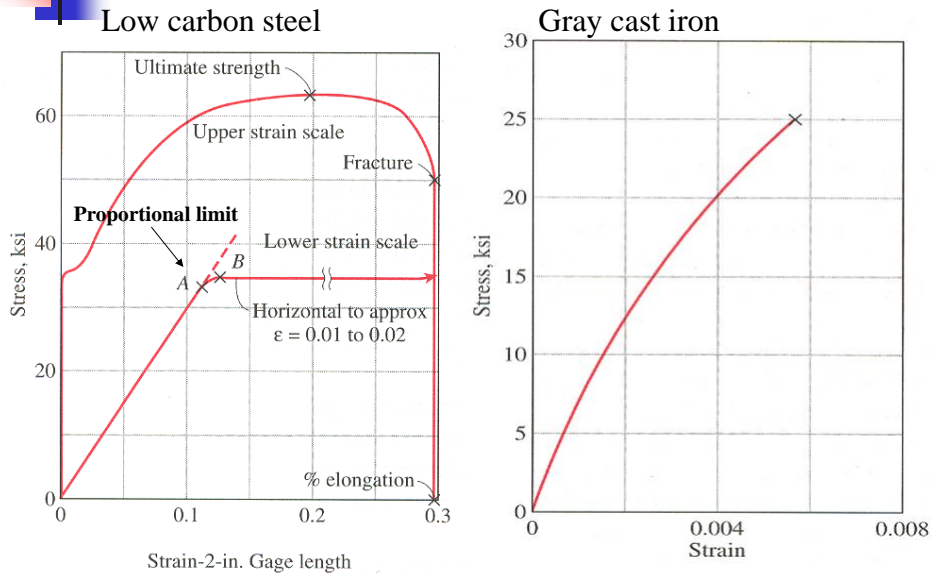
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p. 156

## Example Stress–Strain Diagrams



## Example Stress–Strain Diagrams



## Modulus of Elasticity

- Hooke's law (1678 by Robert Hooke)
 
$$\sigma = E\varepsilon$$
- Young's modulus or **modulus of elasticity** (1807 by Thomas Young) or "*stiffness*"
 
$$E = \sigma/\varepsilon \quad (\text{normal stress-strain})$$
- Shear modulus (**modulus of rigidity**)
 
$$G = \tau/\gamma \quad (\text{shear stress-strain})$$
- Proportional limit  
max stress for which stress and strain are proportional

## Elastic Limit

- **Elastic**

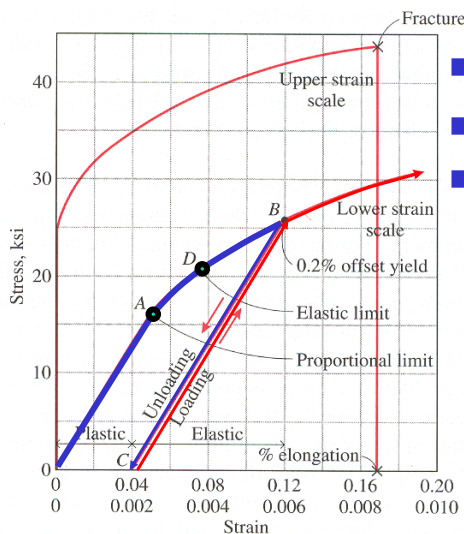
the strain resulting from loading **disappears** when the load is removed.

- Elastic limit

the max stress for which the material acts elastically.

If  $\sigma >$  elastic limit  $\Rightarrow$  **plastic** deformation

## Elastic/Plastic deformation: Elastic Limit



- loading/unloading

- strain/work hardening

- plastic deformation

- **(Crystals) Slip:**

depends on load, independent of time

- **Creep (滑變) :**

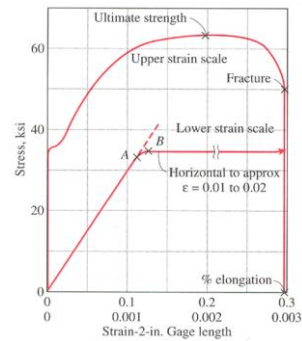
continues to increase under **long-term constant load** (time-dependent permanent deformation)  
often occurs at high T

p. 158

## Yield Point (降伏點)

- The stress at which there is an **appreciable** increase in strain with **no** increase in stress.  
⇒ the load of the testing machine ceases to rise  
If straining is continued, the stress will again increase.  
⇒ kink or knee in the  $\sigma$ - $\epsilon$  diagram.

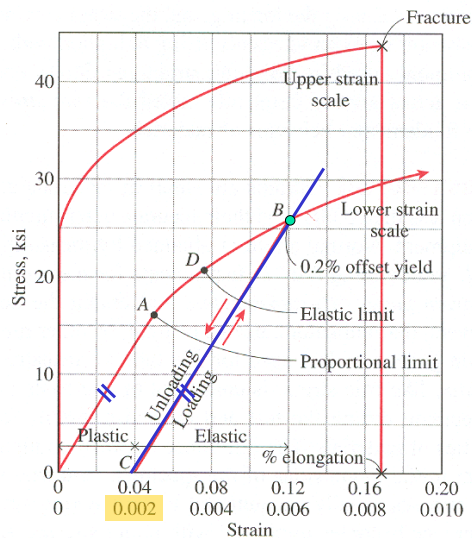
- Used as the proportional limit
- Example: low-carbon steel



p. 158

## Yield Strength (降伏強度)

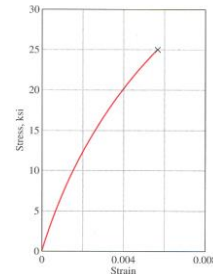
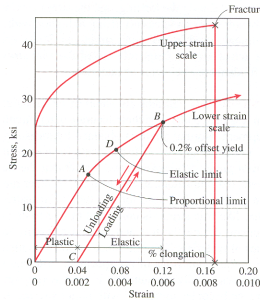
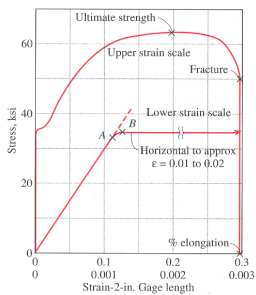
- The stress that will induce a specified permanent set usually 0.05 to 0.3% (**0.2% is most commonly used**)
- Used as the proportional limit



p. 158

## Ultimate Strength (I)

- The maximum stress (nominal stress) developed in a material before rupture. (may be tensile, compressive or shearing strength)
- **Ductile materials** undergo considerable plastic deformation before rupture.

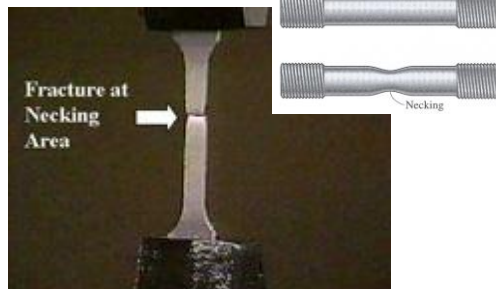
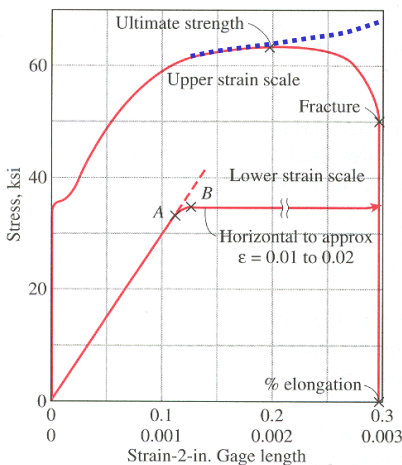


p. 158

## Ultimate Strength (II)

- Necking phenomenon (ductile material)

nominal stress  $\downarrow$  beyond the ultimate strength, but true stress continues  $\uparrow$  until rupture.



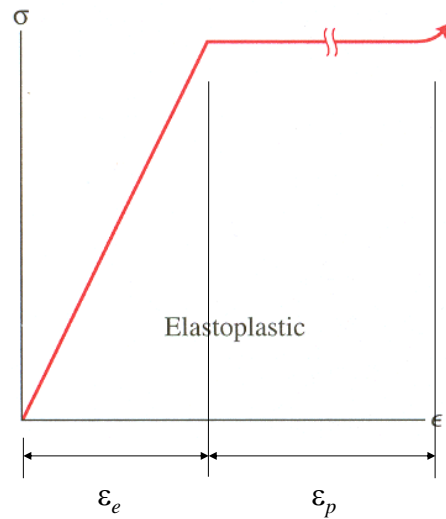
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p. 158



## Elasto-Plastic Materials



■ mild steel

$$\varepsilon_p \cong 16 \sim 20 \varepsilon_e$$

p. 158



## Ductility (延展性)

- Capacity of plastic deformation in tension or shear
- Indices:
  - ultimate **elongation** (expressed as a percent elongation of the gage length at rupture)
  - the **reduction** (in %) of **cross-sectional area** at the section where rupture occurs.

## Creep Limit

- **Creep Limit:** The maximum stress for which the plastic strain will not exceed a specified amount during a specified time interval at a specified temperature.
- The resistance of a material to failure by creep
- Lead, rubber and some plastics may creep at ordinary temperature
- Important for polymeric materials or metal parts subjected to high temperatures (35~50% of melting temperature).
- **Creep strength:** the stress which, at a given temperature, will result in a creep rate of **1% deformation within a specified time** (e.g. 1000 ~ 100,000 hours).

## Viscoelastic Behavior -Creep

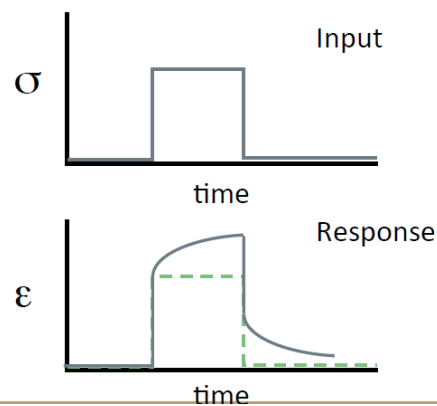
### Creep Test

A constant stress is applied to the web.

Response is strain.

Elastic Response

Viscoelastic Response



# Viscoelastic Behavior – Stress Relaxation

## Stress Relaxation

A constant strain is applied to the web.

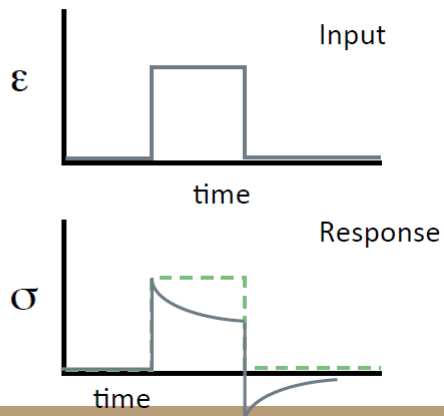
Response is stress.

Elastic Response

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Viscoelastic Response

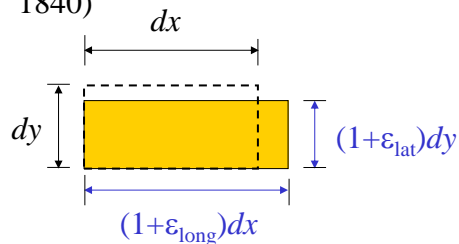
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p. 159

# Poisson's Ratio

- Simeon D. Poisson, 1811 (French mathematician (1781-1840))



$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}} = -\frac{\epsilon_t}{\epsilon_n}$$

- $E = 2(1+\nu)G$  (shown in section 4-3)
- $\nu = \text{constant for } \sigma < \text{proportional limit}$
- $\nu = 1/4 \sim 1/3$  for most metals



## Effect of Composition

- Various alloy contents for steels

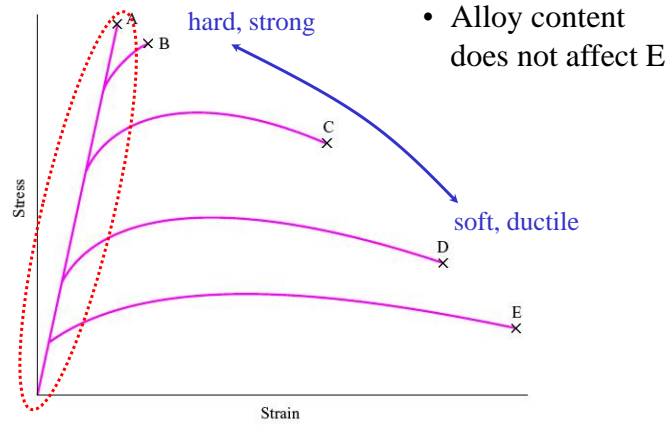
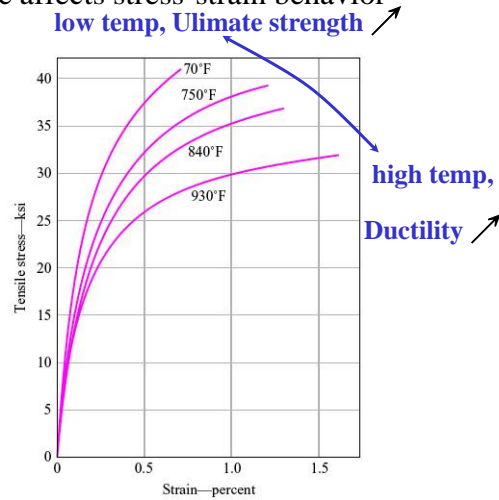


Figure 4-7



## Effect of Temperature

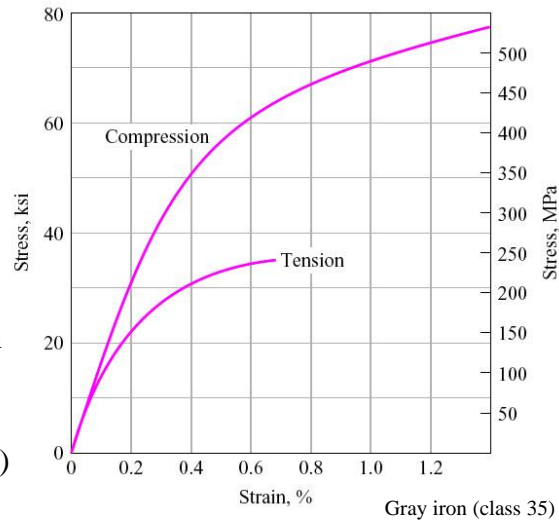
- Temperature affects stress-strain behavior



p. 161

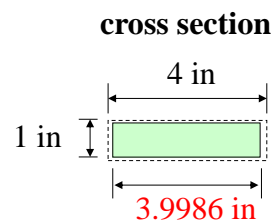
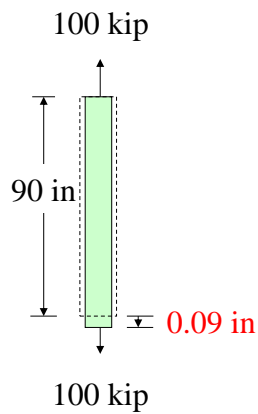
## Effect of Tension or Compression

- For *ductile material*, the tension and compression behavior are usually assumed to be *the same*.
- *Brittle* materials, e.g., gray iron, behave differently on load conditions
- Material properties: Appendix B (A42/43)



p. 161

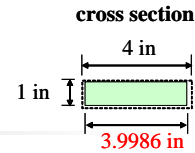
## Example Problem 4-1



- Determine  $\nu$ ,  $E$ ,  $G = ?$

p. 161

## Example Problem 4-1



$$\delta_{\text{lat}} = 3.9986 - 4 = -0.0014 \text{ in} \quad \varepsilon_{\text{lat}} = \frac{\delta_{\text{lat}}}{L} = \frac{-0.0014}{4} = -0.00035$$

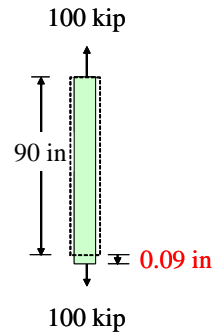
$$\varepsilon_{\text{long}} = \frac{\delta_{\text{long}}}{L} = \frac{0.09}{90} = 0.00100$$

$$\nu = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = -\frac{0.00035}{0.00100} = 0.35$$

$$\sigma = \frac{P}{A} = \frac{100}{4(1)} = 25 \text{ ksi}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{25}{0.00100} = 25,000 \text{ ksi}$$

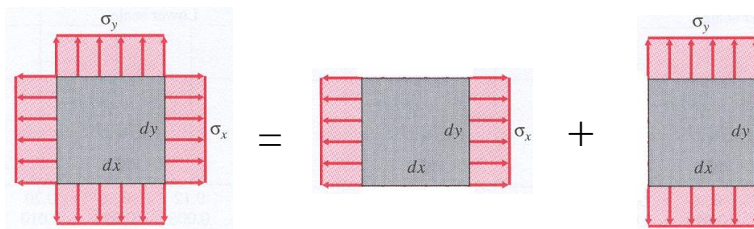
$$G = \frac{E}{2(1+\nu)} = \frac{25,000}{2(1+0.35)} = 9260 \text{ ksi}$$



p. 164

## 4-3 Generalized Hooke's Law

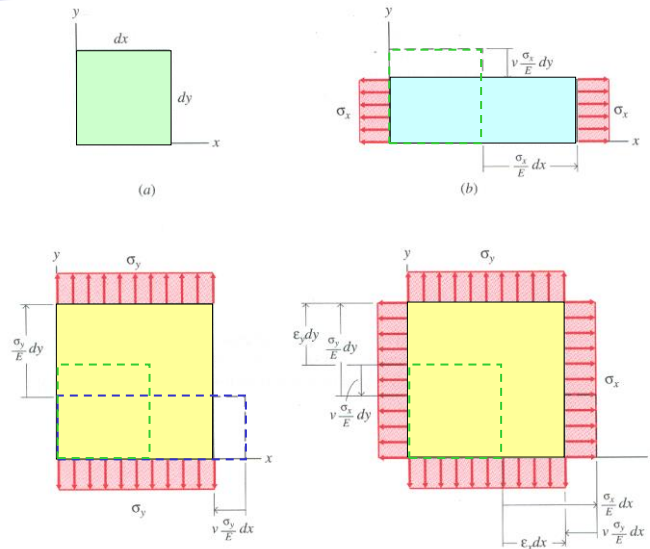
Principle of **superposition**



- Valid under conditions of
  - $\sigma \sim \varepsilon$  linear ( $\sigma < \text{proportional limit}$ )
  - effect of  $\sigma_x$  does not significantly change the effect of  $\sigma_y$  (if  $\varepsilon$  small  $\rightarrow \Delta A$  small)



## 4-3 Generalized Hooke's Law



$$\sigma_z = 0$$



## Generalized Hooke's Law – Biaxial Stress

- **Isotropic** material: material properties **independent of direction**

$$d\delta_x = \epsilon_x dx = \frac{\sigma_x}{E} dx - \nu \frac{\sigma_y}{E} dx$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y)$$

$$d\delta_y = \epsilon_y dy = \frac{\sigma_y}{E} dy - \nu \frac{\sigma_x}{E} dy$$



$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x)$$

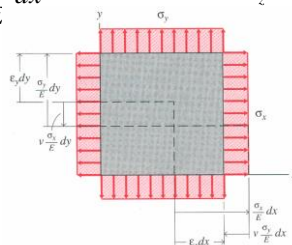
$$d\delta_z = \epsilon_z dz = -\nu \frac{\sigma_x}{E} dz - \nu \frac{\sigma_y}{E} dz$$

$$\epsilon_z = -\frac{\nu}{E} (\sigma_x + \sigma_y)$$



$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y)$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x)$$



p. 166

## Generalized Hooke's Law – **Triaxial** Stress

For  $\sigma_z = 0$ ,  $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$   $\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_z + \sigma_x)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$



$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z)]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_y + \nu(\varepsilon_z + \varepsilon_x)]$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_z + \nu(\varepsilon_x + \varepsilon_y)]$$

p. 166

## Hooke's Law for **Shear Stress**

$$\tau = G\gamma$$

$$\tau_{xy} = G\gamma_{xy}$$

$$\tau_{yz} = G\gamma_{yz}$$

$$\tau_{zx} = G\gamma_{zx}$$



p. 167



## Relationship between $E$ , $\nu$ , and $G$

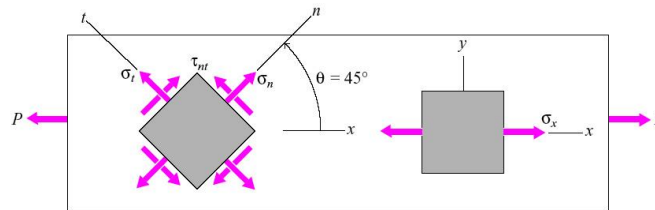


Figure 4-12

(from Eq. 2-13a)

(from Eq. 3-8a)

$$\begin{aligned} \tau_{nt} &= -(\sigma_x - \sigma_y) \sin \theta \cos \theta + \tau_{xy} (\cos^2 \theta - \sin^2 \theta) & \gamma_{nt} &= -2(\epsilon_x - \epsilon_y) \sin \theta \cos \theta + \gamma_{xy} (\cos^2 \theta - \sin^2 \theta) \\ &= -(\sigma_x - 0) \sin(45^\circ) \cos(45^\circ) + 0 & &= -2(\epsilon_x - \epsilon_y) \sin(45^\circ) \cos(45^\circ) + 0 \\ &= -\frac{\sigma_x}{2} & &= -(\epsilon_x - \epsilon_y) \end{aligned}$$

p. 166



## $\epsilon_z$ in Plane Stress ( $\sigma_z = 0$ )

$$\epsilon_z = -\frac{\nu}{E} (\sigma_x + \sigma_y)$$

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) \quad \sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x)$$

$$\epsilon_z = -\frac{\nu}{E} \left[ \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) + \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) \right]$$

$$= -\frac{\nu}{E} \left( \frac{E}{1-\nu^2} \right) [(\epsilon_x + \nu \epsilon_y) + (\epsilon_y + \nu \epsilon_x)]$$

$$= -\frac{\nu}{1-\nu} (\epsilon_x + \epsilon_y)$$

see Chapter 3 Eq 3-13.

For  $\nu = 1/3 \rightarrow \nu/(1-\nu) = 1/2$ ,  
 $\epsilon_z$  is simply the average of  $\epsilon_x$  and  $\epsilon_y$ .

p. 167

## Relationship between $E$ , $\nu$ , and $G$

For the element subjected to an axial load,

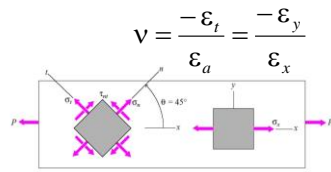


Figure 4-12

$$\nu = \frac{-\varepsilon_y}{\varepsilon_x} = \frac{-\varepsilon_y}{\varepsilon_x} \quad \Rightarrow \quad \varepsilon_y = -\nu\varepsilon_x$$

$$\gamma_{nt} = -(\varepsilon_x - \varepsilon_y) \quad \Rightarrow \quad \gamma_{nt} = -\varepsilon_x(1 + \nu)$$

$$\tau_{nt} = G\gamma_{nt} = -G\varepsilon_x(1 + \nu)$$

$$\begin{aligned} &= -G \frac{\sigma_x}{E} (1 + \nu) \quad \Rightarrow \quad \boxed{G = \frac{E}{2(1 + \nu)}} \\ (\because \tau_{nt} = -\frac{\sigma_x}{2}) &= -G \frac{-2\tau_{nt}}{E} (1 + \nu) \end{aligned}$$

p. 167

## Relationship between $E$ , $\nu$ , and $G$

$$\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1 + \nu)} \gamma_{xy}$$

$$\Rightarrow \tau_{yz} = G\gamma_{yz} = \frac{E}{2(1 + \nu)} \gamma_{yz}$$

$$\tau_{zx} = G\gamma_{zx} = \frac{E}{2(1 + \nu)} \gamma_{zx}$$

## Biaxial Stress for isotropic materials

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$$

$$= -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\sigma_z = 0$$

$$\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$$

$$\tau_{yz} = G\gamma_{yz} = \frac{E}{2(1+\nu)}\gamma_{yz}$$

$$\tau_{zx} = G\gamma_{zx} = \frac{E}{2(1+\nu)}\gamma_{zx}$$

$$G = \frac{E}{2(1+\nu)}$$

p. 168

## Example Problem 4-2 (I)

### ■ Biaxial stress

■  $E = 210 \text{ GPa}, \nu = 0.30$

■  $\varepsilon_x = +1394 \text{ } \mu\text{m/m}$

$\varepsilon_y = -660 \text{ } \mu\text{m/m}$

$\gamma_{xy} = 2054 \text{ } \mu\text{rad}$

### Determine

■  $\sigma_x, \sigma_y, \tau_{xy} = ?$

■  $\sigma_p, \tau_{\max} = ?$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{210(10^9)}{1-0.3^2}[1394 + 0.3(-660)](10^{-6})$$

$$= +276.0(10^6) \text{ N/m}^2 \cong 276 \text{ MPa (T)}$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{210(10^9)}{1-0.3^2}[-660 + 0.3(1394)](10^{-6})$$

$$= -55.80(10^6) \text{ N/m}^2 \cong 55.8 \text{ MPa (C)}$$

p. 169

## Example Problem 4-2 (II)

$$\tau_{xy} = \frac{E}{2(1+\nu)} \gamma_{xy} = \frac{210(10^9)}{2(1+0.30)} (2054)(10^{-6}) = 165.90(10^6) \text{ N/m}^2 \cong 165.9 \text{ MPa}$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{276.0 - 55.80}{2} \pm \sqrt{\left(\frac{276.0 + 55.80}{2}\right)^2 + 165.90^2}$$

$$\sigma_{p1} = 110.10 + 234.62 = 344.72 \text{ MPa} \cong 345 \text{ MPa (T)}$$

$$\sigma_{p2} = 110.10 - 234.62 = -124.52 \text{ MPa} \cong 124.5 \text{ MPa (C)}$$

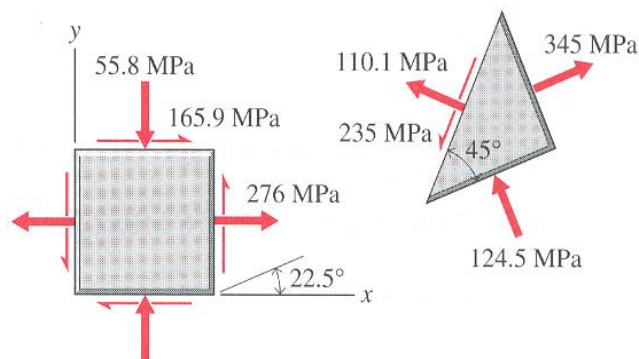
$$\sigma_{p3} = \sigma_z = 0$$

p. 170

## Example Problem 4-2 (III)

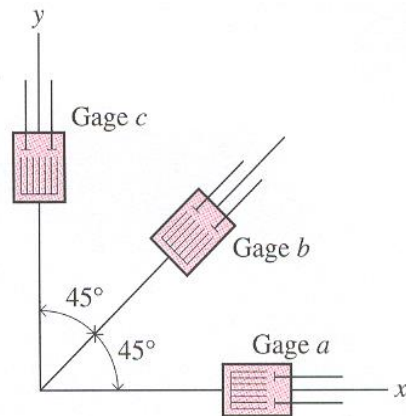
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(165.90)}{276.0 + 55.80} = 1.000$$

$$2\theta_p = 45.00^\circ \quad \theta_p = 22.5^\circ$$



p. 170

## Example Problem 4-3 (I)



■ Plane stress

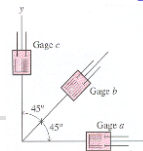
- $E = 30,000$  ksi
- $\nu = 0.30$
- $\epsilon_a = +650$   $\mu\text{in/in}$
- $\epsilon_b = +475$   $\mu\text{in/in}$
- $\epsilon_c = -250$   $\mu\text{in/in}$

Determine

- $\sigma_x, \sigma_y, \tau_{xy} = ?$
- $\epsilon_p, \gamma_{\max} = ?$
- $\sigma_p, \tau_{\max} = ?$

p. 171

## Example Problem 4-3 (II)



■ Plane stress

- $E = 30,000$  ksi
- $\nu = 0.30$
- $\epsilon_a = +650$   $\mu\text{in/in}$
- $\epsilon_b = +475$   $\mu\text{in/in}$
- $\epsilon_c = -250$   $\mu\text{in/in}$

$$\epsilon_a = \epsilon_x = +650 \mu \text{ (measured)}$$

$$\epsilon_c = \epsilon_y = -250 \mu \text{ (measured)}$$

$$\epsilon_b = \epsilon_x \cos^2 \theta_b + \epsilon_y \sin^2 \theta_b + \gamma_{xy} \sin \theta_b \cos \theta_b$$

$$= 650 \mu \cos^2 45^\circ - 250 \mu \sin^2 45^\circ + \gamma_{xy} \sin 45^\circ \cos 45^\circ = +475 \mu$$

$$\Rightarrow \gamma_{xy} = 550 \mu$$

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = \frac{30,000}{1-0.3^2} [650 + 0.3(-250)] (10^{-6})$$

$$= +18.956 \text{ ksi} \cong 18.96 \text{ ksi (T)}$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) = \frac{30,000}{1-0.3^2} [-250 + 0.3(650)] (10^{-6})$$

$$= -1.8132 \text{ ksi} \cong 1.813 \text{ ksi (C)}$$

$$\tau_{xy} = \frac{E}{2(1+\nu)} \gamma_{xy} = \frac{30,000}{2(1+0.30)} (550) (10^{-6}) = 6.346 \text{ ksi} \cong 6.35 \text{ ksi}$$

p. 171



### Example Problem 4-3 (III)

$$\begin{aligned}\varepsilon_{p1}, \varepsilon_{p2} &= \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \\ &= \frac{650\mu - 250\mu}{2} \pm \sqrt{\left(\frac{650\mu + 250\mu}{2}\right)^2 + \left(\frac{550\mu}{2}\right)^2} \\ &= 200\mu \pm 527.4\mu\end{aligned}$$

$$\varepsilon_{p1} \cong 727\mu \quad \varepsilon_{p2} \cong -327\mu$$

$$\varepsilon_{p3} = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{0.3}{1-0.3}(650\mu - 250\mu) = -171.4\mu$$

$$\gamma_{\max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 724.4\mu + 327.4\mu \cong 1055\mu$$

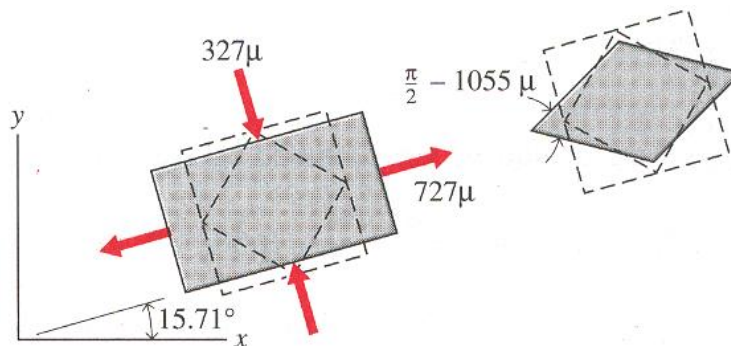
p. 171



### Example Problem 4-3 (IV)

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{550\mu}{650\mu + 250\mu} = 0.61111$$

$$2\theta_p = 31.429^\circ \quad \theta_p \cong 15.71^\circ$$



p. 172

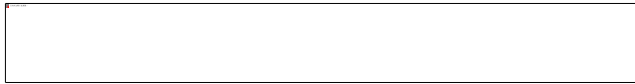
### Example Problem 4-3 (V)

$$\sigma_{p1} = \frac{E}{1-\nu^2} (\epsilon_{p1} + \nu\epsilon_{p2}) = \frac{30,000}{1-0.3^2} [727.4 + 0.3(-327.4)](10^{-6})$$
$$= +20.742 \text{ ksi} \cong 20.76 \text{ ksi (T)}$$

$$\sigma_{p2} = \frac{E}{1-\nu^2} (\epsilon_{p2} + \nu\epsilon_{p1}) = \frac{30,000}{1-0.3^2} [-327.4 + 0.3(727.4)](10^{-6})$$
$$= -3.599 \text{ ksi} \cong 3.60 \text{ ksi (C)}$$

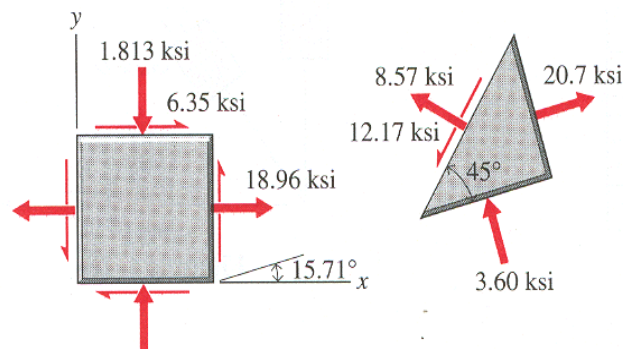
$$\sigma_{p3} = \sigma_z = 0$$

$$\tau_{\max} = \frac{E}{2(1+\nu)} \gamma_{\max} = \frac{30,000}{2(1+0.30)} (1054.8)(10^{-6}) = 12.171 \text{ ksi} \cong 12.17 \text{ ksi}$$



p. 172

### Example Problem 4-3 (VI)



■ Note:

The principal directions for stress and strain are the same for isotropic material.

p. 176



## 4-4 Thermal Strain

$$\varepsilon_T = \alpha \Delta T \quad \alpha: \text{coefficient of thermal expansion}$$

↙ **normal strain**

$$\varepsilon_{\text{total}} = \varepsilon_{\sigma} + \varepsilon_T = \frac{\sigma}{E} + \alpha \Delta T$$

- Homogeneous, isotropic materials expand uniformly in all directions when heated. **NO shear strain.**
- $\alpha$  is constant for a large range of temperature.

p. 177

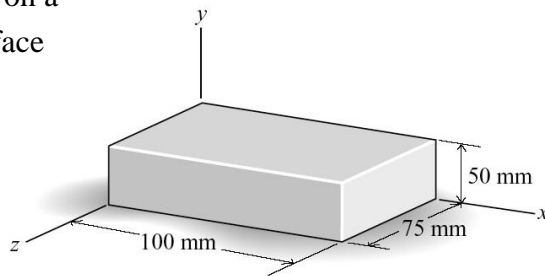


## Example Problem 4-6

- Aluminum block rests on a smooth horizontal surface

- $E = 70 \text{ GPa}$

- $\alpha = 22.5 \times 10^{-6} / ^\circ\text{C}$



Find:

- $\varepsilon_{Tx}, \varepsilon_{Ty} = ?$  for  $\Delta T = +20^\circ\text{C}$
- $\delta_x, \delta_y, \delta_z = ?$
- $\gamma_{xy} = ?$



p. 177

## Example Problem 4-6

Sol: (a) Thermal strain  $\varepsilon_T = \alpha \Delta T$

$$\begin{aligned}\varepsilon_T &= \varepsilon_{Tx} = \varepsilon_{Ty} = \varepsilon_{Tz} = \alpha \Delta T \\ &= 22.5 \cdot 10^{-6} \cdot 20 \\ &= 450 \cdot 10^{-6} \text{ m/m} = 450 \mu\text{m/m}\end{aligned}$$

(b) Deformation  $\delta = \varepsilon_T L = \alpha \cdot \Delta T \cdot L$

$$\begin{aligned}\delta_x &= \varepsilon_{Tx} L_x = 450 \cdot 10^{-6} \cdot 0.1 = 45 \cdot 10^{-6} \text{ m} \\ \delta_y &= \varepsilon_{Ty} L_y = 450 \cdot 10^{-6} \cdot 0.05 = 22.5 \cdot 10^{-6} \text{ m} \\ \delta_z &= \varepsilon_{Tz} L_z = 450 \cdot 10^{-6} \cdot 0.075 = 33.8 \cdot 10^{-6} \text{ m}\end{aligned}$$

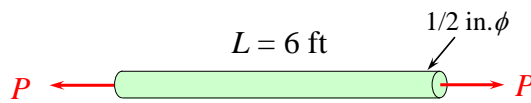
(c) Shearing strain  $\gamma_{xy} = 0$ , since the block remains a rectangular parallelepiped when unconstrained.

p. 178

## Example Problem 4-7

Given

:



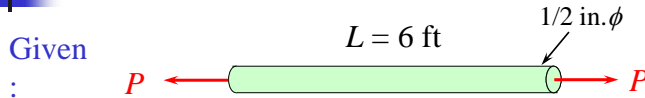
$$\begin{aligned}\text{steel: } E &= 30,000 \text{ ksi, } \alpha = 6.5(10^{-6})/\text{°F} \\ P &= 5000 \text{ lb, } \Delta T = -50 \text{ °F}\end{aligned}$$

Find  $\delta = ?$

$$\text{Sol: } \varepsilon = \frac{\delta}{L} = \frac{\sigma}{E} + \alpha \Delta T = \frac{P}{EA} + \alpha \Delta T$$

$$\begin{aligned}\delta &= \left( \frac{P}{EA} + \alpha \Delta T \right) L = \left[ \frac{5000}{30(10^6)(\pi/4)(1/2)^2} + 6.5(10^{-6})(-50) \right] (6) \\ &= 0.003143 \text{ ft} \approx 0.0377 \text{ in} \quad \delta = 0.0845 \text{ in for } \Delta T = +50 \text{ °F}\end{aligned}$$

## Example Problem 4-7



steel:  $E = 30,000 \text{ ksi}$ ,  $\alpha = 6.5(10^{-6})/^{\circ}\text{F}$

$P = 5000 \text{ lb}$ ,  $\Delta T = -50 \text{ }^{\circ}\text{F}$

Find  $\delta_d = ?$  (change in diameter)

Sol:  $\epsilon_{dP} = -\nu\epsilon_P$ ;  $\delta_{dP} = -\nu\delta_P$

$$\delta_d = \delta_{dP} + \delta_{dT} = -\nu \frac{Pd}{AE} + \alpha d \Delta T \quad \text{Note: multiply by } d$$

$$= -\frac{0.3 \cdot 5000 \cdot 0.5}{(\pi 0.5^2 / 4) \cdot 30 \cdot 10^6} + 6.5 \cdot 10^{-6} \cdot 0.5 \cdot (-50)$$

$$= -0.000290 \text{ in}$$

## 8 Exercises

4-5,            4-6,            4-7,            4-18,  
4-20,          4-32,          4-49,          4-53