Mechanics of Materials

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Chapter 5

Axial Loading Applications and Pressure Vessels

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Axial Loading Applications and Pressure Vessels

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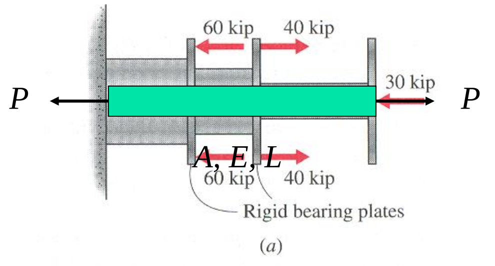
5-2 Deformation of Axially Loaded Members

- Uniform Member

$$\delta = \varepsilon L = \frac{\sigma L}{E} = \frac{PL}{AE}$$



- Multiple Loads/Sizes

$$\delta = \sum_{i=1}^{n} \delta_i = \sum_{i=1}^{n} \frac{\sigma_i L_i}{E_i} = \sum_{i=1}^{n} \frac{P_i L_i}{E_i A_i}$$

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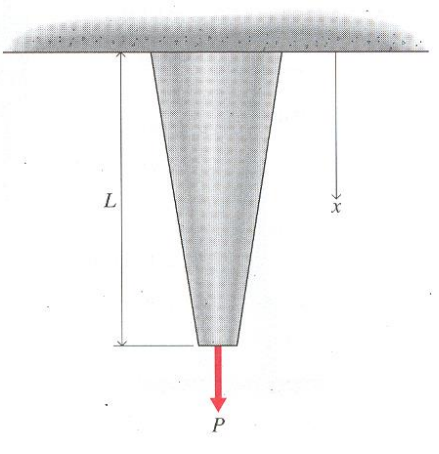
Nonuniform Deformation



$$\varepsilon = \frac{d\delta}{dx}$$

$$d\delta = \varepsilon dx = \frac{\sigma}{E} dx = \frac{P_x}{EA_x} dx$$

$$\delta = \int_0^L d\delta = \int_0^L \frac{P_x}{EA_x} dx$$

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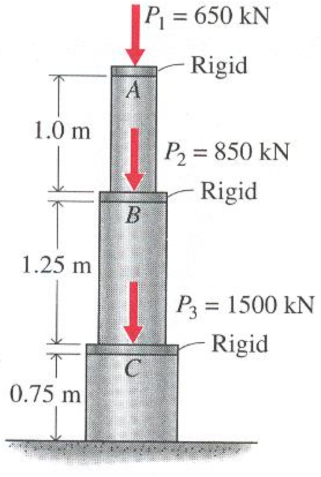
Example Problem 5-1 (I)



- Solid aluminum bar A
  - d = 100mm
  - E = 73 Gpa
- Brass tube B
  - d_o = 150mm, d_i = 100mm
  - E = 100 Gpa
- Steel pipe C
  - d_o = 200mm , d_i = 125mm
  - E = 210 Gpa

- Determine δ_{total} = ?

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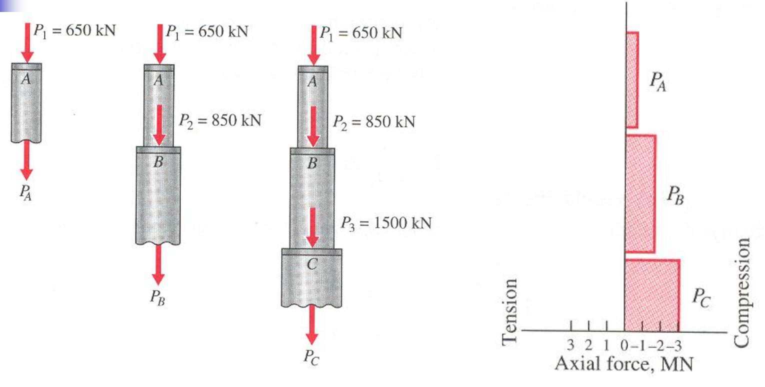
Example Problem 5-1(II)



$$\sum F = -P_A - 650 = 0 \qquad P_A = -650 \text{ kN} = 650 \text{ kN (C)}$$

$$\sum F = -P_B - 650 - 850 = 0 \qquad P_B = -1500 \text{ kN} = 1500 \text{ kN (C)}$$

$$\sum F = -P_c - 650 - 850 - 1500 = 0 \qquad P_C = -3000 \text{ kN} = 3000 \text{ kN (C)}$$

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Example Problem 5-1 (III)

$$A_A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(100)^2 = 7584\text{mm}^2 = 0.007584\text{m}^2$$

$$A_B = \frac{\pi}{4}\left(d_o^2 - d_i^2\right)^2 = \frac{\pi}{4}\left(150^2 - 100^2\right) = 9817\text{mm}^2 = 0.009817\text{m}^2$$

$$A_C = \frac{\pi}{4}\left(d_o^2 - d_i^2\right)^2 = \frac{\pi}{4}\left(200^2 - 125^2\right) = 19{,}144\text{mm}^2 = 0.019144\text{m}^2$$

$$\delta_A = \frac{P_A L_A}{E_A A_A} = \frac{-650\left(10^3\right)(1.0)}{73\left(10^9\right)(0.007854)} = -1.1337\left(10^{-3}\right)\text{m} = -1.1337\text{mm}$$

$$\delta_B = \frac{P_B L_B}{E_B A_B} = \frac{-1500\left(10^3\right)(1.25)}{100\left(10^9\right)(0.009817)} = -1.9100\left(10^{-3}\right)\text{m} = -1.9100\text{mm}$$

$$\delta_C = \frac{P_C L_C}{E_C A_C} = \frac{-3000\left(10^3\right)(0.75)}{210\left(10^9\right)(0.019144)} = -0.5597\left(10^{-3}\right)\text{m} = -0.5597\text{mm}$$

$$\delta_{\text{total}} = \delta_A + \delta_B + \delta_C = -3.6034\text{mm} \cong -3.60\,\text{mm}$$

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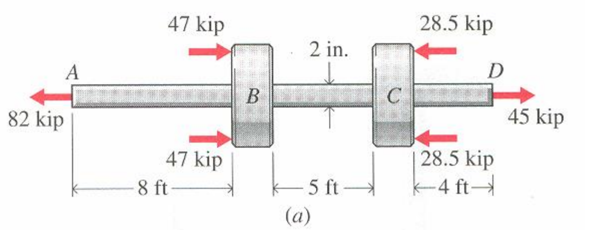
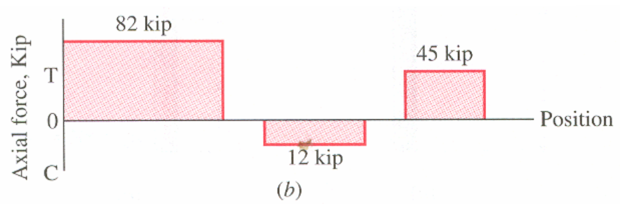
Example Problem 5-2 (I)



(a)



(b)

- Yokes (軛) B and C :
  - rigid
- Steel bar *AD*
  - A = 2 in × 2 in
  - E = 30,000 ksi

Determine

- σ_{max} = ?
- δ_{AB} = ?
- δ_{BC} = ?
- δ_{AD} = ?

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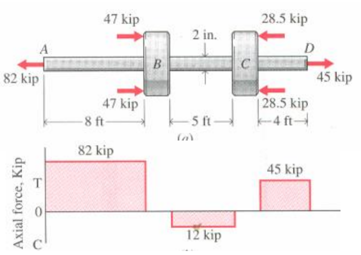
Example Problem 5-2 (II)



$$\sigma_{max} = \frac{P_{max}}{A} = \frac{82}{4} = 20.5 \text{ ksi (T)}$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{E_{AB}A_{AB}} = \frac{+82(8)(12)}{30{,}000(4)} = +0.06560 \text{ in} \cong +0.0656 \text{ in}$$

$$\delta_{BC} = \frac{P_{BC}L_{BC}}{E_{BC}A_{BC}} = \frac{-12(5)(12)}{30{,}000(4)} = -0.00600 \text{ in}$$

$$\delta_{CD} = \frac{P_{CD}L_{CD}}{E_{CD}A_{CD}} = \frac{+45(4)(12)}{30{,}000(4)} = +0.01800 \text{ in}$$

- Determine
  - σ_{max} = ?
  - δ_{AB} = ?
  - δ_{BC} = ?
  - δ_{AD} = ?

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$
$$= +0.06560 - 0.00600 + 0.01800 = +0.07760 \text{in} \cong +0.0776 \text{in}$$

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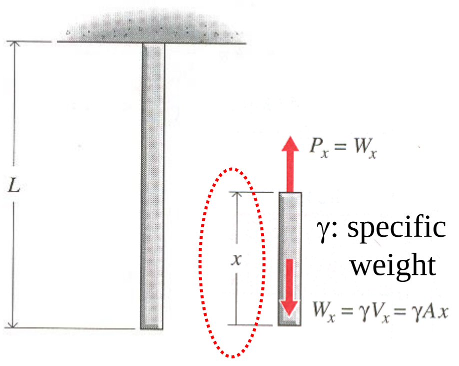
Example Problem 5-3



γ: specific weight

$$\delta = \int_0^L \frac{P_x}{EA_x} dx$$

$$= \int_0^L \frac{1}{EA} \gamma A x \, dx = \frac{\gamma}{E} \int_0^L x \, dx$$

$$= \frac{\gamma x^2}{2E}\bigg|_0^L = \frac{\gamma L^2}{2E}$$

$$= \frac{W}{AL}\left[\frac{L^2}{2E}\right] = \frac{WL}{2AE}$$

- Homogeneous bar
  - *W, L, A, E*
- Determine δ = ?

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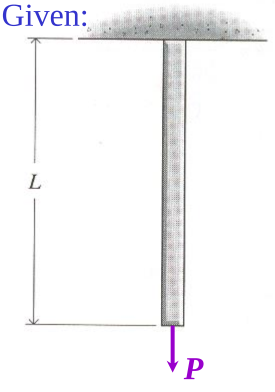
Example Problem 5-3

Given:



Find: (b) δ = ? for an applied *P* at the end

Sol: Method of superposition is applicable for linear problems

$$\delta = \delta_W + \delta_P$$

$$= \frac{WL}{2EA} + \frac{PL}{EA} = \frac{L}{EA}\left(\frac{W}{2} + P\right)$$

For your reference,

$$\boldsymbol{F} = \boldsymbol{kx}$$

$$\boldsymbol{k} = \frac{\boldsymbol{EA}}{\boldsymbol{L}}$$

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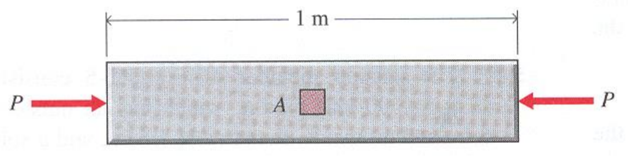
Example Problem 5-4 (I)



=180 kN

- Steel bar
  - E = 200 Gpa
  - *Area* = 30 × 30 mm

Determine

- δ = ?
- σ_x, σ_y, τ_{xy}, on element A = ?

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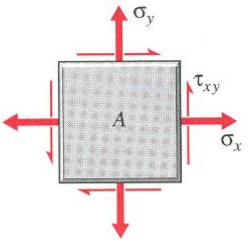
Example Problem 5-4 (II)

$$\delta = \frac{PL}{EA} = \frac{-180(10^3)(1)}{200(10^9)(0.030)^2} = -1.000(10^{-3})\,\text{m} = -1.000\,\text{mm}$$



$$\sigma_x = \frac{P}{A} = \frac{-180(10^3)}{(0.030)^2}$$
$$= -200(10^6)\,\text{N/m}^2 = 200\,\text{MPa}\,(\text{C})$$
$$\sigma_y = 0$$
$$\tau_{xy} = 0$$

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Example Problem 5-4 (III)

The bar is subjected to an axial load.

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{-200 \cdot 10^6}{200 \cdot 10^9} = -0.00100 = -1000\,\mu\text{m/m}$$

$$\varepsilon_y = -\nu\varepsilon_x = -0.3 \cdot (-1000) = 300\,\mu\text{m/m}$$

$$\varepsilon_z = -\nu\varepsilon_x = -0.3 \cdot (-1000) = 300\,\mu\text{m/m}$$

The bar is subjected to an axial load only, no shearing force exists.

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0\,\mu\text{rad}$$

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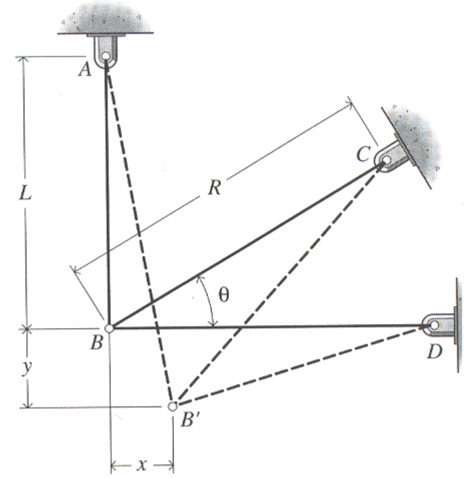
5-3 Deformations in a System of Axially Loaded Bars



- Unknowns:
  - P_{AB}, P_{BC}, P_{BD}
- Equations
  - $\Sigma F_x = 0$
  - $\Sigma F_y = 0$

⇨ statically indeterminate

⇨ Another Eq. is need (compatibility)

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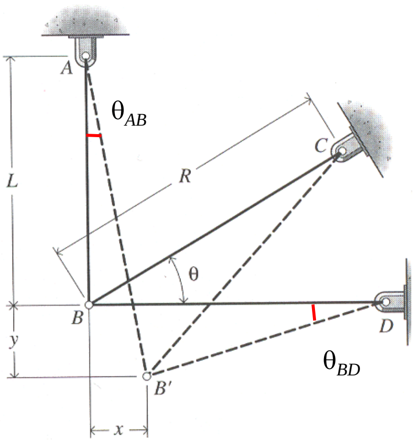
Compatibility of Strains



$$\delta_{AB} = \sqrt{(L+y)^2 + x^2} - L$$

$$\cancel{\delta_{AB}^2} + 2L\delta_{AB} + \cancel{L^2} = \cancel{L^2} + 2Ly + \cancel{y^2} + \cancel{x^2}$$

$$\delta_{AB} \cong y$$

Similarly, $\delta_{BD} \cong x$

$$\theta_{AB} \cong \tan\theta_{AB} = \frac{x}{L_{AB}}$$

$$\theta_{BD} \cong \tan\theta_{BD} = \frac{y}{L_{BD}}$$

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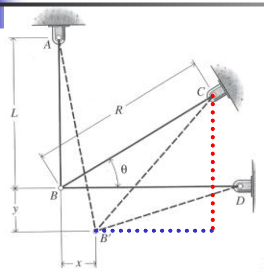
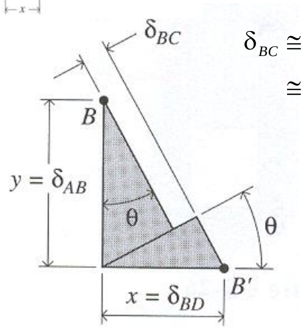
Compatibility of Strains





$$\delta_{BC} = \sqrt{(R\cos\theta - x)^2 + (R\sin\theta + y)^2} - R$$

$$\delta_{BC}^2 + 2R\delta_{BC} + R^2 = R^2\cos^2\theta - 2Rx\cos\theta + x^2 + R^2\sin^2\theta + 2Ry\sin\theta + y^2$$

$$\delta_{BC} \cong y\sin\theta - x\cos\theta \cong \delta_{AB}\sin\theta - \delta_{BD}\cos\theta$$

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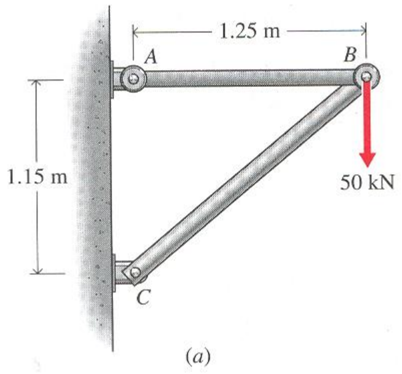
Example Problem 5-5 (I)



(a)

- A_{AB} = 650 mm^2
- A_{BC} = 925 mm^2
- E = 200 GPa

Determine

- σ_{AB} , σ_{BC} = ?
- δ_{AB} , δ_{BC} = ?
- δ_h , δ_v at B = ?
- θ_{AB} , θ_{BC} = ?

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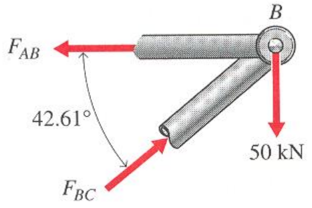
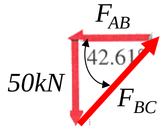
Example Problem 5-5 (II)

- A_{AB} = 650 mm²
- A_{BC} = 925 mm²
- E = 200 GPa



(b)



$$\boldsymbol{F_{BC}} = 50/\sin 42.61° = 73.85 \text{ kN (C)}$$

$$\boldsymbol{F_{AB}} = 50/\tan 42.61° = 54.36 \text{ kN (T)}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{+54.36(10^3)}{650(10^{-6})}$$
$$= +83.63(10^6)\,\text{N/m}^2 \cong 83.6\,\text{MPa (T)}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{-73.85(10^3)}{925(10^{-6})}$$
$$= -79.84(10^6)\,\text{N/m}^2 \cong 79.8\,\text{MPa (C)}$$

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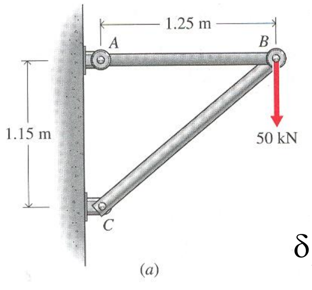
Example Problem 5-5 (III)



(a)

$$\delta_{AB} = \frac{\sigma_{AB} L_{AB}}{E} = \frac{+83.63(10^6)(1.25)}{200(10^9)}$$
$$= +0.5227(10^{-3})\,\text{m} \cong +0.523\text{mm}$$

$$\delta_{BC} = \frac{\sigma_{BC} L_{BC}}{E} = \frac{-79.84(10^6)(1.699)}{200(10^9)}$$
$$= -0.6782(10^{-3})\,\text{m} \cong -0.678\text{mm}$$

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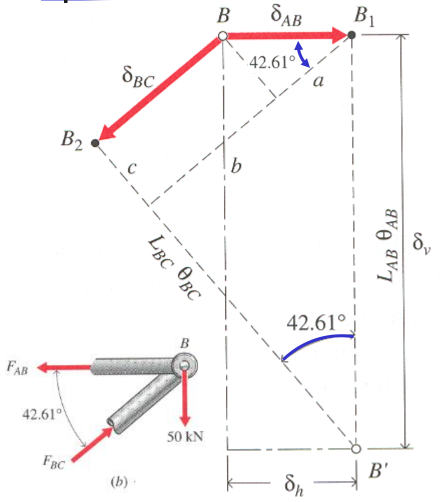
Example Problem 5-5 (IV)



$$\delta_h = \delta_{AB} = +0.5227\text{mm} \cong 0.523\text{mm}$$

$$a = \delta_{AB}\cos 42.61° = 0.5227\cos 42.61°$$

$$= 0.3847\text{mm}$$

$$\sin 42.61° = \frac{b+a}{\delta_v} = \frac{\delta_{BC}+a}{\delta_v}$$

$$= \frac{0.6782+0.3847}{\delta_v} = \frac{1.0629}{\delta_v}$$

$$\delta_v = \frac{1.0629}{\sin 42.61°} = 1.5700\text{mm}$$

$$\cong 1.570\text{mm}$$

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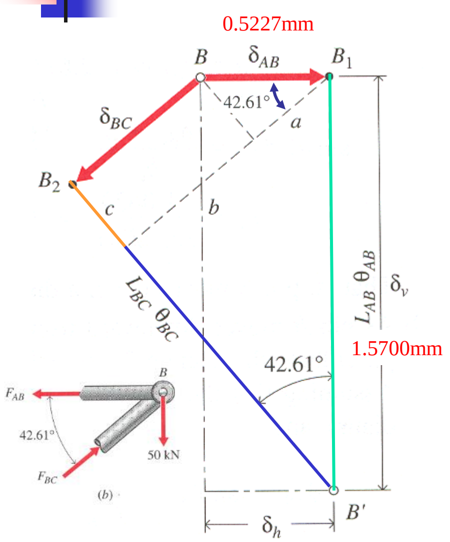
Example Problem 5-5 (V)



$$\theta_{AB} \cong \tan\theta_{AB} = \frac{\delta_v}{L_{AB}} = \frac{1.570}{1250}$$

$$= 0.001256\text{rad} \cong 0.0720°$$

$$\theta_{BC} \cong \tan\theta_{BC} = \frac{c+\delta_v\cos 42.61°}{L_{BC}}$$

$$= \frac{0.5227\sin 42.61° + 1.5700\cos 42.61°}{\sqrt{1150^2+1250^2}}$$

$$= 0.000889\text{rad} \cong 0.0509°$$

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5-4 Statically Indeterminate Axially Loaded Members

- Statically determinate (靜定)

  Equilibrium eqs. = unknowns (no. of member force + reactions)

- Statically indeterminate (靜不定)

  No. of equilibrium eqs. < unknowns (no. of member force + reactions)

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5-4 Statically Indeterminate Axially Loaded Members

- Procedure
  - Draw a free-body diagram.
  - Note the number of unknowns involved.
  - Note the number of independent equations.
  - If no. unknowns > no. equil. eqs. $\Rightarrow$ stat. indeterminate
    $\Rightarrow$ Write deformation equations.
  - Solve equilibrium (and deformation if needed) equations.

- Assumption
  - The body is rigid when solving the equilibrium equations

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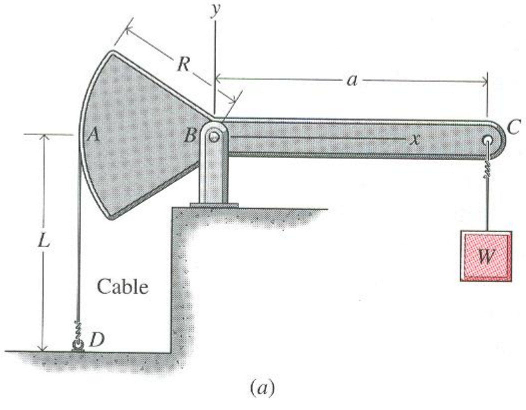
Example



- Condition 1
  Cable rigid
- Condition 2
  Cable deformable

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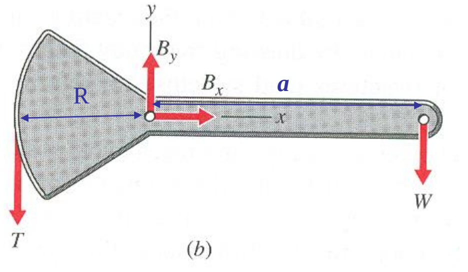
Condition 1 – Cable Rigid



$$\sum M_B = TR - Wa = 0$$

$$T = \frac{Wa}{R}$$

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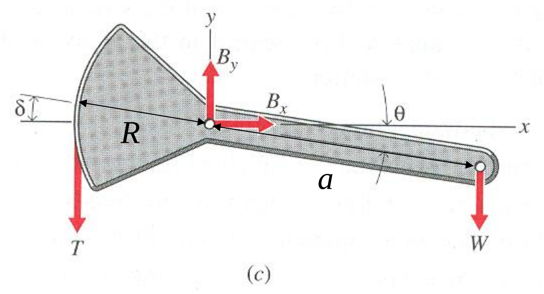
Condition 2 - Cable Deformable



$$\delta = \frac{TL}{EA}$$

$$\frac{\delta EA}{L} = T = \frac{Wa}{R}\cos\theta$$

$$\delta = R\theta$$

$$\sum M_B = TR - Wa\cos\theta = 0$$

$$T = \frac{Wa}{R}\cos\theta$$

$$R^2 EA\theta = WaL\cos\theta$$

Solved by iterations

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Example – W = 100 lb, a = 30 in, R = 15 in

Case	θ (°)	T (lb)	%D = 100 % × (T_{rigid}-T)/T
rigid cable	0	200	0
steel cable d = 3/32 in. E = 29,000 ksi	0.1717	199.999	0.0005%
aluminum cable d = 3/32 in. E = 10,600 ksi	0.4698	199.993	0.0035%

error acceptable, cable can be assumed rigid

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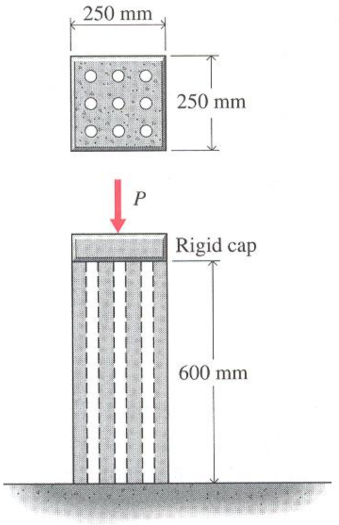
Example Problem 5-7 (I)



- Concrete pier
  - E = 30 GPa
- Steel bars (25mm ϕ)
  - E = 200 GPa
- P = 650 kN

Determine

- σ_C (concrete) = ?
- σ_R (steel) = ?
- δ (pier) = ?

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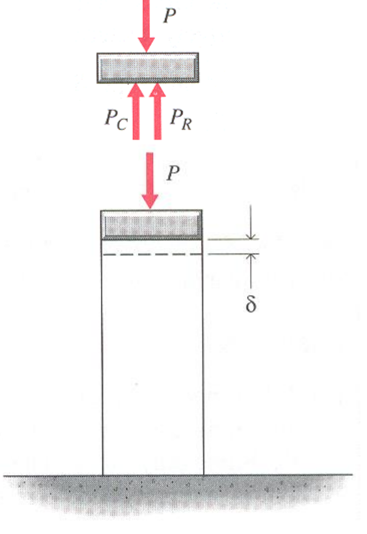
Example Problem 5-7 (II)



$$\sum F_y = P_R + P_C - P = 0$$ equilibrium

$$\delta_R = \delta_C \Rightarrow \frac{P_R L_R}{E_R A_R} = \frac{P_C L_C}{E_C A_C}$$ deformation

$$A_R = 9\left(\pi/4\right)(25)^2 = 4418\text{mm}^2$$

$$A_C = (250)^2 - 4418 = 58{,}080\text{mm}^2$$

$$\frac{P_R(0.600)}{200(10^9)(4418)(10^{-6})} = \frac{P_C(0.600)}{30(10^9)(58{,}080)(10^{-6})}$$

$$P_R = 0.5071 P_C$$

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Example Problem 5-7 (III)

$$\begin{cases} P_R + P_C - 650(10^3) = 0 \\ P_R = 0.5071 P_C \end{cases} \Rightarrow \begin{matrix} P_R = 218.7(10^3)\,\text{N} \cong 219\,\text{kN (C)} \\ P_C = 431.3(10^3)\,\text{N} \cong 431\,\text{kN (C)} \end{matrix}$$

$$\Rightarrow \quad \sigma_R = \frac{P_R}{A_R} = \frac{218.7(10^3)}{4418(10^{-6})} \cong 49.5\,\text{MPa (C)}$$

$$\sigma_C = \frac{P_R}{A_R} = \frac{431.3(10^3)}{58{,}080(10^{-6})} \cong 7.43\,\text{MPa (C)}$$

$$\delta = \delta_C = \delta_R = \frac{\sigma_R L_R}{E_R} = \frac{49.5(10^6)(0.600)}{200(10^9)} \cong 0.1485\text{mm}$$

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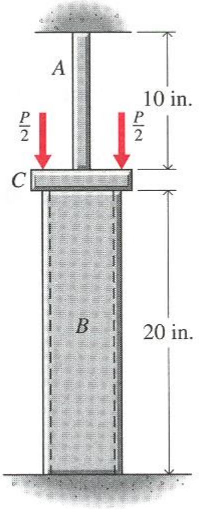
Example Problem 5-8 (I)



- Plate C : rigid
- Steel Rod A
  - E = 30,000 ksi
  - A = 0.800 in^2
- Aluminum alloy pipe B
  - E = 10,000 ksi
  - A = 3.00 in^2
- P = 20 kip

Determine:

- σ_A = ?
- σ_B = ?
- δ_C = ?

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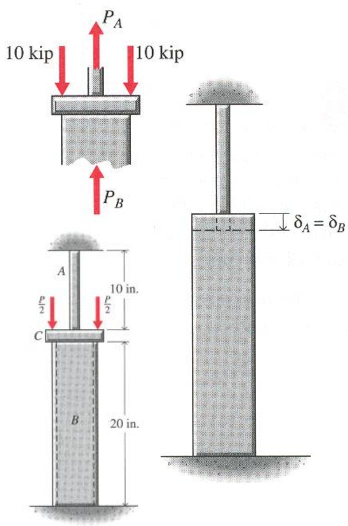
Example Problem 5-8 (II)

- Steel Rod *A*
  - $E = 30{,}000$ ksi
  - $A = 0.800\ \text{in}^2$
- Aluminum alloy pipe *B*
  - $E = 10{,}000$ ksi
  - $A = 3.00\ \text{in}^2$



$$\sum F_y = P_A + P_B - 20 = 0$$ equilibrium

$$\Rightarrow \quad 0.800\sigma_A + 3.00\sigma_B = 20$$

$$\delta_A = \delta_B \quad \Rightarrow \quad \frac{\sigma_A L_A}{E_R} = \frac{\sigma_B L_B}{E_B}$$ deformation

$$\frac{\sigma_A(10)}{30{,}000} = \frac{\sigma_B(20)}{10{,}000}$$

$$\Rightarrow \quad \sigma_A = 6\sigma_B$$

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Example Problem 5-8 (III)

$$\begin{cases} 0.800\sigma_A + 3.00\sigma_B = 20 \\ \sigma_A = 6\sigma_B \end{cases}$$

$$\Rightarrow \quad \begin{aligned} \sigma_A &= 15.384\,\text{ksi} \cong 15.38\,\text{ksi (T)} \\ \sigma_B &= 2.564\,\text{ksi} \cong 2.56\,\text{ksi (C)} \end{aligned}$$

$$\delta = \delta_A = \delta_B = \frac{\sigma_A L_A}{E_A} = \frac{15.384(10)}{30{,}000} \cong 0.00513\text{in}\ (\downarrow)$$

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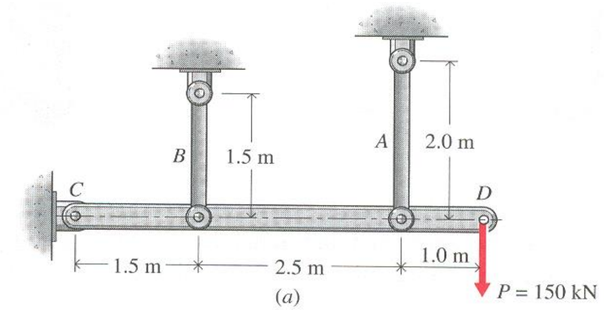
Example Problem 5-9 (I)



(a)

- Member *CD* : rigid
- Aluminum alloy bar *A*
  - $E = 75$ GPa
  - $A = 1000$ mm^2
- Steel bar *B*
  - $E = 200$ GPa
  - $A = 500$ mm^2

Determine:

- $\sigma_A = ?$
- $\sigma_B = ?$
- $\delta_D = ?$

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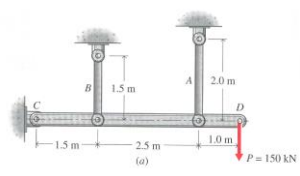
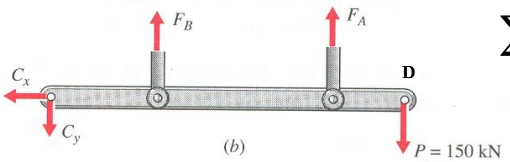
Example Problem 5-9 (II)

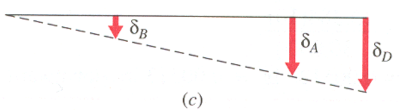




(b)

$$\sum M_C = -5P + 4F_A + 1.5F_B = 0$$

$$\Rightarrow \quad 4F_A + 1.5F_B = 750\left(10^3\right)$$



(c)

$$\frac{\delta_A}{4} = \frac{\delta_B}{1.5} \Rightarrow \frac{F_A L_A}{4E_A A_A} = \frac{F_B L_B}{1.5E_B A_B}$$

$$\frac{F_A(2)}{4(75)\left(10^9\right)(1000)\left(10^{-6}\right)} = \frac{F_B(1.5)}{1.5(200)\left(10^9\right)(500)\left(10^{-6}\right)} \quad \Rightarrow \quad F_A = 1.5F_B$$

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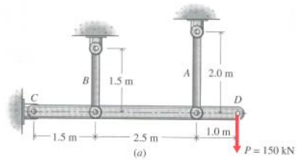
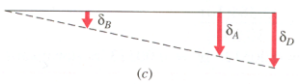
Example Problem 5-9 (III)



$$\begin{cases} 4F_A + 1.5F_B = 750\left(10^3\right) \\ F_A = 1.5F_B \end{cases} \Rightarrow \begin{aligned} F_A &= 150.0\left(10^3\right)\text{N} = 150.0\text{ kN} \\ F_B &= 100.0\left(10^3\right)\text{N} = 100.0\text{ kN} \end{aligned}$$

$$\Rightarrow \quad \sigma_A = \frac{F_A}{A_A} = \frac{150.0\left(10^3\right)}{1000\left(10^{-6}\right)} = 150.0\text{ MPa (T)}$$

$$\sigma_B = \frac{F_B}{A_B} = \frac{100.0\left(10^3\right)}{500\left(10^{-6}\right)} = 200\text{ MPa (T)}$$



$$\delta_D = \frac{5}{4}\delta_A = \frac{5\sigma_A L_A}{4E_A} = \frac{5(150.0)\left(10^6\right)(2)}{4(75)} = 5.000\left(10^{-3}\right)\text{m} = 5.00\text{ mm}(\downarrow)$$

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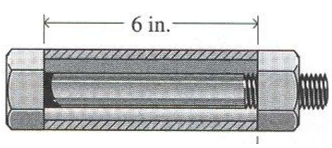
Example Problem 5-10 (I)



- Alloy-steel bolt
  - E = 30,000 ksi
  - d = 1/2 in
- Cold-rolled brass sleeve
  - E = 15,000 ksi
  - A = 0.375 in^2
- Tightening nut ¼ turn (0.02 in.)

Determine:

- σ_B = ?
- σ_S = ?

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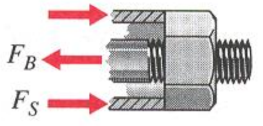
Example Problem 5-10 (II)



- Alloy-steel bolt
  - d = 1/2 in
- Cold-rolled brass sleeve
  - A = 0.375 in^2

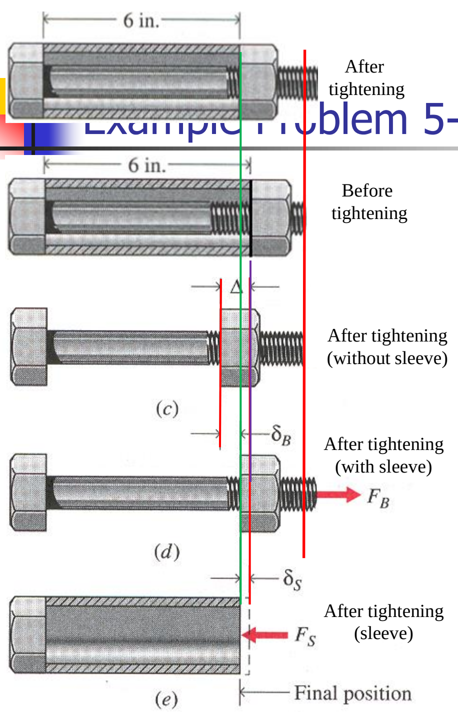
$$\sum F_x = F_S - F_B = 0$$

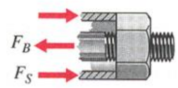
⇨ $$0.375\sigma_S = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 \sigma_B$$

➡ $$\sigma_S = 0.523\sigma_B$$

p.

Example Problem 5-10 (III)





$$\sigma_S = 0.523\sigma_B$$

$$\delta_S + \delta_B = \Delta$$ deformation

⇨ $$\frac{\sigma_B L_B}{E_B} + \frac{\sigma_S L_S}{E_S} = \Delta$$

$$\frac{\sigma_B(6)}{30{,}000} + \frac{\sigma_S(6)}{15{,}000} = 0.020$$

⇨ $$\sigma_B + 2\sigma_S = 100$$

➡ $$\sigma_S = 0.523\sigma_B$$

⇨ $$\sigma_B = 48.84\,\text{ksi} \cong 48.8\,\text{ksi (T)}$$

$$\sigma_S = 25.58\,\text{ksi} \cong 25.6\,\text{ksi (C)}$$

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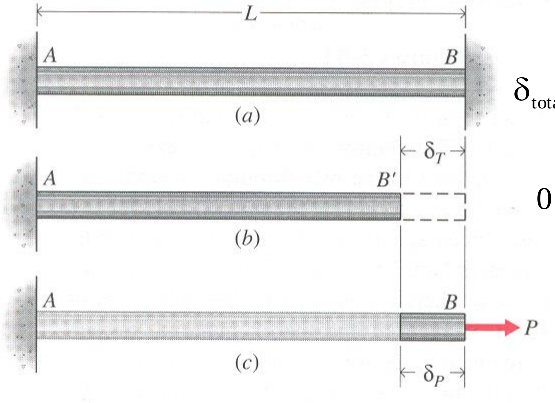
5-5 Thermal Effects



for fixed ends.

$$\delta_{\text{total}} = \delta_T + \delta_\sigma = \varepsilon_T L + \varepsilon_\sigma L$$

$$0 = \alpha\,\Delta T\,L + \frac{\sigma}{E}L$$

↑ Thermal induced stress

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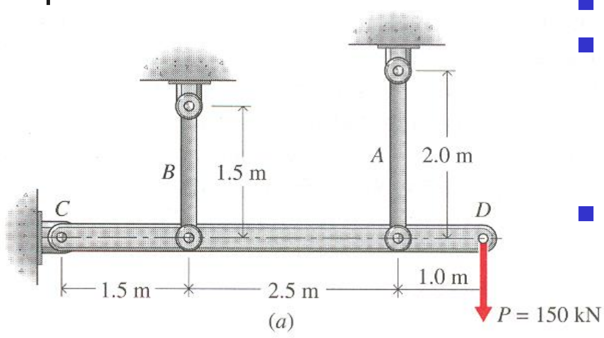
Example Problem 5-12 (I)



- Member *CD* : rigid
- Aluminum alloy bar *A*
  - E = 75 GPa
  - A = 1000 mm^2
  - $\alpha = 22(10^{-6})/℃$
- Steel bar *B*
  - E = 200 GPa
  - A = 500 mm^2
  - $\alpha = 12(10^{-6})/℃$

ΔT = 100℃

Determine

- σ_A , σ_B = ?
- δ_D = ?

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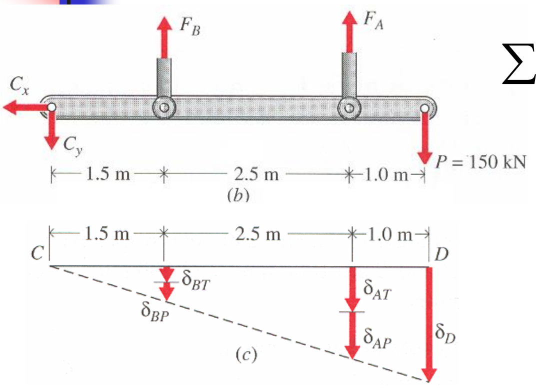
Example Problem 5-12 (II)



$$\sum \boldsymbol{M_C} = -5\boldsymbol{P} + 4\boldsymbol{F_A} + 1.5\boldsymbol{F_B} = 0$$

$$\Rightarrow \quad 4F_A + 1.5F_B = 750(10^3)$$

$$\frac{\delta_A}{4} = \frac{\delta_B}{1.5}$$

$$\delta_A = \delta_{AP} + \delta_{AT}$$

$$\delta_B = \delta_{BP} + \delta_{BT}$$

$$\frac{F_A L_A}{4E_A A_A} + \frac{\alpha_A L_A \Delta T}{4} = \frac{F_B L_B}{1.5E_B A_B} + \frac{\alpha_B L_B \Delta T}{1.5}$$

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Example Problem 5-12 (III)

$$\frac{F_A L_A}{4E_A A_A} + \frac{\alpha_A L_A \Delta T}{4} = \frac{F_B L_B}{1.5E_B A_B} + \frac{\alpha_B L_B \Delta T}{1.5}$$

$$\Rightarrow \quad \frac{F_A(2.0)}{4(75)(10^9)(1000)(10^{-6})} + \frac{22(10^{-6})(2.0)(100)}{4}$$

$$= \frac{F_B(1.5)}{1.5(200)(10^9)(500)(10^{-6})} + \frac{12(10^{-6})(1.5)(100)}{1.5}$$

$$\Rightarrow \quad F_A = 1.5F_B + 15(10^3)$$

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Example Problem 5-12 (IV)

$$4F_A + 1.5F_B = 750(10^3)$$
$$F_A = 1.5F_B + 15(10^3)$$

⇨

$$F_A = 153.00(10^3)\,\text{N} = 153.00\ \text{kN}$$
$$F_B = 92.00(10^3)\,\text{N} = 92.00\ \text{kN}$$

⇨

$$\sigma_A = \frac{F_A}{A_A} = \frac{153.00(10^3)}{1000(10^{-6})} = 153.0\ \text{MPa (T)}$$
$$\sigma_B = \frac{F_B}{A_B} = \frac{92.00(10^3)}{500(10^{-6})} = 184.0\ \text{MPa (T)}$$

$$\boldsymbol{\delta_D} = \frac{5}{4}\boldsymbol{\delta_A} = \frac{5}{4}[\frac{\boldsymbol{\sigma_A L_A}}{\boldsymbol{E_A}} + \boldsymbol{\alpha_A L_A \Delta T}] = \frac{5}{4}[\frac{(150.0)(10^6)(2.0)}{(75)} + (22)(10^{-6})(2.0)(100)]$$
$$= 10.60(10^{-3})\ \text{m} = 10.60\ \text{mm}\ (\downarrow)$$

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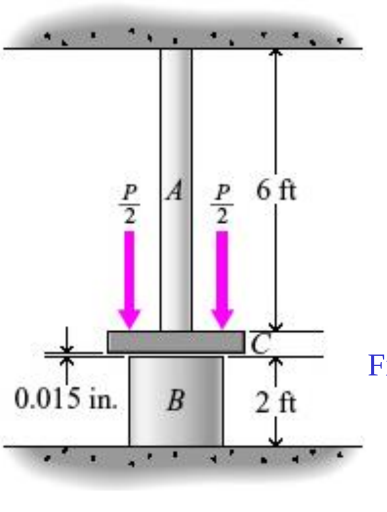
Example Problem 5-13



Given:

- Steel rod A
  - E_A = 30,000 ksi
  - A_A = 2.50 in^2
  - $\alpha_A = 6.6(10^{-6})$/°F
- Member C : rigid
- Bronze bar B
  - E_B = 15,000 ksi
  - A_B = 3.75 in^2
  - $\alpha_B = 9.4(10^{-6})$/°F
- δ_{BC} = 0.015 in.
- P = 5kip

Find:

σ_A , σ_B as a function of temperature increase for 0°F < ΔT < 50°F

Please do by yourself!!

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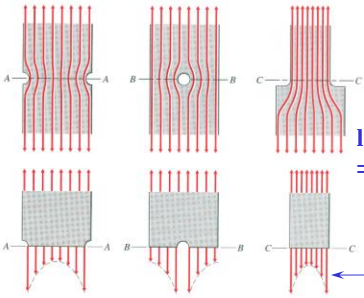
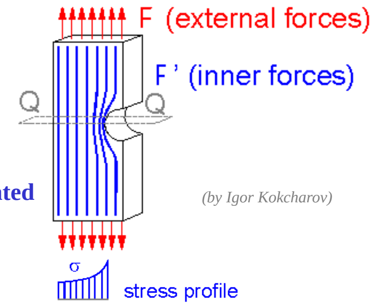
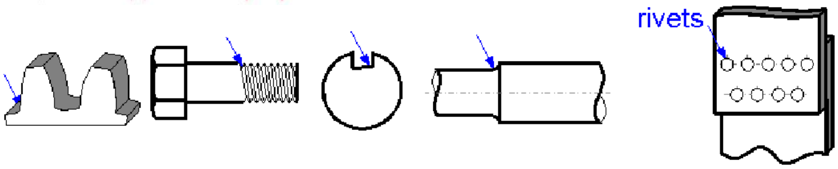
5-6 Stress Concentrations

- Discontinuity that interrupts the stress path (*stress trajectory*)
  → Stress concentration

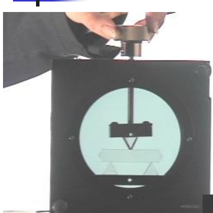




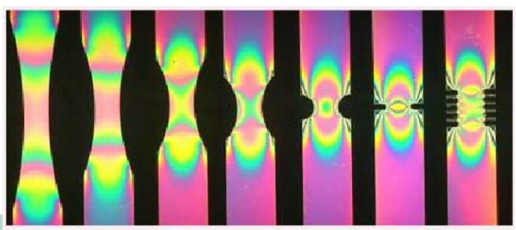
(by Igor Kokcharov)



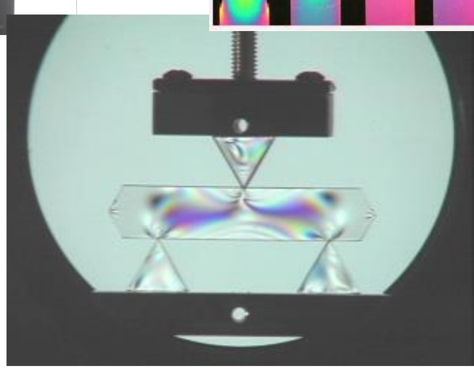
Stress Concentrations by Photoelastic experiments



www.physics.montana.edu



www.ndt.net/article/v08n09/asundi/fig15.jpg



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Stress Concentration factor

- Stress Concentration Factor K

$$\sigma = K\frac{P}{A}$$

maximum stress (σ) — nominal stress (P/A)

- Ascertaining K is based on

  $A = A_t$ (net section) for $K = K_t$

  $A = A_g$ (gross section) for $K = K_g$

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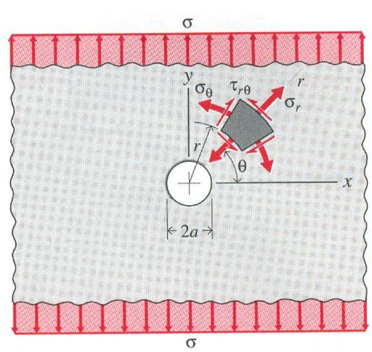
A wide plate under uniform unidirectional tension

From elasticity theory:

$$\sigma_r = \frac{\sigma}{2}\left(1-\frac{a^2}{r^2}\right)-\frac{\sigma}{2}\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{\sigma}{2}\left(1+\frac{a^2}{r^2}\right)+\frac{\sigma}{2}\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\tau_{r\theta} = \frac{\sigma}{2}\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta$$



At $r = a$

$$\sigma_r = 0$$

$$\sigma_\theta = \sigma(1+2\cos 2\theta)$$

$$\tau_{r\theta} = 0$$

⇨ ***at $\theta = 0$,***
max. $\sigma = 3\sigma$

⇨ $\sigma = K\frac{P}{A}$

***K* = 3**

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A wide plate under uniform unidirectional tension (cont.)

- At $r = 3a$ *(1 diameter away from hole)*, $\sigma_\theta = 1.074\sigma$
  $\Rightarrow$ rapid decay
- Stress concentration factor is **significant** for
  - **brittle** material under **static** loading
  - any material under **impact** or **repeated loading**
- **Saint-Venant's principal**

In the regions of support and load application, **the stress distribution varies from the nominal value**. However, these localized effects disappear at a short distance from such locations.

"... the difference between the effects of two different but statically equivalent load becomes very small at sufficiently large distances from load." (From Wikipedia)

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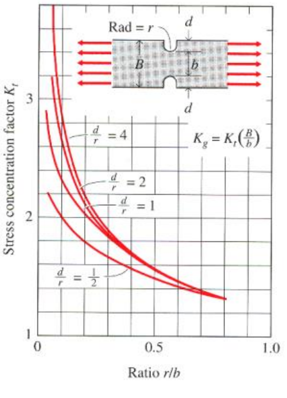
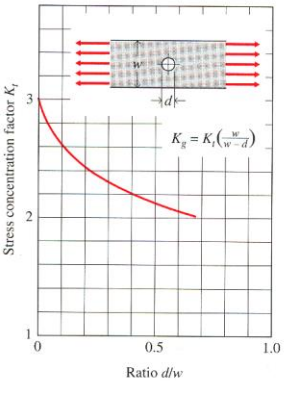
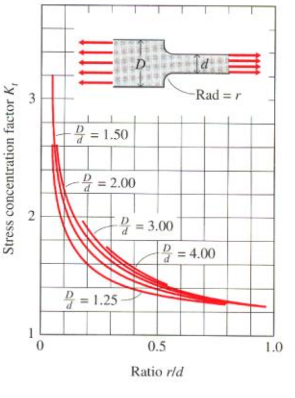
Stress Concentration factors for grooves, holes and fillets







強化玻璃怎麼打也打不破？流言追追追-【實驗精華片段】

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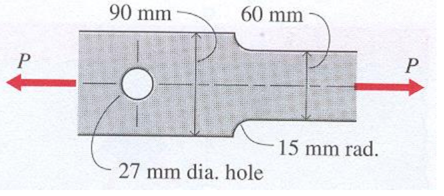
Example Problem 5-14 (I)



- 0.4 percent carbon hot-rolled steel
  - Yield strength = 360 MPa (from Appendix B, A43)
- Thickness = 20mm
- Factor of safety = 2.5 ⇨ $\sigma_{all} = 360/2.5 = 144\,\text{MPa}$
- Determine P_{max} = ?

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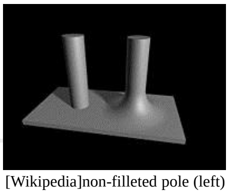
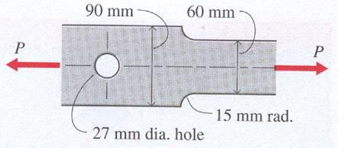
Example Problem 5-14 (II)



[Wikipedia]non-filleted pole (left) and a filleted pole (right)



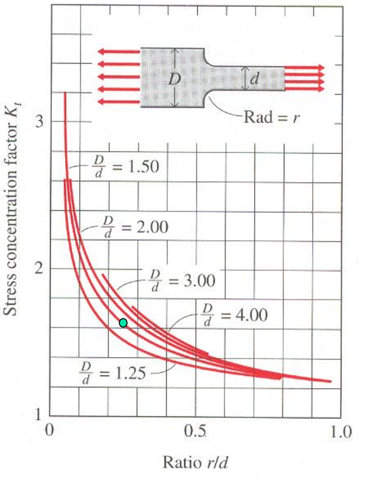
- Fillet

$$\frac{D}{d} = \frac{90}{60} = 1.5 \qquad \frac{r}{d} = \frac{15}{60} = 0.25$$

$$K_t = 1.62$$

$$P = \frac{\sigma_{all} A_t}{K_t} = \frac{144(10^6)(60)(20)(10^{-6})}{1.62}$$

$$\cong 106.7\ \text{kN}$$



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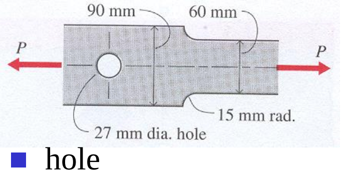
Example Problem 5-14 (III)



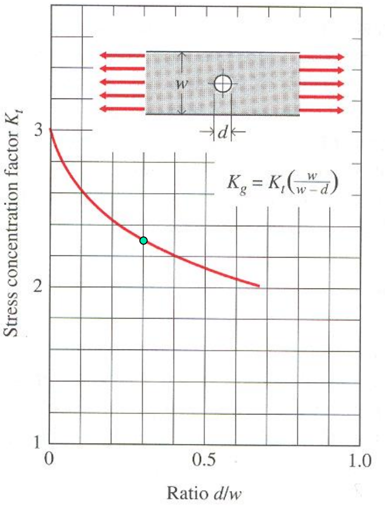
- hole

$$\frac{d}{w}=\frac{27}{90}=0.3 \qquad K_t=2.30$$

$$P=\frac{\sigma_{all}A_t}{K_t}=\frac{144(10^6)(90-27)(20)(10^{-6})}{2.30}$$

$$\cong 78.9\ \text{kN} \longleftarrow P_{max}$$

$$P=\frac{\sigma_{all}A_t}{K_t}\cong 106.7\ \text{kN}$$



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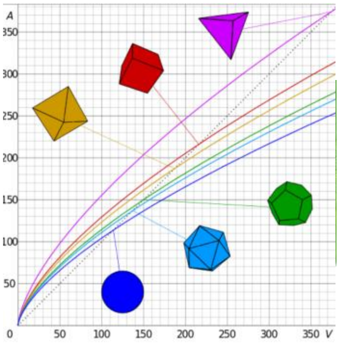
5-9 Thin-Walled Pressure Vessels

- Boilers
- Gas storage tanks
- Pipelines
- Metal tires

Why spherical shape?



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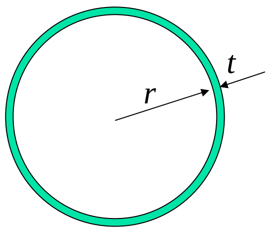
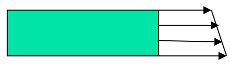
5-9 Thin-Walled Pressure Vessels

- Boilers
- Gas storage tanks
- Pipelines
- Metal tires



- If $t/r << 1$



almost uniform

- If $t/r < 0.1$, $\sigma_{max} < 1.05\sigma_{avg}$

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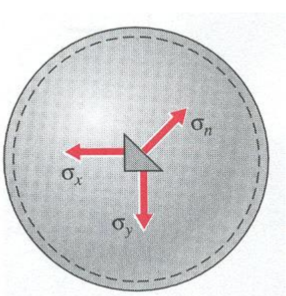
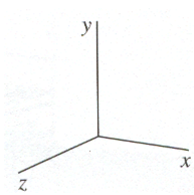
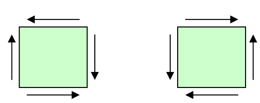
Spherical Pressure Vessels





- Weights of gas and vessel negligible
- symmetry $\sigma_x = \sigma_y = \sigma_n, \quad \tau_{nt} = 0$



∵ symmetry

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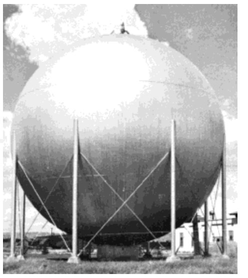
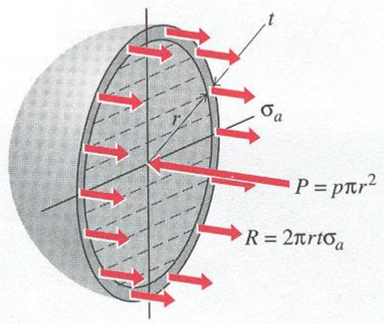
Spherical Pressure Vessels





$$R - P = 0$$

$$2\pi r t \sigma_a = p \pi r^2$$

$$\sigma_a = \frac{pr}{2t}$$

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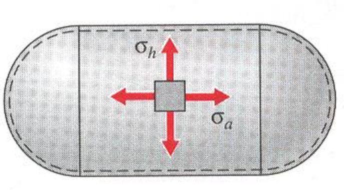
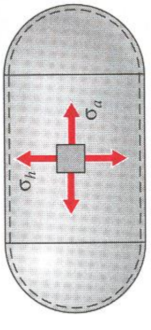
Cylindrical Pressure Vessels





- Meridional (子午線的) or axial stress

  σ_m or σ_a

hoop(箍), tangential, or circumferential stress

(σ_h, σ_t or σ_c)

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Cylindrical Pressure Vessels

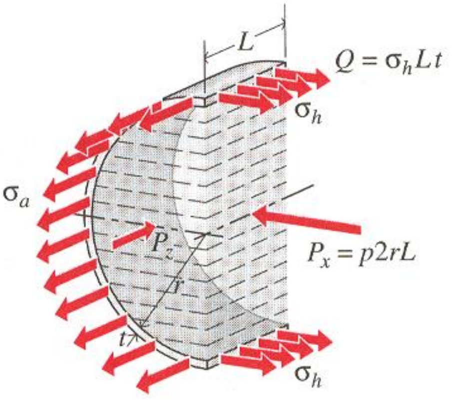
- From FBD of hemisphere

$$\sigma_a = \frac{pr}{2t}$$

- From FBD of a cylindrical section

$$P_x - 2Q = 0$$
$$p2rL = 2\sigma_h Lt$$

$$\sigma_h = \frac{pr}{t} (= 2\sigma_a)$$



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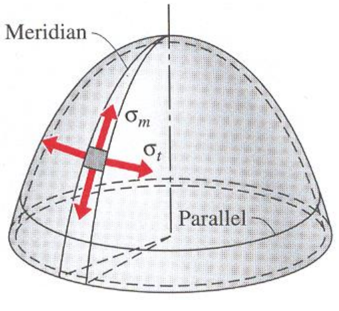
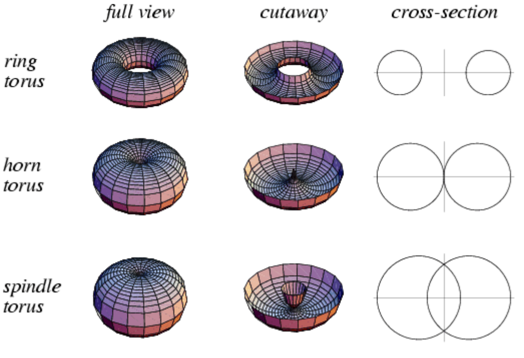
Thin Shells of Revolution

- Thin shell of revolution

  generated by rotating a plane curve, called the meridian, about an axis lying in the plane of the curve

  - sphere, hemisphere, torus, cylinder, cone, ellipsoid





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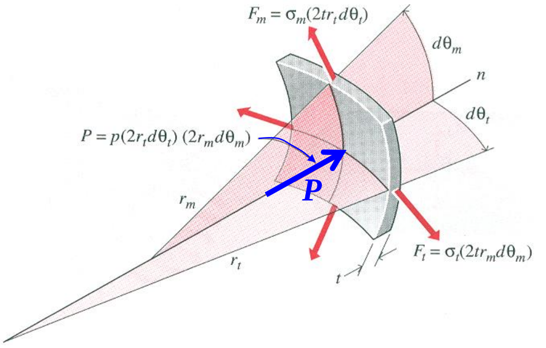
Thin Shells of Revolution

- equilibrium in the *n*-direction



$$P = 2F_m \sin d\theta_m + 2F_t \sin d\theta_t$$

$$p(2r_t d\theta_t)(2r_m d\theta_m) = 2\sigma_m(2tr_t d\theta_t)\sin d\theta_m + 2\sigma_t(2tr_m d\theta_m)\sin d\theta_t$$

$$\boxed{\frac{\sigma_m}{r_m} + \frac{\sigma_t}{r_t} = \frac{p}{t}}$$

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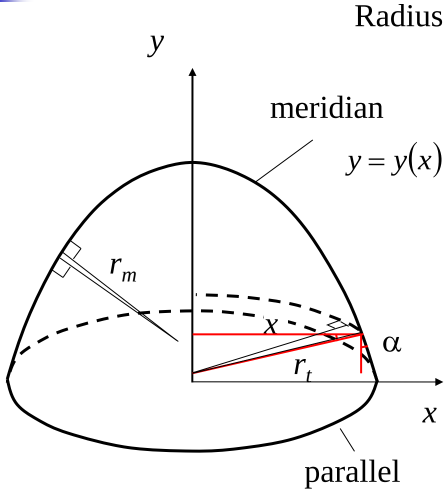
Thin Shells of Revolution

Radius of curvature for $y = y(x)$?



$$r_m = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1.5}}{\frac{d^2y}{dx^2}}$$

$$r_t = \frac{x}{\cos\alpha}$$

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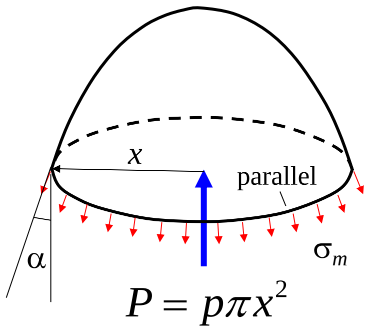
Thin Shells of Revolution

- equilibrium of a portion of the vessel above a paraboloid



$$P = p\pi x^2$$

$$R - P = 0$$

$$p\pi x^2 = 2\pi x t \sigma_m \cos\alpha$$

$$\sigma_m = \frac{px}{2t\cos\alpha}$$

Subst. into $\frac{\sigma_m}{r_m} + \frac{\sigma_t}{r_t} = \frac{p}{t}$ ⇒ σ_t

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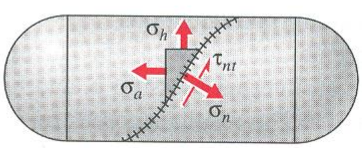
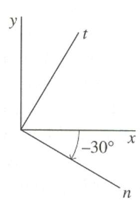
Example Problem 5-16 (I)





- d_i = 1.50 m
- t = 15 mm
- P = 1500 kPa

Determine

- σ_n , τ_{nt} = ? along the weld

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Example Problem 5-16 (II)

- d_i = 1.50 m
- t = 15 mm
- P = 1500 kPa

$$\sigma_h = \frac{pr}{t} = \frac{1500(10^3)(0.75)}{0.015} = 75.0(10^6)\,\text{N/m}^2 = 75.0\,\text{MPa}$$

$$\sigma_a = \frac{pr}{2t} = \frac{1500(10^3)(0.75)}{2(0.015)} = 37.5(10^6)\,\text{N/m}^2 = 37.5\,\text{MPa}$$

$$\begin{aligned}\sigma_n &= \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \\ &= 37.5\cos^2(-30°) + 75.0\sin^2(-30°) \cong 46.9\,\text{MPa}\end{aligned}$$

$$\begin{aligned}\tau_{nt} &= -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) \\ &= -(37.5 - 75.0)\sin(-30°)\cos(-30°) \cong -16.24\,\text{MPa}\end{aligned}$$

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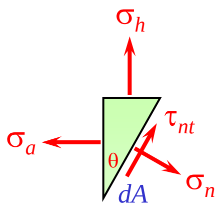
Example Problem 5-16 (III)

Alternatively,



$$\sum F_n = 0: \quad \sigma_n dA - \sigma_a dA\cos\theta\cos\theta - \sigma_h dA\sin\theta\sin\theta = 0$$

$$\Rightarrow \quad \sigma_n = 46.88\,\text{MPa} = 46.9\,\text{MPa}$$

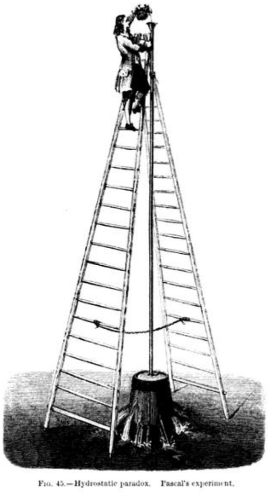
$$\sum F_t = 0: \quad \tau_{nt} dA - \sigma_a dA\cos\theta\sin\theta + \sigma_h dA\sin\theta\cos\theta = 0$$

$$\Rightarrow \quad \tau_{nt} = -16.238\,\text{MPa} = -16.24\,\text{MPa}$$

Units of Pressure

- 1 atm = 760 torr
- The SI unit for pressure is the **Pascal** (**Pa**), **1 Pa = 1** N/m^2 or $kg \cdot m^{-1} \cdot s^{-2}$ (**one** **Newton per square metre**).
- 1 bar (巴**)** = 10^5 Pa = 14.504 lb_f/in^2.
- 1 atm = 101325 N/m^2 ~ 10^5 Pa ~ 10^4 mm H_2O
- Some meteorologists prefer the **hecto**pascal (**hPa; hecto : 10**^{2}) for atmospheric air pressure, which is equivalent to the older unit millibar (**mbar** 毫巴).

Blaise Pascal (19 June 1623 – 19 August 1662), a French mathematician, physicist, inventor, writer and Christian philosopher.



From Wikipedia, the free encyclopedia

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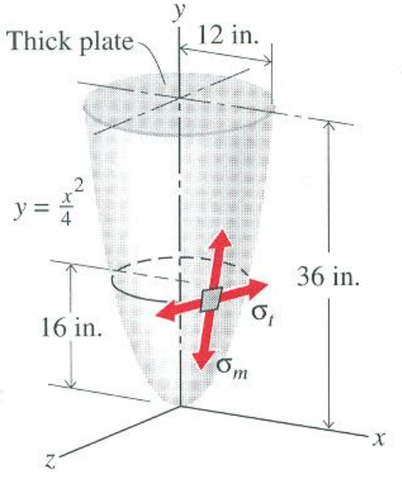
Example Problem 5-17 (I)



A pressure vessel

- t = 1/4 in.
- $y = x^2/4$
- p = 250 psi

Determine

- σ_m , σ_t = ? at y = 16 in.

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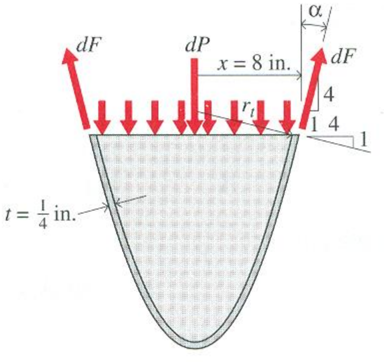
Example Problem 5-17 (II)

- t = 1/4 in.
- $y = x^2/4$
- p = 250 psi



- At y = 16 in.

$$x = \sqrt{4y} = \sqrt{64} = 8 \text{ in.}$$

$$\cot\alpha = \frac{dy}{dx} = \left.\frac{x}{2}\right|_{x=8} = 4 \Rightarrow \cos\alpha = \frac{4}{\sqrt{17}}$$

$$\int_{A_p} dP = \int_{A_p} dF\cos\alpha$$

$$p\pi x^2 = 2\pi x t\sigma_m \cos\alpha$$

$$\sigma_m = \frac{250(8)}{2(1/4)\left(4/\sqrt{17}\right)} \cong 4120\text{psi (T)}$$

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Example Problem 5-17 (III)

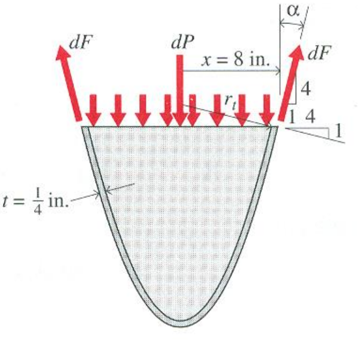
- At y = 16 in. $\quad y = \frac{x^2}{4} \Rightarrow \frac{dy}{dx} = \frac{x}{2}$ and $\frac{d^2y}{dx^2} = \frac{1}{2}$

$$\Rightarrow \quad r_m = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1.5}}{\frac{d^2y}{dx^2}} = \frac{\left(1+4^2\right)}{1/2} = 140.19\text{in}$$

$$r_t = \frac{x}{\cos\alpha} = \frac{8}{\frac{4}{\sqrt{17}}} = 8.246 \qquad \left(\cos\alpha = \frac{4}{\sqrt{17}}\right)$$

$$\frac{\sigma_m}{r_m} + \frac{\sigma_t}{r_t} = \frac{p}{t} \qquad \frac{4123}{140.19} + \frac{\sigma_t}{8.246} = \frac{250}{1/4}$$

$$\sigma_t = 8003\text{psi} \cong 8000\text{psi (T)}$$



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5-11 Design (Modes of failure and Factor of Safety)

- Failure: a member or structure no longer functions as intended
- Modes of Failures depends on **material, load conditions**, …
  - **Elastic failure** – excessive elastic deformation
    - Significant property: E, ν
  - **Yielding (slip failure)** – excessive plastic deformation
    - Significant property: yield strength, yield point

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5-11 Design

- Modes of Failures
  - **Creep failure** – excessive plastic deformation over a long time under constant stress
    - E.g. machine under high σ, T
    - Significant property: **creep limit**
  - **Fracture** – complete separation of the material
    - E.g. brittle material, crack, flaw, repeated loading
    - Significant property: **ultimate strength**
- Mathematical Analysis
  - Allowable stress design:
    - **Strength ≥ Stress**

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5-11 Design

- Uncertainties
  - Loads – varied, future time
  - Material properties – only test specimen, local defect
  - Stress – model error, non-uniformity
- **Factor of Safety**; allowable stress
  - Strength ≥ (Factor of Safety) · (Stress)

$$FS = \frac{\text{failure load}}{\text{actual computed load}} \quad \text{or} \quad = \frac{\text{failure stress}}{\text{actural computed stress}}$$

$$\text{Allowable stress} = \frac{\text{failure stress}}{FS}$$

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5-11 Design

- Example: axially loaded rod
  - Given: F, σ_y, FS
  - Find: d

$$\left(\sigma_{y(\text{actual})} = \frac{F}{A} = \frac{F}{\pi d^2/4}\right) \cdot (\text{FS}) \leq \sigma_{y(\text{allowable})}$$

$$d \geq \sqrt{4 \cdot (\text{FS}) \cdot F / (\pi \sigma_y)} = d_{\min}$$

Please Study Example Problems 5-20 ~ 5-23 by yourself

8 Exercises

- **5-32, 5-72, 5-84, 5-89,**
  5-92, 5-104, 5-106 5-142