Mechanics of Materials

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Chapter 6

Torsional Loading of Shafts

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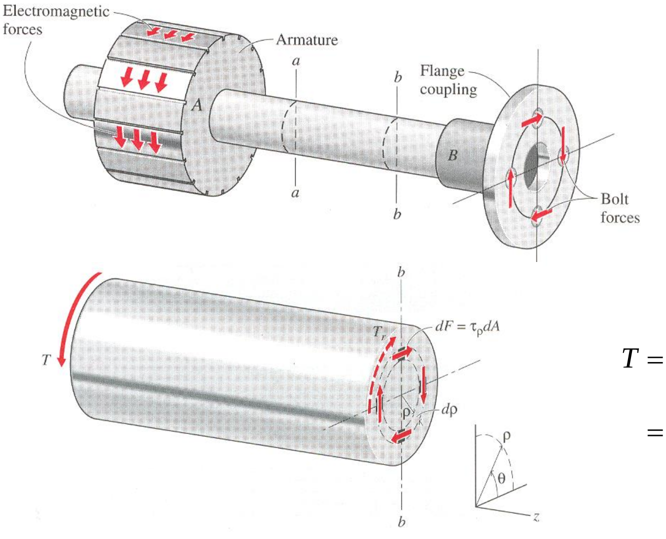
6-1 Introduction



$$T = T_r = \int_{area} \rho dF$$

$$= \int_{area} \rho \tau_\rho dA$$

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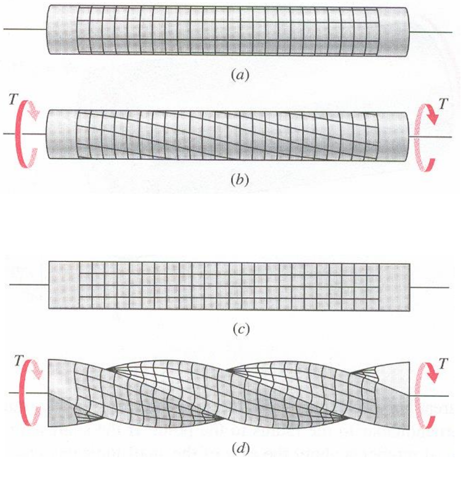
6-1 Introduction

- 1784, Charles-Augustin Coulomb
  - **θ ~ T (experiment)**
- 1820, A. Duleau (French)
  - **θ ~ T (analytical)**
    by assuming:
  - **plane remains plane**
  - **diameter remains straight**

○ for circular shaft

✕ for other shaft,

warping (翹曲) occurs



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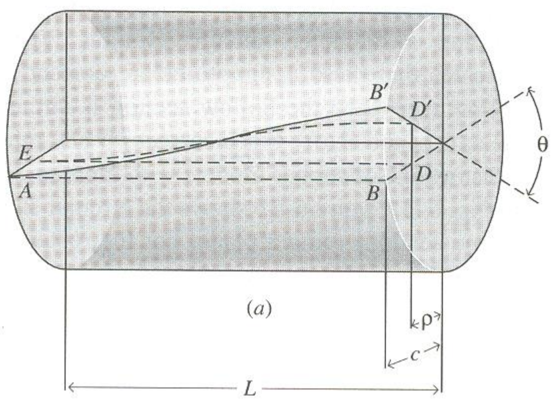
6-2 Torsional Shearing Strain



straight shaft of constant diameter

- plane remains plane
- diameter remains straight
- θ : angle of twist
- $B \rightarrow B'$, $D \rightarrow D'$

  BB', DD' in the same plane

  B'& B and D & D' on the same radius

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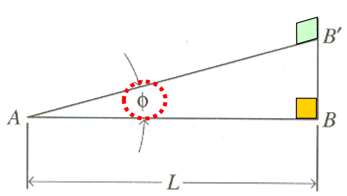
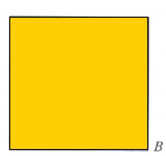
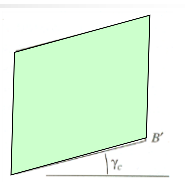
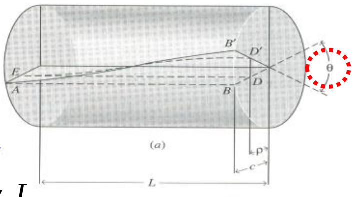
6-2 Torsional Shearing Strain









Two torsional angles: ϕ and θ

small strain

$$\tan\gamma_c = \tan\phi = \frac{BB'}{AB} = \frac{c\theta}{L} \cong \gamma_c$$

$$\tan\gamma_\rho = \frac{DD'}{ED} = \frac{\rho\theta}{L} \cong \gamma_\rho$$

$$\theta = \frac{\gamma_c L}{c} = \frac{\gamma_\rho L}{\rho} \Rightarrow \gamma_\rho = \frac{\rho}{c}\gamma_c$$

valid for elastic **or inelastic** homogeneous **or heterogeneous** materials

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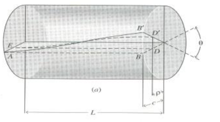
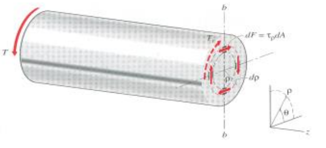
6-3 Torsional Shearing Stress – The Elastic Torsion Formula

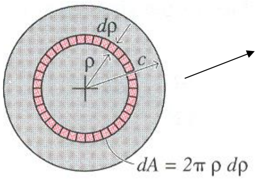
$$\gamma_\rho = \frac{\rho}{c}\gamma_c \quad \xrightarrow{\tau = G\gamma} \quad \boxed{\tau_\rho = \frac{\rho}{c}\tau_c}$$



$$T = T_r = \int_A \rho\tau_\rho dA = \frac{\tau_c}{c}\int_A \rho^2 dA = \frac{\tau_\rho}{\rho}\int_A \rho^2 dA$$



$$J = \int_A \rho^2 dA$$ polar second moment of area



$$J = \int_0^c \rho^2 (2\pi\rho) d\rho = \frac{\pi c^4}{2}$$

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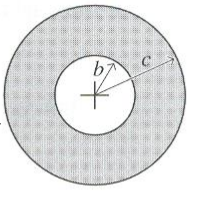
6-3 Torsinal Shearing Stress – The Elastic Torsion Formula

$$T = T_r = \frac{\tau_c J}{c} = \frac{\tau_\rho J}{\rho}$$

$$\boxed{\tau_c = \frac{Tc}{J} \qquad \tau_\rho = \frac{T\rho}{J}}$$

- valid for linearly elastic, homogeneous, isotropic material ($\tau = G\gamma$)
- $\boldsymbol{\tau_{max} = \tau_c}$
- $\boldsymbol{\tau_\rho = 0}$ at center

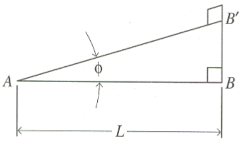
$$J = \int_b^c \rho^2 (2\pi\rho) d\rho = \frac{\pi c^4}{2} - \frac{\pi b^4}{2}$$

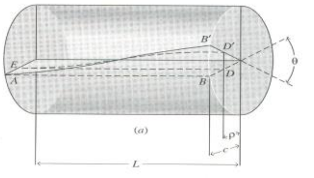


6-4 Torsional Displacements

$$\begin{cases} \theta = \dfrac{\gamma_\rho L}{\rho} \\ \tau_\rho = \dfrac{T\rho}{J} \end{cases}$$

(or $\gamma_\rho = \rho \dfrac{d\theta}{dL}$, if T or J varies along L)





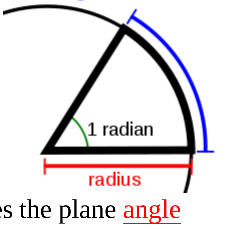
$$\Rightarrow \quad \theta = \frac{\gamma_\rho L}{\rho} = \frac{\tau_\rho L}{G\rho}$$

$$\Rightarrow \quad \boxed{\theta = \frac{TL}{GJ}}$$

Comparison: $\delta = \dfrac{PL}{EA}$

Unit of θ

$\boxed{\theta = \dfrac{TL}{GJ}}$ ⇨ unit: **radian**(s), ***not*** **degree**!



From Wikipedia:

The **radian** is the standard unit of angular measure. It describes the plane angle subtended by a circular arc as the length of the arc divided by the radius of the arc. The SI unit of solid angle measurement is the steradian.

The radian is represented by the symbol "rad" or, more rarely, by the superscript c (for "circular measure"). **For example, an angle of 1.2 radians would be written as "1.2 rad" or "1.2^c" (the second symbol is often mistaken for a degree: "1.2^o").** As the ratio of two lengths, the radian is a "pure number" that needs no unit symbol, and **in mathematical writing the symbol "rad" is almost always omitted**. In the absence of any symbol radians are assumed, and when degrees are meant the symbol **"°"** is used.

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6-4 Torsional Displacements

- $T = T_r$ must be obtained from a free-body diagram.
- If T, G , or J is not constant along the length of the shaft,

$$\theta = \sum_{i=1}^{n} \frac{T_i L_i}{G_i J_i}$$

or

$$\theta = \int_0^L \frac{T_r(x)}{G(x)J(x)} dx$$

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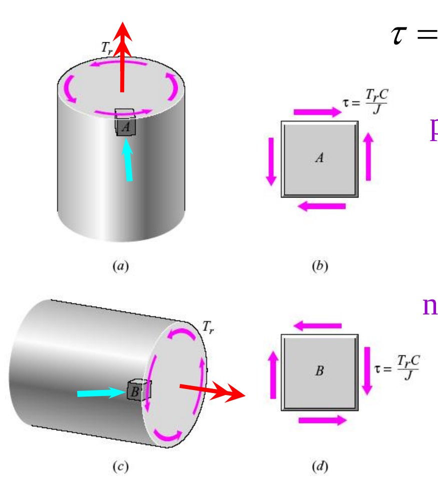
6-4 Torsional Displacements

positive outward for T and θ

- Sign convention

$$\tau = \frac{T\rho}{J}$$

Direction of τ from ***T***



positive shear?

Not always consistent!

negative shear?

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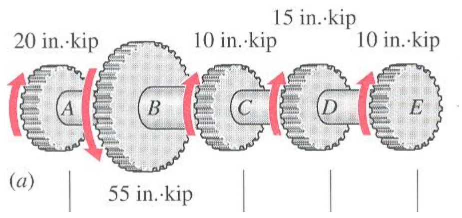
Example Problem 6-1 (I)



Determine

- T_{AB} , T_{BC} , T_{CD} , T_{DE} = ?
- Draw a **torque diagram**.

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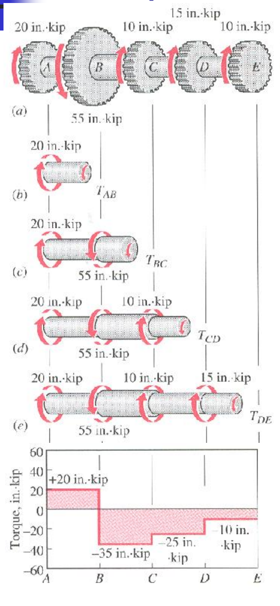
Example Problem 6-1 (II)



$$\sum M = T_{AB} - 20 = 0$$

$$\sum M = T_{BC} - 20 + 55 = 0$$

$$\sum M = T_{CD} - 20 + 55 - 10 = 0$$

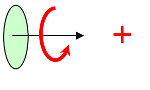
$$\sum M = T_{AB} - 20 + 55 - 10 - 15 = 0$$

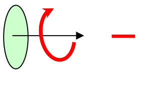
$$T_{AB} = +20.0 \text{ in} \cdot \text{kip}$$

$$T_{BC} = -35.0 \text{ in} \cdot \text{kip}$$

$$T_{CD} = -25.0 \text{ in} \cdot \text{kip}$$

$$T_{DE} = -10.0 \text{ in} \cdot \text{kip}$$





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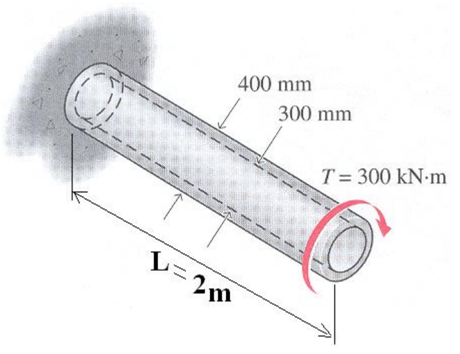
Example Problem 6-2 (I)



- r_i = 150 mm
- r_o = 200 mm
- G = 80 Gpa
- L = 2 m

Determine

- τ_{max} = ?
- τ_ρ(inside surface)= ?
- θ = ?

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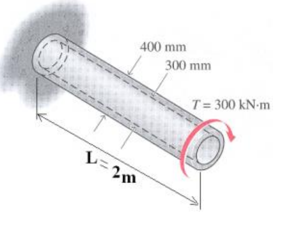
Example Problem 6-2 (II)



$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = \frac{\pi}{2}\left(200^4 - 150^4\right) \cong 1718.1\left(10^{-6}\right)\mathrm{m}^4$$

$$\tau_{max} = \tau_c = \frac{Tc}{J} = \frac{300\left(10^3\right)\left(200\right)\left(10^{-3}\right)}{1718.1\left(10^{-6}\right)} \cong 34.9\ \mathrm{MPa}$$

$$\tau_\rho = \frac{T\rho}{J} = \frac{300\left(10^3\right)\left(150\right)\left(10^{-3}\right)}{1718.1\left(10^{-6}\right)} \cong 26.2\ \mathrm{MPa}$$

$$\theta = \frac{TL}{GJ} = \frac{300\left(10^3\right)\left(2\right)}{80\left(10^9\right)\left(1718.1\right)\left(10^{-6}\right)} \cong 0.00437\mathrm{rad}$$

(***not*** degree!)

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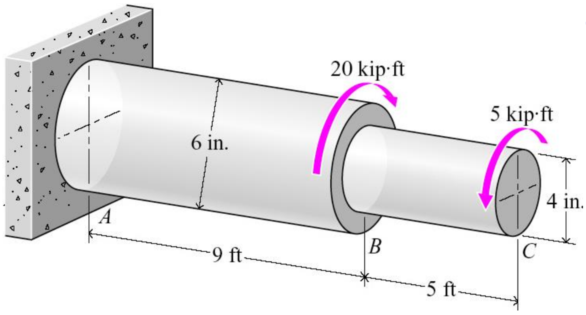
Example Problem 6-3 (I)



Given:

- G = 12,000 ksi

Find:

- τ_{max} = ?
- $\theta_{B/A}$ = ?
- $\theta_{C/B}$ = ?
- $\theta_{C/A}$ = ?

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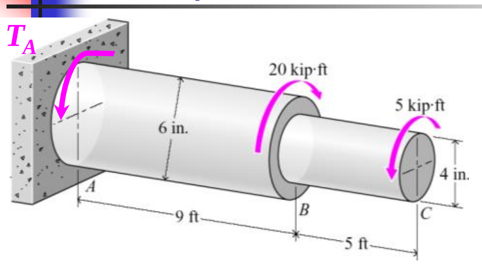
Example Problem 6-3 (II)

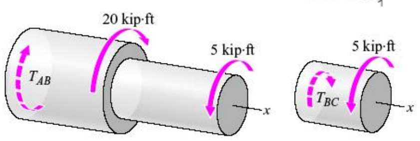


$$\sum M_x = 0: \quad -T_{AB} - 20 + 5 = 0$$

$$T_{AB} = -15 \text{ kip}\cdot\text{ft}$$

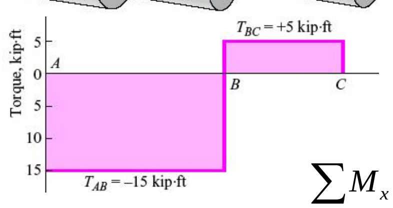
$$\sum M_x = 0: \quad -T_{BC} + 5 = 0$$

$$T_{BC} = 5 \text{ kip}\cdot\text{ft}$$



$$T_{AB} = -15 \text{ kip}\cdot\text{ft}$$

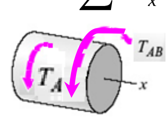
$$T_{BC} = 5 \text{ kip}\cdot\text{ft}$$



Or Alternatively,

$$\sum M_x = 0: \quad T_A - 20 + 5 = 0$$

$$T_A = 15 \text{ kip}\cdot\text{ft}$$



$$\sum M_x = 0: \quad T_A + T_{AB} = 0 \,;\, T_{AB} = -15 \text{ kip}\cdot\text{ft}$$

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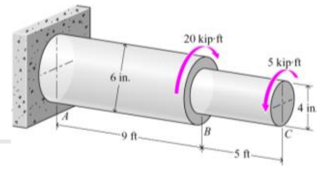
Example Problem 6-3 (III)



$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(3^4\right) = 127.23 \text{ in.}^4$$

$$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(2^4\right) = 25.13 \text{ in.}^4$$

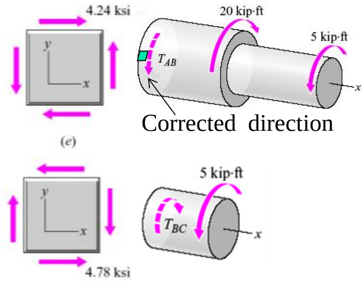
$$T_{AB} = -15 \text{ kip}\cdot\text{ft}$$

$$T_{BC} = 5 \text{ kip}\cdot\text{ft}$$

$$\tau_{AB} = \frac{T_{AB}c_{AB}}{J_{AB}} = \frac{-15(12)(3)}{127.23} = -4.244 \text{ ksi}$$

$$\tau_{BC} = \frac{T_{BC}c_{BC}}{J_{BC}} = \frac{5(12)(2)}{25.13} = 4.775 \text{ ksi}$$

$$\tau_{\max} = \tau_{BC} = 4.775\text{ksi} \cong 4.78\text{ksi}$$



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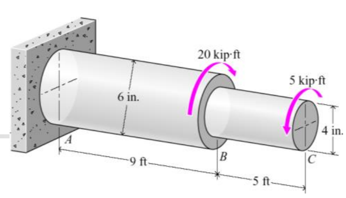
Example Problem 6-3 (IV)



$$\theta_{B/A} = \frac{T_{AB}L_{AB}}{G_{AB}J_{AB}} = \frac{-15(12)(9)(12)}{12,000(127.23)} \cong -0.01273 \text{ rad}$$

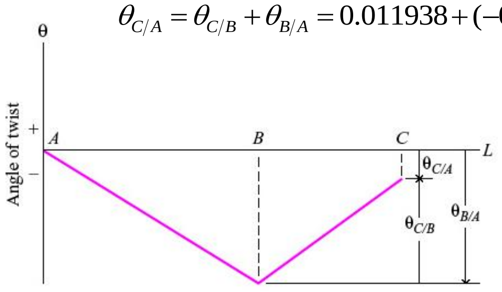
= - 0.73°

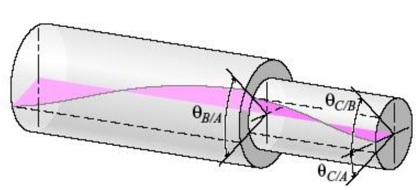
$$\theta_{C/B} = \frac{T_{BC}L_{BC}}{G_{BC}J_{BC}} = \frac{5(12)(5)(12)}{12,000(25.13)} \cong 0.01194 \text{ rad}$$

= 0.68°

$$\theta_{C/A} = \theta_{C/B} + \theta_{B/A} = 0.011938 + (-0.012733) = -0.000795 \text{ rad}$$

= -0.0456°





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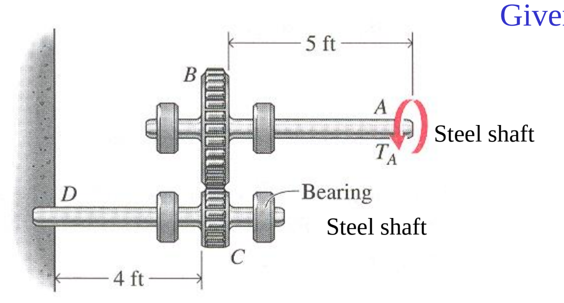
Example Problem 6-4 (I)



Given:

- d_s = 1.50 in.
- d_B = 10 in.
- d_C = 6 in.
- G = 12,000 ksi
- T_A = 750 ft·lb

Find:

- τ_{max} of shaft CD = ?
- θ_A = ?

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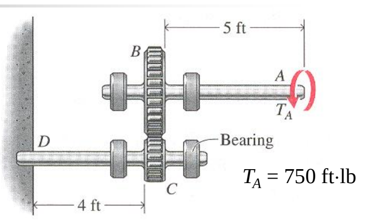
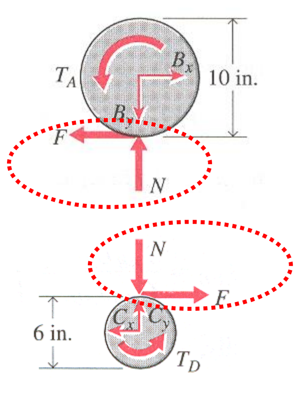
Example Problem 6-4 (II)

- d_s = 1.50 in.
- d_B = 10 in.
- d_C = 6 in.





$$T_A = r_B F$$
$$T_D = r_C F$$

$$T_D = \frac{r_C}{r_B} T_A = \left(\frac{3}{5}\right) 750 = 450\,\text{ft}\cdot\text{lb}$$

$$\tau_{\max} = \frac{T_{CD} c_{CD}}{J_{CD}} = \frac{450(12)(0.75)}{(\pi/2)(0.75)^4} \cong 8150\,\text{psi}$$

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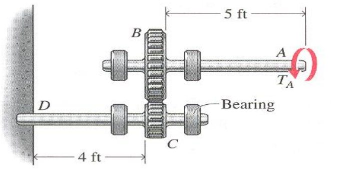
Example Problem 6-4 (III)

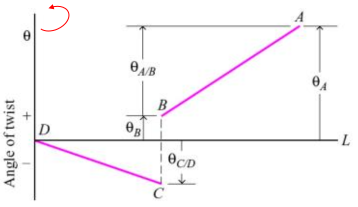


$$\theta_{C/D} = \frac{-T_{CD}L_{CD}}{G_{CD}J_{CD}} = \frac{-450(12)(4)(12)}{12(10^6)(\pi/2)(0.75^4)} = -0.04346 \text{ rad}$$

$= -2.49°$

$$s = r_B\theta_B = r_C\theta_{C/D}$$

Gears move the **same arc length** i.e., the same number of teeth



$$\theta_B = \frac{r_C}{r_B}\theta_{C/D} = \frac{3}{5}(0.04346) = 0.02608 \text{ rad}$$

$= 1.49°$

$$\theta_{A/B} = \frac{T_{AB}L_{AB}}{G_{AB}J_{AB}} = \frac{750(12)(5)(12)}{12(10^6)(\pi/2)(0.75^4)} = 0.09054 \text{ rad}$$

$= 5.19°$

$$\theta_A = \theta_B + \theta_{A/B} = 0.02608 + 0.09054 = 0.11662 \text{ rad} = 6.68°$$

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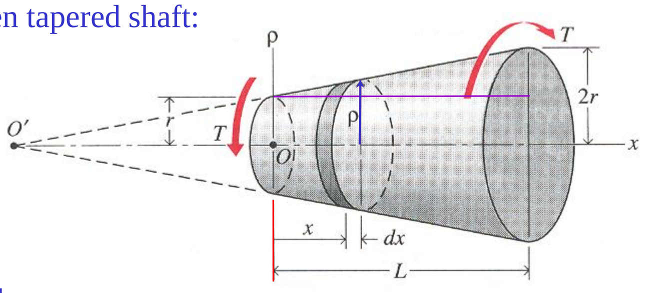
Example Problem 6-5 (I)

Given tapered shaft:



Find:

- θ(T, L, G, r)= ?

$$\rho = r + \frac{2r - r}{L}x = \frac{r}{L}(L + x) = r + \frac{r}{L}x$$

$\tau(\rho)$, $J(\rho)$

$$\theta = \frac{\gamma_c L}{c} = \frac{\gamma_\rho L}{\rho}$$

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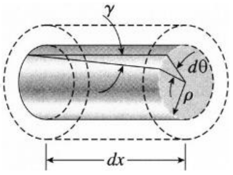
Example Problem 6-5 (II)



$$d\theta = \frac{\gamma}{\rho}dx \qquad \gamma = \frac{\tau}{G} \qquad \tau = \frac{T_r \rho}{J} = \frac{T\rho}{\pi\rho^4/2} = \frac{2T}{\pi\rho^3}$$

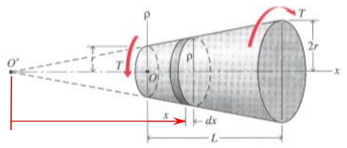
$$d\theta = \frac{\tau}{G\rho}dx = \frac{2T}{G\pi\rho^4}dx = \frac{2TL^4}{G\pi r^4 (L+x)^4}dx$$

$$\rho = \frac{r}{L}(L+x)$$

$$\theta = \frac{2TL^4}{G\pi r^4}\int_0^L \frac{dx}{(L+x)^4} = -\frac{2TL^4}{3G\pi r^4}\left(\frac{1}{8L^3} - \frac{1}{L^3}\right) = \frac{7TL}{12G\pi r^4}$$

$$\theta = \int_0^L \frac{T}{GJ(x)}dx \qquad J(x) = \frac{\pi}{2}\rho^4 = \frac{\pi}{2}\left[\frac{r}{L}(L+x)\right]^4$$

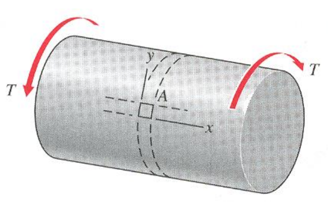
- Alternative : place the origin at O'

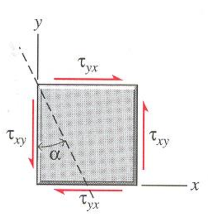


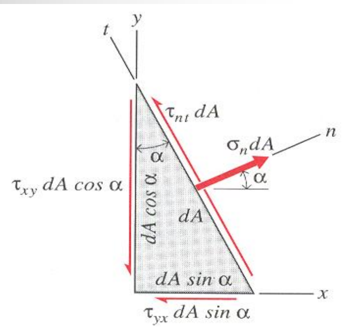
$$\rho = \frac{r}{L}x \qquad \theta = \frac{2TL^4}{G\pi r^4}\int_L^{2L}\frac{dx}{x^4} = \frac{7TL}{12G\pi r^4}$$

p.295

6-5 Stresses on Oblique Planes







$$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = \tau_{yx} \qquad \theta = \alpha$$

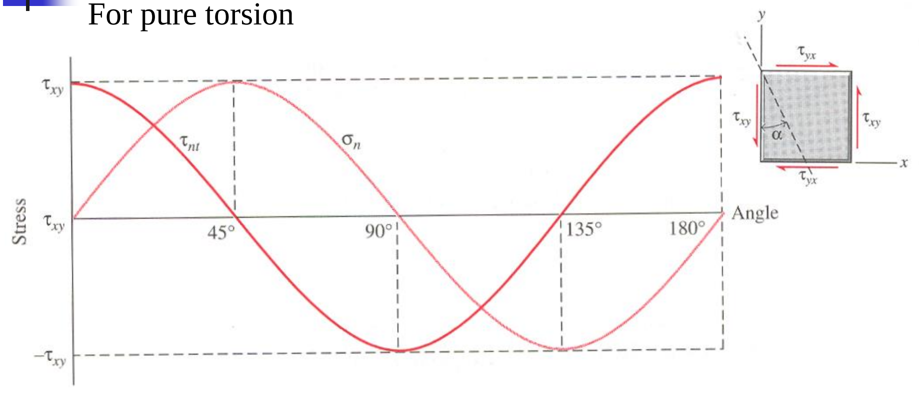
$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta = \tau_{xy}\sin 2\alpha$$

$$\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = \tau_{xy}\cos 2\alpha$$

p.296

6-5 Stresses on Oblique Planes

For pure torsion

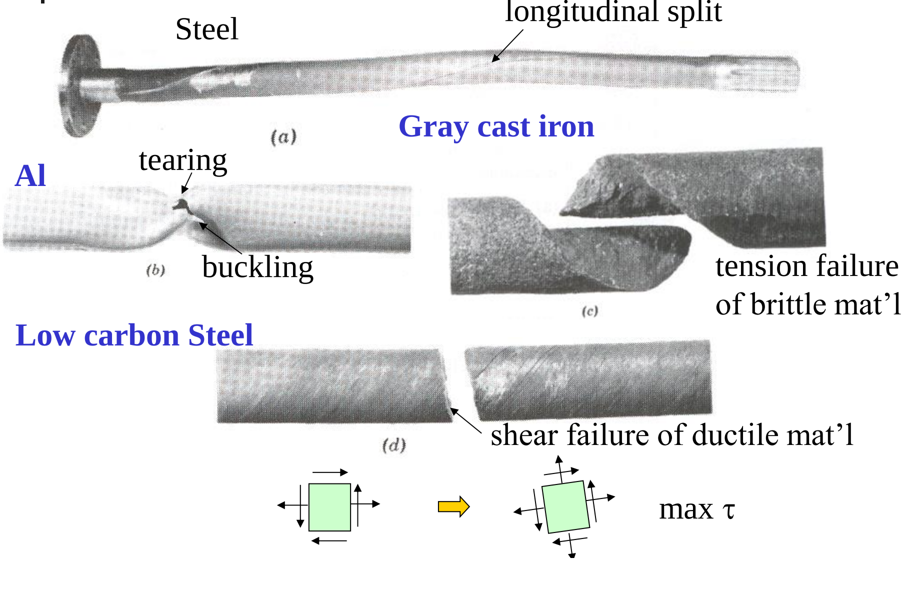


$\max \sigma_n = \tau_{xy}$ $\qquad \alpha = 45° \text{ (T)}, \ 135° \text{ (C)}$

$\max \tau_{nt} = \tau_{xy}$ $\qquad \alpha = 0°, \ 90°$

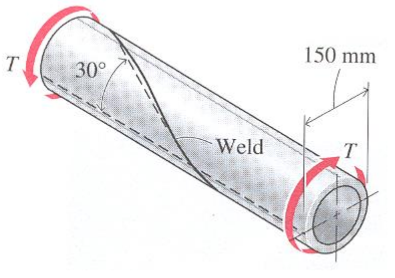
p.297

6-5 Stresses on Oblique Planes



p.297

Example Problem 6-6 (I)



- t = 6 mm,
  σ_{max} (steel tube) = 80 Mpa
- T_{max} = ?
- T = 12 kN · m
  σ_{ult} (weld) = 345 Mpa
  τ_{ult} (weld) = 205 Mpa
  Determine factor of safety
  FS = ?

p.298

Example Problem 6-6 (II)

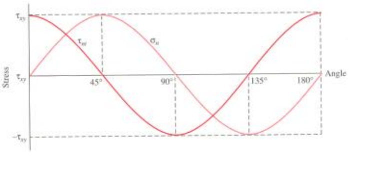
- σ_{max} (tube) = 80 Mpa, T_{max} = ?

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = \frac{\pi}{2}\left(75^4 - 69^4\right) = 14.096\left(10^6\right)mm^4 = 14.096\left(10^{-6}\right)m^4$$

$$\sigma_{max} = \tau_{max} = \frac{T_{max}c}{J} = 80\,\text{MPa} = 80\left(10^6\right)\text{N/m}^2$$

$$T_{max} = \frac{\sigma_{max}J}{c} = \frac{80\left(10^6\right)\left(14.096\right)\left(10^{-6}\right)}{75\left(10^{-3}\right)}$$

$$= 15.036\left(10^3\right)\text{N}\cdot\text{m} \cong 15.04\text{kN}\cdot\text{m}$$



p.298

Example Problem 6-6 (III)

- T = 12 kN·m, σ_{ult} (weld) = 345 Mpa, τ_{ult} (weld) = 205 Mpa

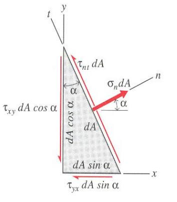
  FS = ?

$$\sigma_n = \tau_{xy} \sin 2\alpha = \frac{Tc}{J} \sin 2\alpha = \frac{12(10^3)(75)(10^{-3})}{14.096(10^{-6})} \sin 2(60°) = 55.29 \text{ MPa (T)}$$

$$\tau_{nt} = \tau_{xy} \cos 2\alpha = \frac{Tc}{J} \cos 2\alpha = \frac{12(10^3)(75)(10^{-3})}{14.096(10^{-6})} \cos 2(60°) = -31.92 \text{ MPa}$$

$$FS_\sigma = \frac{\sigma_{ult}}{\sigma_n} = \frac{345}{55.29} = 6.24 \quad = FS$$

$$FS_\tau = \frac{\tau_{ult}}{\tau_n} = \frac{205}{31.92} = 6.42$$



p.300

6-6 Power Transmission

- Work done by a constant torque T

  $W_k = T\phi$ ϕ : angular displacement (rad)

- Power: rate of doing work

$$\text{Power} = \frac{dW_k}{dt} = T\frac{d\phi}{dt} = T\omega \qquad \omega : \text{angular velocity}$$

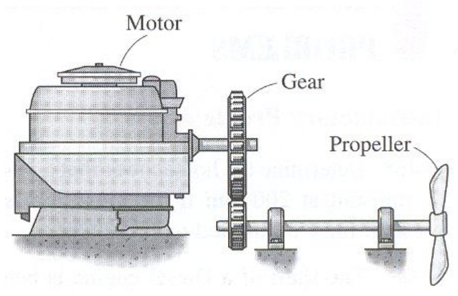
T : N·m — **1 rpm = 2π rad/min**

ω : rad/min — **1 hp = 33000 ft · lb/min**

Power: N ·m /min — **1 watt = 1 N · m/s**

p.301

Example Problem 6-7 (I)



- ω_{motor} = 200 rpm
- Power = 800 hp
- Gear box ratio = 4 : 1
- G = 12,000 ksi
- τ_{ult} = 20 ksi
- θ_{max} (for 10’ propeller shaft) = 4°

Determine

- min diameters of two shafts = ?

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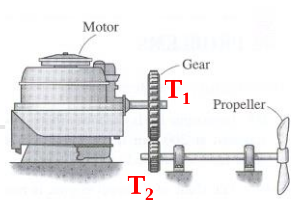
Example Problem 6-7 (II)



- **ω_{motor} = 200 rpm**
- **Power = 800 hp**
- **Gear box ratio = 4 : 1**
- ***G* = 12,000 ksi**

(a) **considering τ_{ult} = 20 ksi**

$$\text{Power} = T\omega \qquad 800(33{,}000) = T_1(200)(2\pi)$$

speed of propeller shaft = 4 × speed of crank shaft

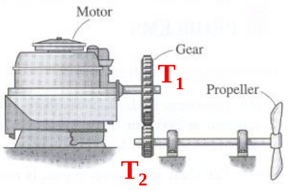
$$T_1(200)(2\pi) = T_2(800)(2\pi)$$

$$T_1 = 21{,}010\,\text{ft}\cdot\text{lb} \qquad T_2 = 5252\,\text{ft}\cdot\text{lb}$$

$$\frac{J_1}{c_1} = \frac{T_1}{\tau} = \frac{21{,}010(12)}{20(10^3)} = \frac{(\pi/2)c_1^4}{c_1} \qquad c_1 = 2.002\,\text{in.} \quad d_1 = 2c_1 \cong 4.00\,\text{in.}$$

p.302

Example Problem 6-7 (III)



- ω_{motor} = 200 rpm
- Power = 800 hp
- Gear box ratio = 4 : 1
- G = 12,000 ksi
- τ_{ult} = 20 ksi

(b) **Considering θ_{max} (propeller shaft) = 4°**

$$\theta = \frac{TL}{GJ}$$

$$\frac{\pi}{180}(4) = \frac{5252(12)(10)(12)}{(12)(10^6)(\pi c_2^4/2)}$$

$$c_2^4 = 5.747 \qquad c_2 = 1.5483$$

$$\frac{J_2}{c_2} = \frac{T_2}{\tau} = \frac{5252(12)}{20(10^3)} = \frac{(\pi/2)c_2^4}{c_2} \qquad c_2 = 1.2612\,in. \quad (d_2 = 2c_2 \cong 2.32\,in.)$$

$$c_2 = 1.5483 > 1.2612$$

$$d_2 = 2c_2 = 2(1.5483) = 3.0966\text{in.} \cong 3.10\,\text{in.}$$

p. 303

6-7 Statically Indeterminate Members

- Statically indeterminate members
  - Equations of equilibrium
  - Distortion equations

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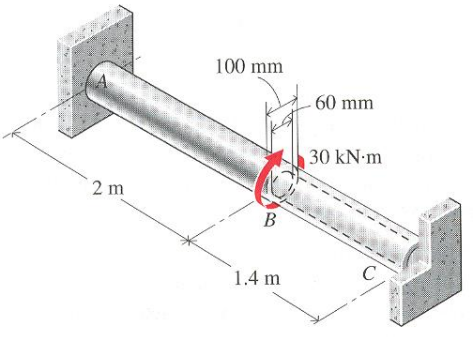
Example Problem 6-8 (I)



- Solid section *AB*
  - Annealed bronze
  - G_{AB} = 45 GPa
- Hollow section *BC*
  - Aluminum alloy
  - G_{BC} = 28 GPa

Determine

- τ_{max} = ? in both sections

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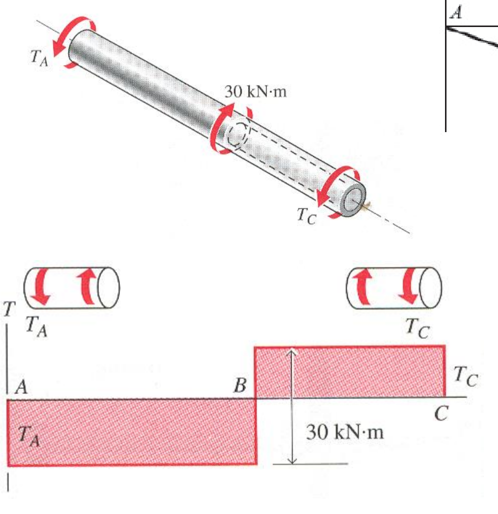
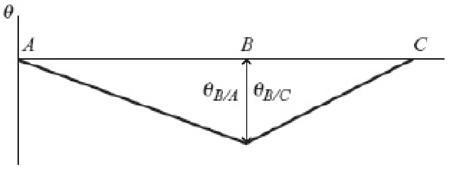
Example Problem 6-8 (II)





$$T_A + T_C = 30(10^3)$$

$$\theta_{B/A} = \theta_{B/C}$$

$$\Rightarrow \frac{T_A L_{AB}}{G_{AB} J_{AB}} = \frac{T_C L_{BC}}{G_{BC} J_{BC}}$$

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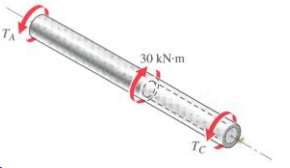
Example Problem 6-8 (III)



$$T_A + T_C = 30(10^3) \qquad \frac{T_A L_{AB}}{G_{AB} J_{AB}} = \frac{T_C L_{BC}}{G_{BC} J_{BC}}$$

$$J_{AB} = (\pi/2)(50^4) = 9.817(10^6)\,\text{mm}^4 = 9.817(10^{-6})\,\text{m}^4$$

$$J_{BC} = (\pi/2)(50^4 - 30^4) = 8.545(10^6)\,\text{mm}^4 = 8.545(10^{-6})\,\text{m}^4$$

$$\frac{T_A(2)}{45(10^9)(9.817)(10^{-6})} = \frac{T_C(1.4)}{28(10^9)(8.545)(10^{-6})} \Rightarrow T_C = 0.7737 T_A$$

$\Rightarrow$ $T_A = 16.914\,\text{kN}\cdot\text{m}$ $\qquad T_C = 13.086\,\text{kN}\cdot\text{m}$

$$\tau_{AB} = \frac{T_A c_{AB}}{J_{AB}} = \frac{16.914(10^3)(50)(10^{-3})}{9.817(10^{-6})} \cong 86.1\,\text{MPa}$$

$\Rightarrow$

$$\tau_{BC} = \frac{T_C c_{BC}}{J_{BC}} = \frac{13.086(10^3)(50)(10^{-3})}{8.545(10^{-6})} \cong 76.6\,\text{MPa}$$

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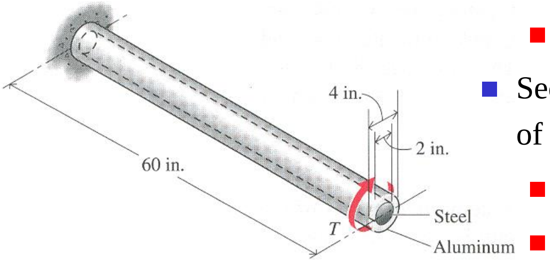
Example Problem 6-9 (I)



- Aluminum alloy tube
  - G_a = 4000 ksi
  - $\tau_{allowable}$ = 10 ksi
- Securely connected at the end of a Steel core
  - G_s = 11,600 ksi
  - $\tau_{allowable}$ = 14 ksi

Determine

- Max. T = ?
- θ = ? under max. T

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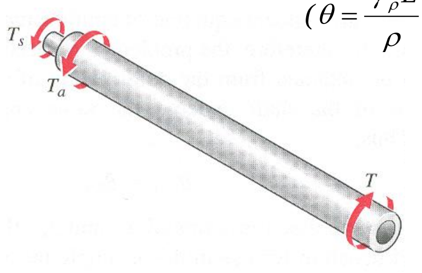
Example Problem 6-9 (II)

- Aluminum alloy tube
  - $G_a = 4000$ ksi
  - $\tau_{allowable} = 10$ ksi
- Securely connected at the end of a Steel core
  - $G_s = 11{,}600$ ksi
  - $\tau_{allowable} = 14$ ksi



$$(\theta = \frac{\gamma_\rho L}{\rho} = \frac{\tau_\rho L}{G\rho})$$

$$T_a + T_s = T$$

$$\theta_a = \theta_s$$

⇨ $\dfrac{\tau_a L_a}{G_a c_a} = \dfrac{\tau_s L_s}{G_s c_s}$

⇨ $\dfrac{\tau_a (60)}{4.0(10^6)(1)} = \dfrac{\tau_s (60)}{11.6(10^6)(2)}$

⇨ $\tau_s = 1.45\tau_a$

$$\tau_s(\text{allowable}) = 14\,\text{ksi}$$
$$= 1.4(10)\,\text{ksi} = 1.4\tau_a(\text{allowable})$$

⇨ τ_s controls

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Example Problem 6-9 (III)

$$\tau_s = 14\,\text{ksi}$$

$$\tau_a = 14/1.45 = 9.655\,\text{ksi} < 10\,\text{ksi}$$

$$T_s = \frac{\tau_s J_s}{c_s} = \frac{14{,}000(\pi/2)(1^4)}{1} = 21{,}990 \text{ in.}\cdot\text{lb}$$

$$T_a = \frac{\tau_a J_a}{c_a} = \frac{9{,}655(\pi/2)(2^4 - 1^4)}{2} = 113{,}750 \text{ in.}\cdot\text{lb}$$

- Max. $T =$?

$$T = T_a + T_s = 113{,}750 + 21{,}990 = 135{,}740 \text{ in.}\cdot\text{lb} \cong 135.74 \text{ in.}\cdot\text{kip}$$

$$\theta = \theta_a = \theta_s = \frac{\tau_s L_s}{G_s c_s} = \frac{14{,}000(60)}{11.6(10^6)(1)} = 0.0724\,\text{rad}$$

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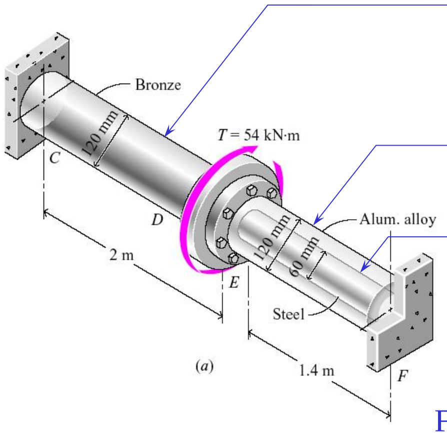
Example Problem 6-10 (I)



Given:

- Section *CD*
  - Solid, bronze, G_B = 45 GPa
- Section *EF*
  - Hollow aluminum alloy tube G_A = 28 GPa
  - Steel core, G_A = 80 GPa
- **Bolt clearance** = 0.03 rad before *EF* carries any load
- *T* = 54 kN·m

Find:

- τ_{max} = ? in each material

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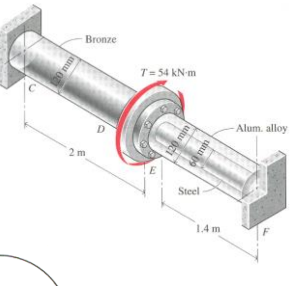
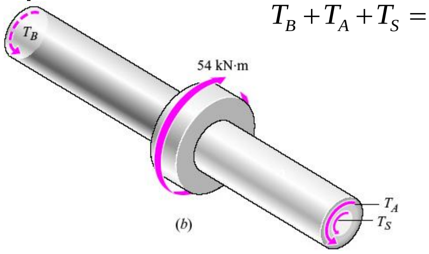
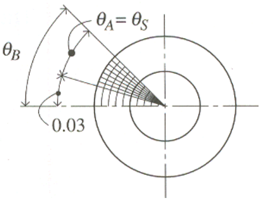
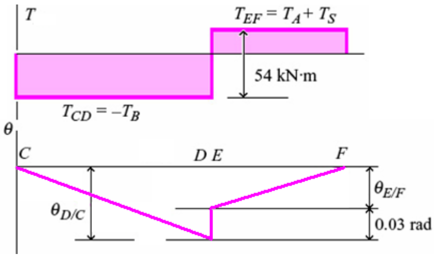
Example Problem 6-10 (II)



$$T_B + T_A + T_S = 54(10^3)\text{ kN•m}$$







$$\theta_{\text{total}} = \theta_{F/E} + \theta_{E/D} + \theta_{D/C} = 0$$

$$\theta_{E/D} = 0.03\text{ rad}$$

$$\left(\theta_{F/E}\right)_A = \left(\theta_{F/E}\right)_S$$

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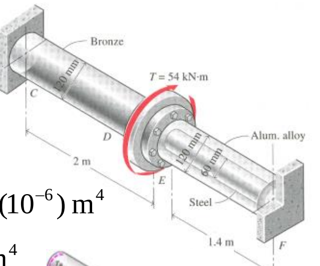
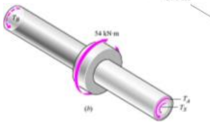
Example Problem 6-10 (III)



$$J_A = (\pi/2)(60^4 - 30^4) = 19.085(10^6)\ \text{mm}^4 = 19.085(10^{-6})\ \text{m}^4$$

$$J_B = (\pi/2)(60^4) = 20.36(10^6)\ \text{mm}^4 = 20.36(10^{-6})\ \text{m}^4$$

$$J_S = (\pi/2)(30^4) = 1.2723(10^6)\ \text{mm}^4 = 1.2723(10^{-6})\ \text{m}^4$$



$$T_B + T_A + T_S = 54(10^3) \Rightarrow \frac{\tau_B J_B}{c_B} + \frac{\tau_A J_A}{c_A} + \frac{\tau_S J_S}{c_S} = 54(10^3)$$

$$\frac{\tau_B(20.36)(10^{-6})}{60(10^{-3})} + \frac{\tau_A(19.085)(10^{-6})}{60(10^{-3})} + \frac{\tau_S(1.2723)(10^{-6})}{30(10^{-3})} = 54(10^3)$$

$$16\tau_B + 15\tau_A + 2\tau_S = 2.546(10^9)$$

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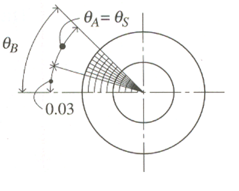
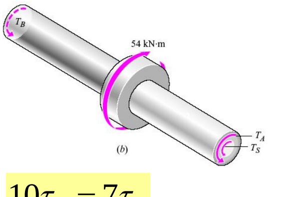
Example Problem 6-10 (IV)



$$\left(\theta = \frac{\gamma_\rho L}{\rho} = \frac{\tau_\rho L}{G\rho}\right) \qquad \frac{-\tau_B L_B}{G_B c_B} + \frac{\tau_A L_A}{G_A c_A} + 0.03 = 0$$

$$\frac{-\tau_B(2)}{45(10^9)(60)(10^{-3})} + \frac{\tau_A(1.4)}{28(10^9)(60)(10^{-3})} + 0.03 = 0 \qquad 8\tau_B = 9\tau_A + 324(10^6)$$



$$\theta_A = \theta_s \qquad \frac{\tau_A L_A}{G_A c_A} = \frac{\tau_S L_S}{G_S c_S}$$

$$\frac{\tau_A(1.4)}{28(10^9)(60)(10^{-3})} = \frac{\tau_S(1.4)}{80(10^9)(60)(10^{-3})} \qquad 10\tau_A = 7\tau_S$$

$\tau_A \cong 52.9$ MPa $\quad \tau_B \cong 100.0$ MPa $\quad \tau_S \cong 75.6$ MPa

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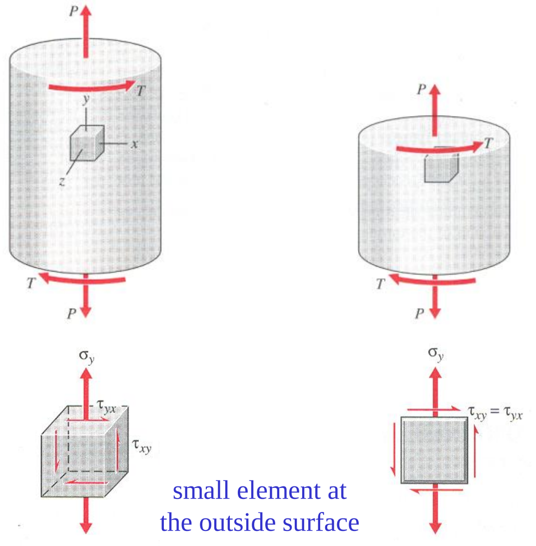
6-8 Combined Effects – Normal and Shearing Stress



small element at the outside surface

$$\sigma_y = \frac{P}{A}$$

$$\tau_{xy} = \frac{Tc}{J}$$

plane stress

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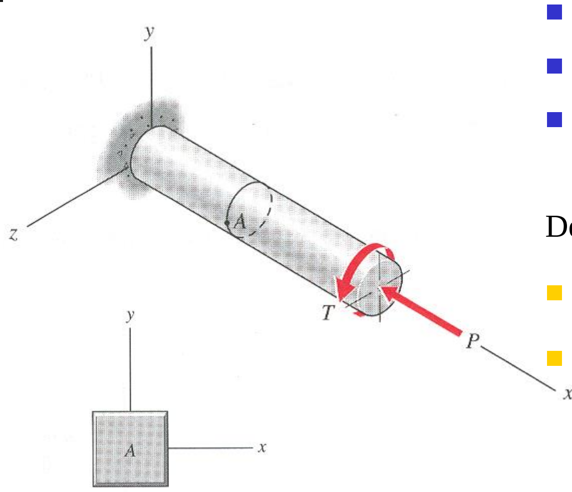
Example Problem 6-11 (I)



- d = 100 mm
- P =200 kN
- T = 30 kN·m

Determine

- stresses at point A
- σ_p , τ_{max} = ?

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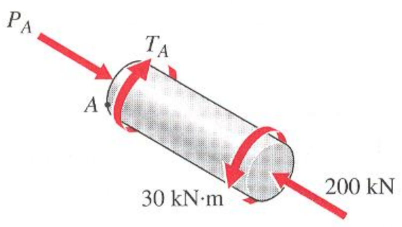
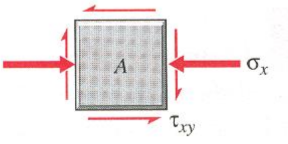
Example Problem 6-11 (II)





$$\sigma_x = \frac{P_A}{A} = \frac{200(10^3)}{(\pi/4)(0.100)^2}\ \text{N/m}^2 = 25.46\ \text{MPa} \cong 25.5\ \text{MPa (C)}$$

$$\tau_{xy} = \frac{T_A c}{J} = \frac{30(10^3)(0.050)}{(\pi/2)(0.050)^4}\ \text{N/m}^2 = 152.79\ \text{MPa} \cong 152.8\ \text{MPa}$$

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Example Problem 6-11(III)

$\sigma_x = -25.46\ \text{MPa}$ $\qquad \sigma_y = 0$ $\qquad \tau_{xy} = -152.79\ \text{MPa}$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-25.46 + 0}{2} \pm \sqrt{\left(\frac{(-25.46) - 0}{2}\right)^2 + (-152.79)^2} = -12.73 \pm 153.32$$

$$\sigma_{p1} = -12.73 + 153.32 \cong 140.6\ \text{MPa}$$

$$\sigma_{p2} = -12.73 - 153.32 \cong -166.1\ \text{MPa}$$

$$\sigma_{p3} = \sigma_z = 0$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{140.59 - (-166.05)}{2} \cong 153.3\ \text{MPa}$$

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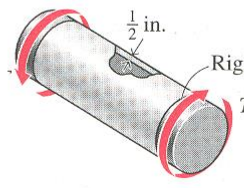
Example Problem 6-12 (I)

Given: a pressure vessel



- d_i = 24 in.
- t = 1/2 in.
- P = 250 psi
- T = 150 ft·kip

Determine

- σ_{max} , τ_{max} = ?

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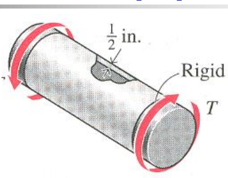
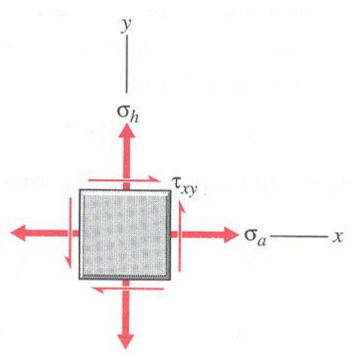
Example Problem 6-12 (II)



- d_i = 24 in.
- t = 1/2 in.



$$\sigma_a = \sigma_x = \frac{pr}{2t} = \frac{250(12)}{2(1/2)} = 3000\text{psi}$$

$$\sigma_h = \sigma_y = \frac{pr}{t} = \frac{250(12)}{1/2} = 6000\text{psi}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{150(10^3)(12)(12.5)}{(\pi/2)(12.5^4 - 12^4)} = 3894\text{psi}$$

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Example Problem 6-12 (III)

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{3000 + 6000}{2} \pm \sqrt{\left(\frac{3000 - 6000}{2}\right)^2 + (3894)^2} = 4500 \pm 4173$$

$$\sigma_{p1} = 4500 + 4173 \cong 8673\text{MPa}$$

$$\sigma_{p2} = 4500 - 4173 \cong 327\text{MPa}$$

$$\sigma_{p3} = \sigma_z = 0$$

$$\sigma_{\max} = \sigma_{p1} \cong 8670\text{psi (T)}$$

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{8673 - 0}{2} \cong 4340\text{MPa}$$

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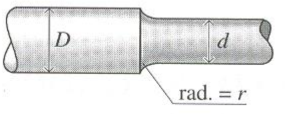
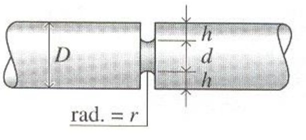
6-9 Stress Concentrations in Circular Shafts Under Torsional Loadings





- Uniform shaft

$$\tau_{\max} = \frac{Tc}{J}$$

- Stress Concentration

$$\tau_{\max} = K\frac{Tc}{J} \qquad K = K\left(\frac{D}{d}, \frac{r}{d}\right)$$

- K can be used to determine $\tau_{\max}$ as long as $\tau_{\max}$ does not exceed the proportional limit.

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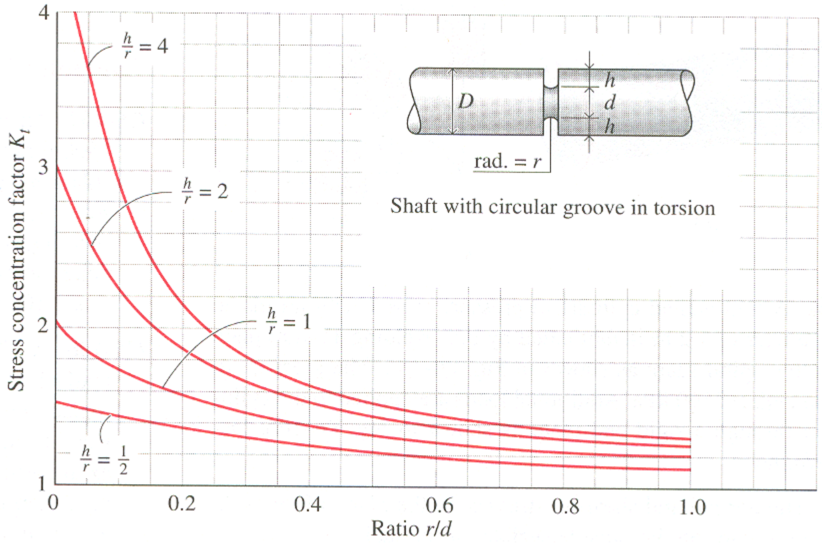
6-9 Stress Concentrations in Circular Shafts Under Torsional Loadings



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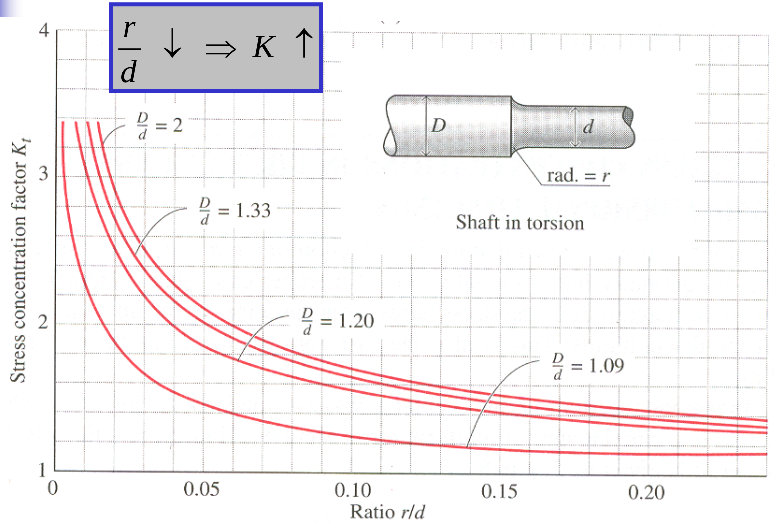
6-9 Stress Concentrations in Circular Shafts Under Torsional Loadings

$$\frac{r}{d} \downarrow \Rightarrow K \uparrow$$



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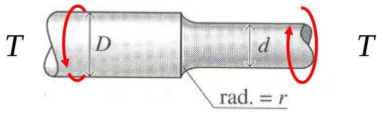
Example Problem 6-13



- D = 4 in.
- d = 2 in.
- T = 6280 in·lb
- $\tau_{allowable}$ = 8 ksi

Determine

(1) r_{min} or (2) r_{max}=?

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Example Problem 6-13

- D = 4 in.
- d = 2 in.
- T = 6280 in·lb
- $\tau_{allowable}$ = 8 ksi

- In the 2in. diameter section

$$\tau_{max} = \frac{Tc}{J} = \frac{6280(1)}{(\pi/2)(1^4)} = 3998\text{psi}$$

- In the fillet

$$\tau_{allowable} = K_t \frac{Tc}{J} = 8\,\text{ksi}$$

$$K_t = 8/3.998 \cong 2$$

- D/d = 2, K_t = 2. From Fig. 6-25(b),

$$r = 0.06d = 0.06(2) = 0.1200\text{in.}$$

8 Exercises

6-18,	**6- 22,**	**6-36,**	**6-47,**
6- 55,	**6- 96,**	**6-139,**	**6- 147**

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6-9 Stress Concentrations in Circular Shafts Under Torsional Loadings