Mechanics of Materials

(http://bernoulli.iam.ntu.edu.tw/)

Chapter 3

Analysis of Strain
Concepts and Definitions

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Analysis of Strain, Concepts and Definitions

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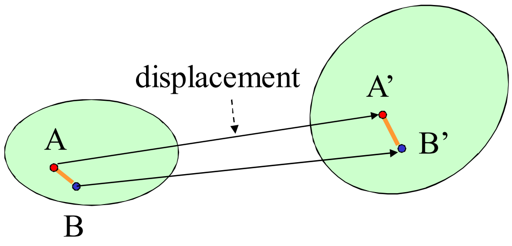
3-2 Displacement, Deformation, and Strain

- Displacement

  a vector indicating the movement of a point with respect to some coordinate system.

  - rigid body motion
  - deformation



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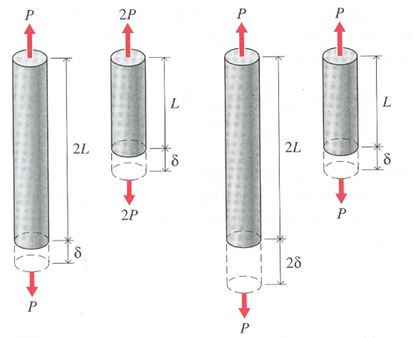
3-2 Displacement, Deformation, and Strain

- Deformation

  Change of size and/or shape of a body.



Note: The elongation induced by the same load may be different.

⇨ Need a quantitative measure of the "**intensity of deformation**".

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3-2 Displacement, Deformation, and Strain

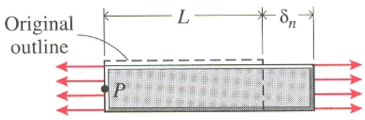
- **Strain**

  Deformation per unit length (Intensity of a deformation)
  - normal strain ε
  - shearing strain γ

❖ Axial Strain

Average

$$\varepsilon_{avg} = \frac{\delta_n}{L}$$



Local point P

$$\varepsilon(P) = \lim_{\Delta L \to 0} \frac{\Delta\delta_n}{\Delta L} = \frac{d\delta_n}{dL}$$

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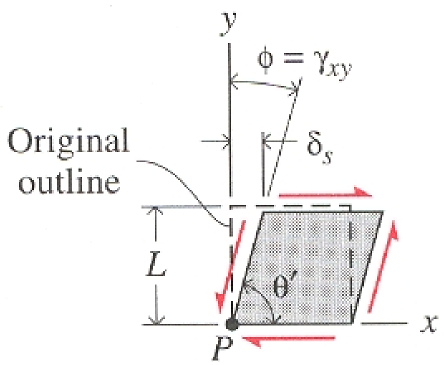
3-2 Displacement, Deformation, and Strain

❖ Shearing Strain



$$\gamma_{avg} = \frac{\delta_s}{L} = \tan\phi \quad \cong \sin\phi \cong \phi$$

$$\gamma_{xy}(P) = \lim_{\Delta L \to 0} \frac{\Delta\delta_s}{\Delta L} = \frac{d\delta_s}{dL} \cong \frac{\pi}{2} - \theta'$$

Note: Usually δ_s/L **< 0.001, tanϕ ≅ sinϕ ≅** ϕ.

γ : decrease in the angle between two reference lines that are orthogonal in the undeformed state.

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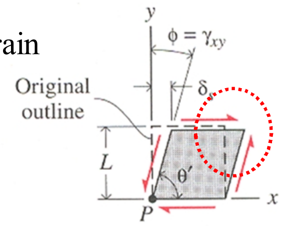
3-2 Displacement, Deformation, and Strain

- Units of Strain : dimensionless
  - Normal strain: m/m (or μm/m)
  - Shear strain: rad (or μ-rad). ?
- Sign Convention for Strains
  - Normal strain:
    - elongation : + tensile strain
    - contraction : – compressive strain
  - Shearing strain
    - θ′ **decreases : +**



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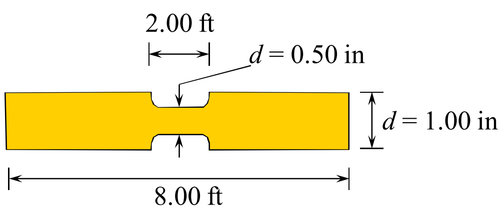
Example Problem 3-1



Given : $(\varepsilon_{avg})_{center}$ = 960 μ in/in

δ_{total} = 0.04032 in

Find:
- $\delta_{Center} = ?$
- $\varepsilon_{End} = ?$

$$\delta_C = \varepsilon_{avg} L = 960(10^{-6})(2)(12) = 0.02304 \text{ in} \cong 0.0230 \text{ in} = 0.585 \text{ mm}$$

$$\delta_E = \delta_{total} - \delta_C = 0.04032\, in - 0.02304\, in = 0.01728\, in = 0.439\, mm$$

$$\varepsilon_E = \frac{\delta_E}{L} = \frac{0.01728}{6(12)} = 240(10^{-6}) = 240\ \mu\text{ in/in} = 240\ \mu\text{mm/mm}$$

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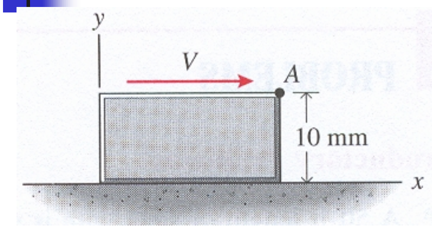
Example Problem 3-2



$\gamma_{avg} = 1000 \mu\text{m/m}$

Determine

- Displacement of A = ?

(1) $\delta_A = 1\ \mu\text{m}$

(2) $\delta_A = 10\ \mu\text{m}$

(3) $\delta_A = 100\ \mu\text{m}$

(4) $\delta_A = 1000\ \mu\text{m}$

$$\delta_A = \gamma_{avg} L = 1000(10^{-6})(10)\text{mm} = 0.01\ \text{mm} = 10\ \mu\text{m}$$

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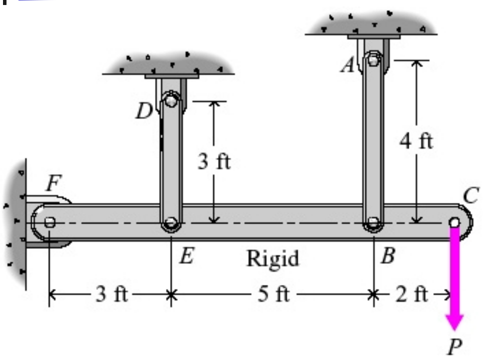
Example Problem 3-3 (I)



$\varepsilon_{DE} = 0.0006$ in/in

Determine

- $\delta_{AB} = ?$ $\varepsilon_{AB} = ?$
- δ_{AB} = ? if there is a 0.001 in clearance in the connection at B between bar *AB* and *CF*.

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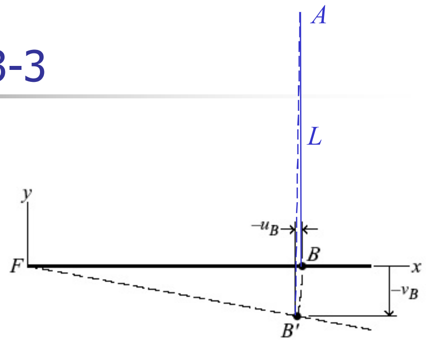
Example Problem 3-3



$$\overrightarrow{BB'} = -u_B\hat{i} - v_B\hat{j}$$

$$\delta_{AB} = AB' - AB = AB' - L$$

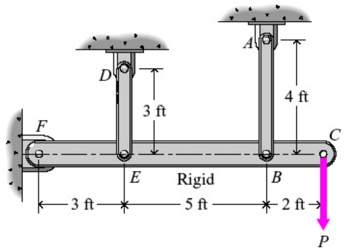
$$\delta_{AB} = \sqrt{(L+v_B)^2 + u_B^2} - L$$

$$\delta_{AB}^2 + 2\delta_{AB}L + \cancel{L^2} = \cancel{L^2} + 2v_BL + v_B^2 + u_B^2$$

Neglect squares of small displacements,

$$\cancel{\delta_{AB}^2} + 2\delta_{AB}L = 2v_BL + \cancel{v_B^2} + \cancel{u_B^2}$$

⇨ $\delta_{AB} \cong v_B$ Similarly, ⇨ $\delta_{DE} \cong v_E$



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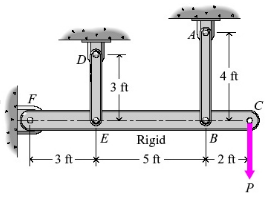
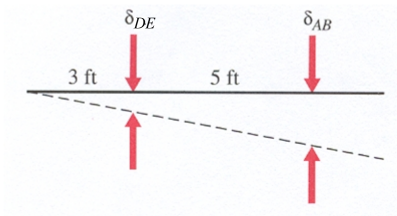
Example Problem 3-3 (II)





Determine

- $\delta_{AB} = ?\ \varepsilon_{AB} = ?$

$$\delta_{DE} = \varepsilon_{DE}L_{DE} = 0.0006(3\text{ft})(12\text{in/ft}) = 0.0216\text{ in}$$

$$\delta_{AB} = \frac{8}{3}\delta_{DE} = \frac{8}{3}(0.0216\text{in}) = 0.0576\text{ in}$$

$$\boldsymbol{\varepsilon_{AB}} = \frac{\boldsymbol{\delta_{AB}}}{\boldsymbol{L_{AB}}} = \frac{0.0576}{4(12)} = 0.001200\text{ in/in} = 1200\mu\text{in/in}$$

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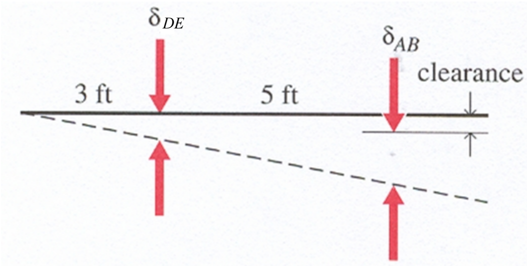
Example Problem 3-3 (III)



Determine

- $\delta_{AB} = ?\ \varepsilon_{AB} = ?$

$$\delta_{AB} = \frac{8}{3}\delta_{DE} - \text{ clearance} = \frac{8}{3}(0.0216) - 0.001 = 0.0566 \text{ in}$$

$$\varepsilon_{AB} = \frac{\delta_{AB}}{L_{AB}} = \frac{0.0566}{4(12)} = 0.001179 \text{ in/in} = 1179\mu\text{in/in}$$

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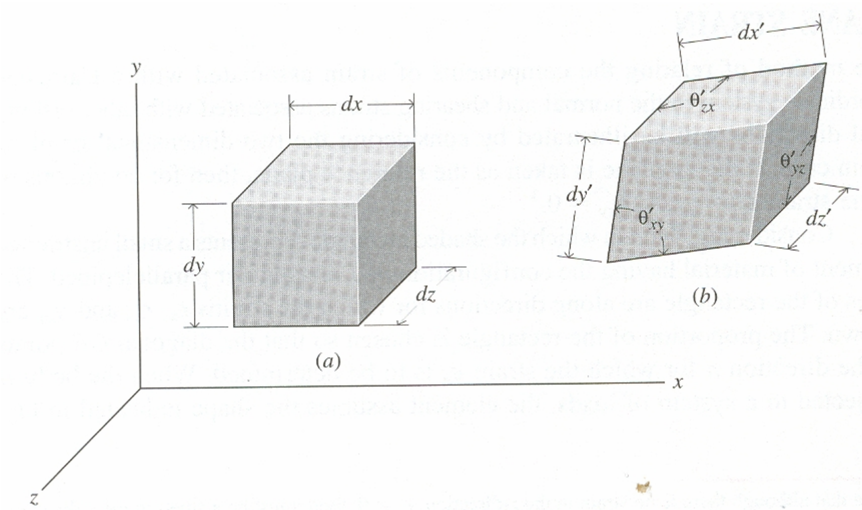
3-3 The State of Strain at a Point



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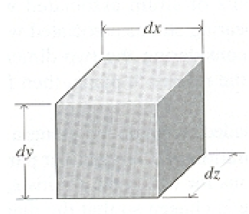
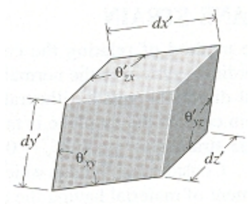
3-3 The State of Strain at a Point

- Strain components associated with the Cartesian coordinates

$$\begin{cases} \varepsilon_x = \dfrac{dx' - dx}{dx} = \dfrac{d\delta_x}{dx} \\ \varepsilon_y = \dfrac{dy' - dy}{dy} = \dfrac{d\delta_y}{dy} \\ \varepsilon_z = \dfrac{dz' - dz}{dz} = \dfrac{d\delta_z}{dz} \end{cases} \Rightarrow \begin{cases} dx' = (1+\varepsilon_x)dx \\ dy' = (1+\varepsilon_y)dy \\ dz' = (1+\varepsilon_x)dx \end{cases}$$



$$\begin{cases} \gamma_{xy} = \dfrac{\pi}{2} - \theta'_{xy} \\ \gamma_{yz} = \dfrac{\pi}{2} - \theta'_{yz} \\ \gamma_{zx} = \dfrac{\pi}{2} - \theta'_{zx} \end{cases} \Rightarrow \begin{cases} \theta'_{xy} = \dfrac{\pi}{2} - \gamma_{xy} \\ \theta'_{yz} = \dfrac{\pi}{2} - \gamma_{yz} \\ \theta'_{zx} = \dfrac{\pi}{2} - \gamma_{zx} \end{cases}$$



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3-3 The State of Strain at a Point

- Normal strain associated with a line in direction n

$$\varepsilon_n = \frac{dn' - dn}{dn} = \frac{d\delta_n}{dn} \quad \text{or} \quad dn' = (1+\varepsilon_n)dn$$

- Shearing strain associated with two orthogonal lines in directions n and t

$$\gamma_{nt} = \frac{\pi}{2} - \theta'_{nt} \qquad \text{or} \qquad \theta'_{nt} = \frac{\pi}{2} - \gamma_{nt}$$

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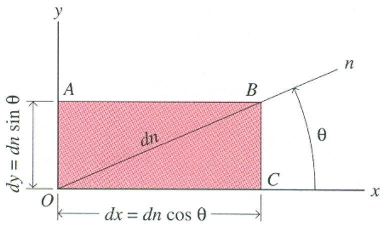
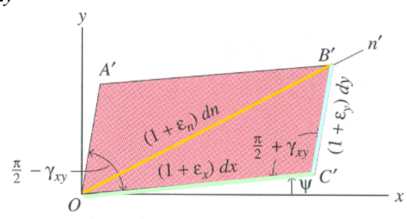
3-4 The Strain Tranformation Equations for Plane Strain

Plane strain: $\varepsilon_z = \gamma_{zx} = \gamma_{zy} = 0$





$$(OB')^2 = (OC')^2 + (C'B')^2 - 2(OC')(C'B')\cos\left(\frac{\pi}{2} + \gamma_{xy}\right)$$

$$[(1+\varepsilon_n)dn]^2 = [(1+\varepsilon_x)dx]^2 + [(1+\varepsilon_y)dy]^2 - 2[(1+\varepsilon_x)dx][(1+\varepsilon_y)dy][-\sin\gamma_{xy}]$$

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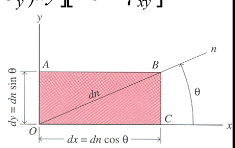
Strain Transformation for **Normal Strain**

$$[(1+\varepsilon_n)dn]^2 = [(1+\varepsilon_x)dx]^2 + [(1+\varepsilon_y)dy]^2 - 2[(1+\varepsilon_x)dx][(1+\varepsilon_y)dy][-\sin\gamma_{xy}]$$

$dx = dn\cos\theta$ $\qquad dy = dn\sin\theta$



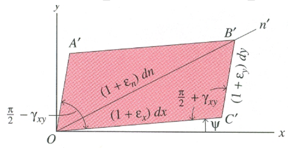
$$(1+\varepsilon_n)^2 (dn)^2 = (1+\varepsilon_x)^2 (dn)^2 \cos^2\theta + (1+\varepsilon_y)^2 (dn)^2 \sin^2\theta + 2(dn)^2 \sin\theta\cos\theta(1+\varepsilon_x)(1+\varepsilon_y)\sin\gamma_{xy}$$

For small strains: $\varepsilon^2 << \varepsilon$ $\qquad \sin\gamma \cong \gamma$ $\qquad$ neglect higher order terms

$$1 + 2\varepsilon_n = (1+2\varepsilon_x)\cos^2\theta + (1+2\varepsilon_y)\sin^2\theta + 2\gamma_{xy}\sin\theta\cos\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\varepsilon_n = \varepsilon_x\cos^2\theta + \varepsilon_y\sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$
$$= \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$



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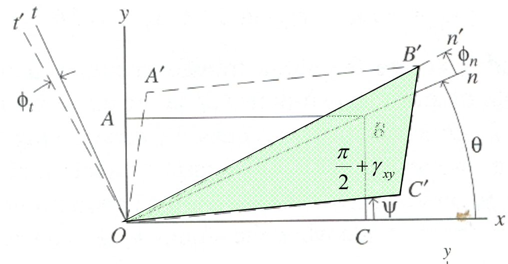
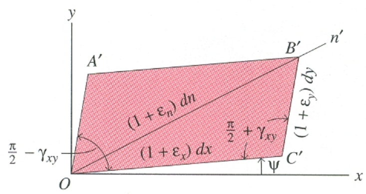
Strain Transformation for **Shearing Strain**



$$\frac{OB'}{\sin\angle OC'B'}=\frac{B'C'}{\sin\angle B'OC'}$$

$$B'C'\sin\angle OC'B'=OB'\sin\angle B'OC'$$

$$\left(1+\varepsilon_y\right)dy\sin\left(\frac{\pi}{2}+\gamma_{xy}\right)=\left(1+\varepsilon_n\right)dn\sin\left(\theta+\phi_n-\psi\right)$$



Greek Symbols

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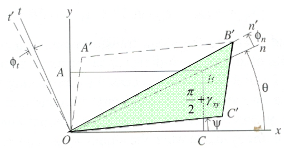
Strain Transformation for **Shearing Strain**

$$\left(1+\varepsilon_y\right)dy\sin\left(\frac{\pi}{2}+\gamma_{xy}\right)=\left(1+\varepsilon_n\right)dn\sin\left(\theta+\left(\phi_n-\psi\right)\right)$$

$$=\cos\gamma_{xy}\cong 1$$

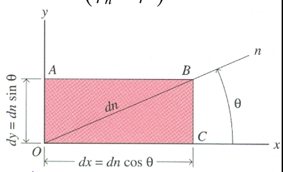


$$\sin\left(\theta+\left(\phi_n-\psi\right)\right)=\sin\theta\cos\left(\phi_n-\psi\right)+\cos\theta\sin\left(\phi_n-\psi\right)\cong\sin\theta+\left(\phi_n-\psi\right)\cos\theta$$

$$\cos\left(\phi_n-\psi\right)\cong 1 \qquad \sin\left(\phi_n-\psi\right)\cong\phi_n-\psi$$

$$\left(1+\varepsilon_y\right)dy\cong\left(1+\varepsilon_n\right)dn\left[\sin\theta+\left(\phi_n-\psi\right)\cos\theta\right]$$

$$dy=dn\sin\theta$$



$$\varepsilon_n \ll \left(\phi_n-\psi\right)$$

$$\left(\varepsilon_y-\varepsilon_n\right)\sin\theta\cong\left(\phi_n-\psi\right)\cos\theta+\varepsilon_n\left(\phi_n-\psi\right)\cos\theta\cong\left(\phi_n-\psi\right)\cos\theta$$

$$=\left(\varepsilon_y-\varepsilon_x\cos^2\theta-\varepsilon_y\sin^2\theta-\gamma_{xy}\sin\theta\cos\theta\right)\sin\theta$$

$$\sin^2 x+\cos^2 x=1$$

$$=-\varepsilon_x\sin\theta\cos^2\theta+\varepsilon_y\sin\theta\cos^2\theta-\gamma_{xy}\sin^2\theta\cos\theta$$

$$\phi_n=-\left(\varepsilon_x-\varepsilon_y\right)\sin\theta\cos\theta-\gamma_{xy}\sin^2\theta+\psi$$

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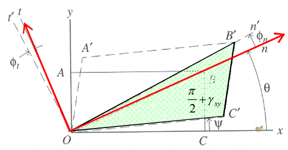
Strain Transformation for **Shearing Strain**

$$\phi_n = -(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta + \psi$$

$$\phi_n = \phi(\theta)$$



$$\phi_t = \phi\left(\theta + \frac{\pi}{2}\right)$$

$$= -(\varepsilon_x - \varepsilon_y)\sin\left(\theta + \frac{\pi}{2}\right)\cos\left(\theta + \frac{\pi}{2}\right) - \gamma_{xy}\sin^2\left(\theta + \frac{\pi}{2}\right) + \psi$$

$$= (\varepsilon_x - \varepsilon_y)\cos\theta\sin\theta - \gamma_{xy}\cos^2\theta + \psi$$

$$\gamma_{nt} = \phi_n - \phi_t$$

$$= -2(\varepsilon_x - \varepsilon_y)\cos\theta\sin\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$\gamma_{nt} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

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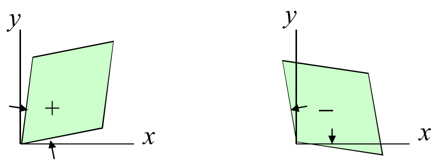
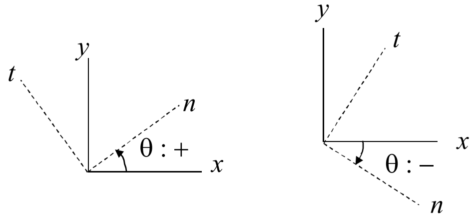
Sign Conventions

- Tensile strains : +, compressive strains : –
- Shearing strains :



- Angle θ



- (n, t, z) axes have the same order as the (x,y,z) axes. Both sets right-hand coordinate system.

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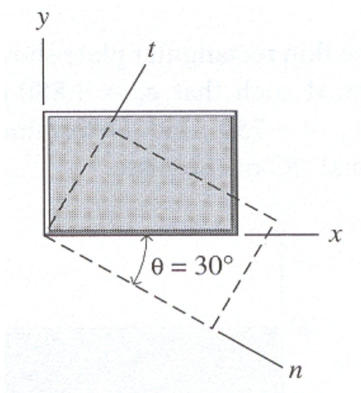
Example Problem 3-4



$\varepsilon_x = +800\mu$

$\varepsilon_y = -1000\mu$

$\gamma_{xy} = -600\mu$

$\theta_n = -30°$

$\theta_t = 60°$

- $\varepsilon_n = ?$
- $\varepsilon_t = ?$
- $\gamma_{nt} = ?$

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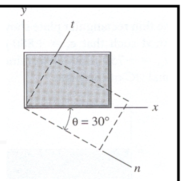
Example Problem 3-4

$\varepsilon_x = +800\mu$

$\varepsilon_y = -1000\mu$

$\gamma_{xy} = -600\mu$



$$\begin{aligned}\varepsilon_n &= \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta \\ &= 800\cos^2(-30°) + (-1000)\sin^2(-30°) + (-600)\sin(-30°)\cos(-30°) \\ &= 609.8\mu \cong 610\mu\end{aligned}$$

$$\begin{aligned}\varepsilon_t &= \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta \\ &= 800\cos^2(+60°) + (-1000)\sin^2(+60°) + (-600)\sin(+60°)\cos(+60°) \\ &= -809.8\mu \cong -810\mu\end{aligned}$$

$$\begin{aligned}\gamma_{nt} &= -2(\varepsilon_x - \varepsilon_y)\cos\theta\sin\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta) \\ &= -2(800 - (-1000))\cos(-30°)\sin(-30°) + (-600)[\cos^2(-30°) - \sin^2(-30°)] \\ &= 1258.8\mu\text{rad} \cong 1259\mu\text{rad}\end{aligned}$$

Check: $\gamma_{nt} = 0.0012588$

$\sin\gamma_{nt} = 0.00125900 \cong \gamma_{nt}$

$\cos\gamma_{nt} = 0.9999992 \cong 1$

Stress vs. Strain Transformation

$$\begin{cases} \sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta \\ \varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta \end{cases}$$

$$\begin{cases} \tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) \\ \gamma_{nt} = -2(\varepsilon_x - \varepsilon_y)\cos\theta\sin\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta) \\ \frac{\gamma_{nt}}{2} = -(\varepsilon_x - \varepsilon_y)\cos\theta\sin\theta + \frac{\gamma_{xy}}{2}(\cos^2\theta - \sin^2\theta) \end{cases}$$

$\Rightarrow$

$\sigma_n \leftrightarrow \varepsilon_n$

$\sigma_x \leftrightarrow \varepsilon_x$

$\sigma_y \leftrightarrow \varepsilon_y$

$\tau_{xy} \leftrightarrow \frac{\gamma_{xy}}{2}$

$\tau_{nt} \leftrightarrow \frac{\gamma_{nt}}{2}$

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3-5 Principal Strains and Maximum Shearing Strains

- Similar to the case of plane stress

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\sigma_p = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \varepsilon_p = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\tau_p = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \frac{\gamma_p}{2} = \pm\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

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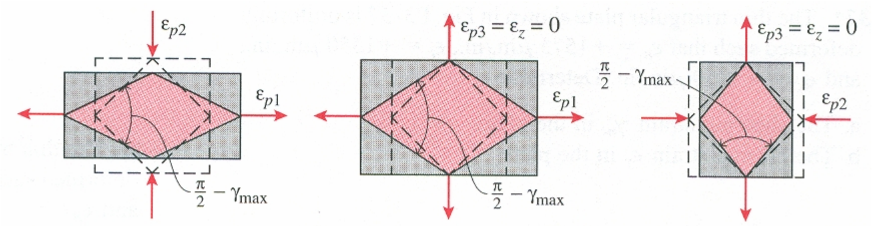
Remarks

- **Plane strain**:

  max. ε_n : ε_{p1} , ε_{p2} , 0

  max. γ_{nt} : $\varepsilon_{p1} - \varepsilon_{p2}$, $\varepsilon_{p1} - 0$, $\varepsilon_{p2} - 0$

- The lines associated with the maximum γ_{nt} bisect the angles between the lines of ε_{max} and ε_{min}



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Example Problem 3-5(I)

Given : $\varepsilon_x = +1200\mu$ $\varepsilon_y = -600\mu$ $\gamma_{xy} = +900\mu$

Find: ε_{p1}, ε_{p2}, γ_{max} = ?

Sol: $$\varepsilon_{p1}, \varepsilon_{p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{1200 + (-600)}{2} \pm \sqrt{\left(\frac{1200 - (-600)}{2}\right)^2 + \left(\frac{900}{2}\right)^2}\right] \times 10^{-6}$$

$$= [300 \pm 1006.2] \times 10^{-6}$$

$$\therefore \varepsilon_{p1} \cong 1306\mu \quad \varepsilon_{p2} \cong -706\mu \quad \varepsilon_{p3} = \varepsilon_z = 0$$

p.137

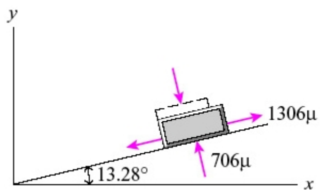
Example Problem 3-5(II)

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{900}{1200-(-600)} = 0.500$$

$$\theta_{p1} = 13.285° \cong 13.28° \quad \rightarrow \quad \varepsilon_{p1} = 1306\,\mu$$

$$\theta_{p2} = \theta_{p1} + 90° \cong 103.28° \quad \rightarrow \quad \varepsilon_{p2} = -706\,\mu$$

$$\begin{aligned}\varepsilon_n &= \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta \\ &= 1200\cdot\cos^2(13.285°) - 600\cdot\sin^2(13.285°) \\ &\quad + 900\cdot\sin(13.285°)\cdot\cos(13.285°) \\ &= 1306.2\,\mu\end{aligned}$$



p.137

Example Problem 3-5(III)

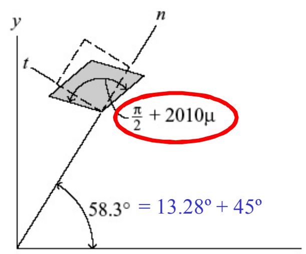
$$\gamma_p = \pm 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\tau_{xy}}{2}\right)^2}$$

$$= \pm 2\sqrt{\left(\frac{1200-(-600)}{2}\right)^2 + \left(\frac{900}{2}\right)^2} \times 10^{-6} = \pm 2012\,\mu$$

$$\because \ \varepsilon_{p2} < 0 < \varepsilon_{p1} \quad \gamma_{\max} = \gamma_p = \pm 2012\,\mu \cong \pm 2010\,\mu$$

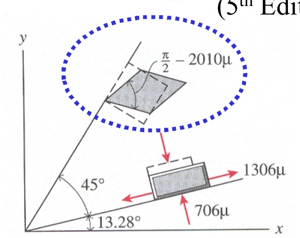
$$\theta_\gamma = 13.285° \pm 45° \cong 58.285° \text{ and } -31.715°$$

$$\begin{aligned}\gamma_{nt} &= -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta) \\ &= -2(1200+600)\sin(58.285°)\cos(58.285°) \\ &\quad + 900[\cos^2(58.285°) - \sin^2(58.285°)] \\ &= -2012.5\,\mu\end{aligned}$$



(b)

(5th Edition)



p.138

Example Problem 3-6 (I)

$\varepsilon_x = +720\mu \qquad \varepsilon_y = +520\mu \qquad \gamma_{xy} = +480\mu$

- $\varepsilon_{p1},\ \varepsilon_{p2},\ \gamma_{max} = ?$

$$\varepsilon_{p1}, \varepsilon_{p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{720+520}{2} \pm \sqrt{\left(\frac{720-520}{2}\right)^2 + \left(\frac{480}{2}\right)^2}\right] \times 10^{-6}$$

$$= [620 \pm 260] \times 10^{-6}$$

$\varepsilon_{p1} \cong 880\mu \quad \varepsilon_{p2} \cong 360\mu \quad \varepsilon_{p3} = \varepsilon_z = 0$

p.138

Example Problem 3-6 (II)

$$\gamma_p = 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\tau_{xy}}{2}\right)^2}$$

$$= 2\sqrt{\left(\frac{720-520}{2}\right)^2 + \left(\frac{480}{2}\right)^2} \times 10^{-6} = 520\mu$$

$\because\ 0 < \varepsilon_{p2} < \varepsilon_{p1} \qquad \varepsilon_{p1} \cong 880\mu \quad \varepsilon_{p2} \cong 360\mu \quad \varepsilon_{p3} = \varepsilon_z = 0$

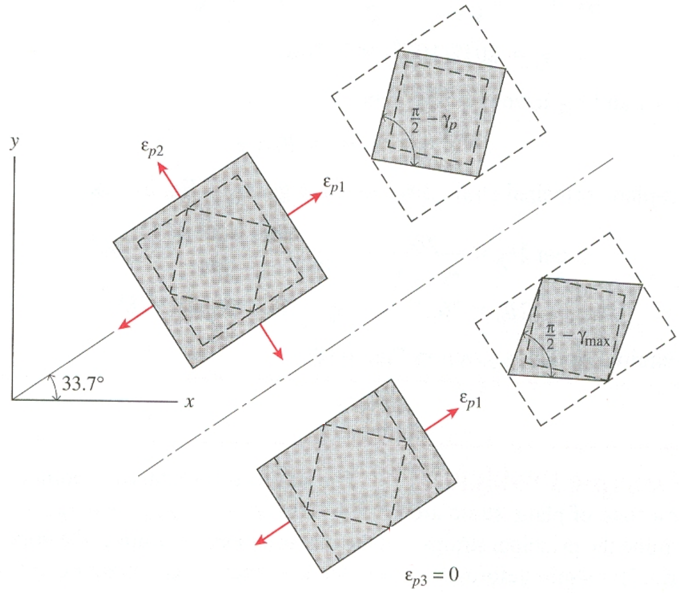
$\gamma_{max} = \varepsilon_{p1} - 0 = 880\mu$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{480}{720-520} = 2.400$$

$\theta_p = 33.69° \cong 33.7°$

p.138

Example Problem 3-6 (III)



p.140

3-6 Mohr's Circle for Plane Strain

$$\left(\sigma_n-\frac{\sigma_x+\sigma_y}{2}\right)^2+\tau_{nt}^2=\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2$$

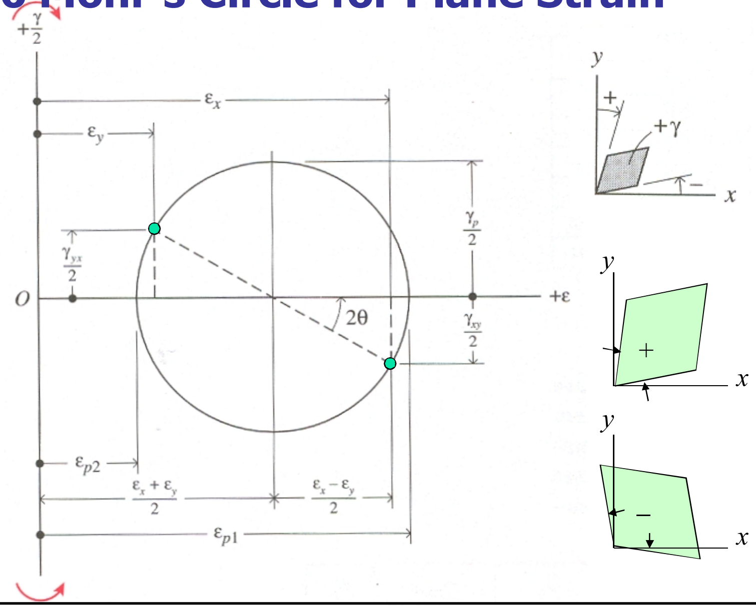
⇨ $$\left(\varepsilon_n-\frac{\varepsilon_x+\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{nt}}{2}\right)^2=\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2$$

Circle : center $\left(\varepsilon_n\,,\ \frac{\gamma_{nt}}{2}\right)=\left(\frac{\varepsilon_x+\varepsilon_y}{2}\,,\ 0\right)$

radius $R=\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}$

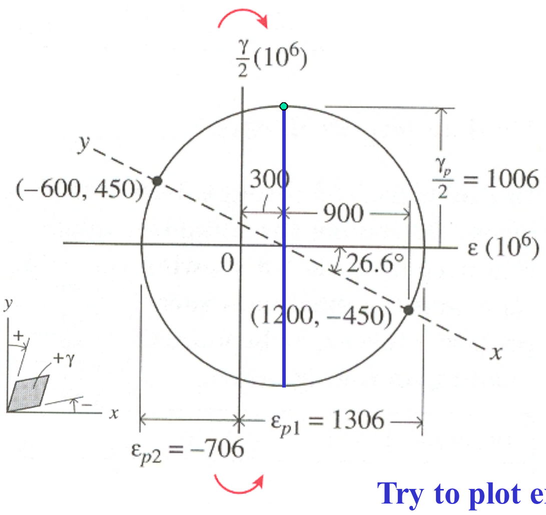
p.141

3-6 Mohr's Circle for Plane Strain



p.141

Plot Mohr's Circle for example problem 3-5



$\varepsilon_x = +1200\mu$

$\varepsilon_y = -600\mu$

$\gamma_{xy} = +900\mu$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = \frac{1200\mu - 600\mu}{2}$$

$$= 300\mu$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\cong 1006\mu$$

Try to plot example 3-6 by yourself!

p.142

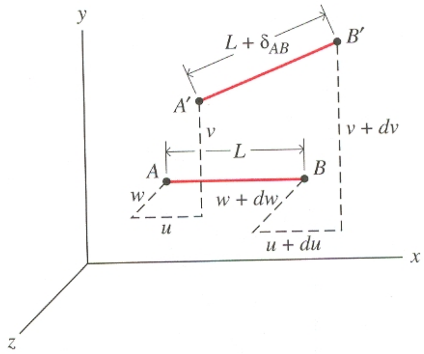
3-7 Strain Measurement and Rosette

- Strain Measurement:

  Strain gages **mounted on a free surface**

  $\sigma_z,\ \tau_{xz},\ \tau_{yz} = 0$

  $\varepsilon_x,\ \varepsilon_y,\ \varepsilon_z,\ \gamma_{xy} \neq 0,\ \ \gamma_{zx},\ \gamma_{zy} = 0$



$$(L+\delta_{AB})^2 = (L+du)^2 + (dv)^2 + (dw)^2$$

$$\cancel{L^2} + 2L\delta_{AB} + \delta_{AB}^{\ 2} = \cancel{L^2} + 2L(du) + (du)^2 + (dv)^2 + (dw)^2$$

neglect 2nd order terms

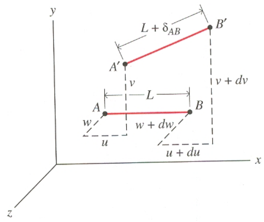
$$\delta_{AB} = du$$

p.142

3-7 Strain Measurement and Rosette

- Normal strain along $AB = \delta_{AB}/L$ is **not** affected by out-of-plane displacement (dv).

- **None** of in-plane strain are affected (dw).



- $\varepsilon_n = \dfrac{\varepsilon_x+\varepsilon_y}{2} + \dfrac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \dfrac{\gamma_{xy}}{2}\sin 2\theta$

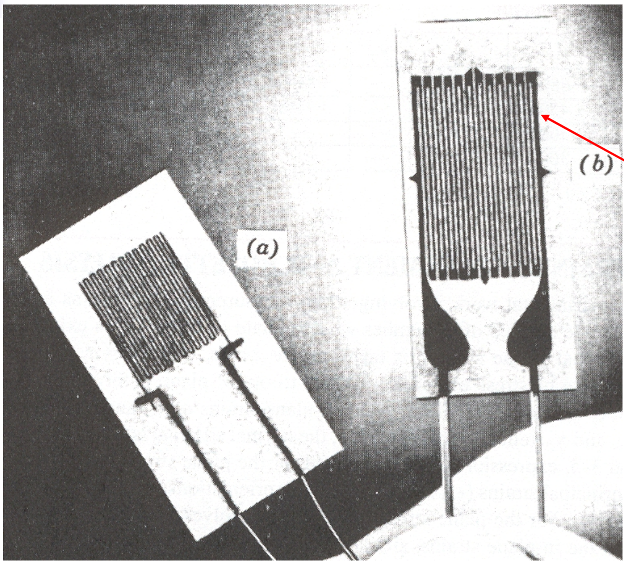
  $\gamma_{nt} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$

  are valid for both plane strain and plane stress states.

- Plane stress: $\varepsilon_z = \varepsilon_{p3} = -\dfrac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$ (section 4-3)

p.143

Strain Gage – Measure Normal Strain

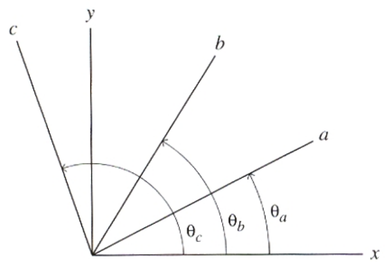


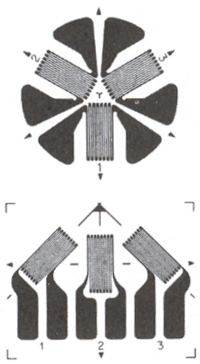
0.001 inϕ

electrical resistence changes

p.144

Strain Rosettes (I)





$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$

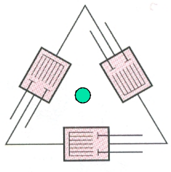
$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b$$

⇨ $\varepsilon_x, \ \varepsilon_y, \gamma_{xy}$

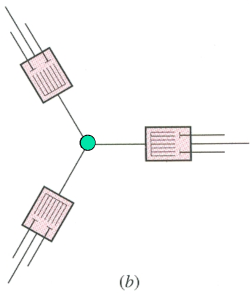
$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c$$

p.144

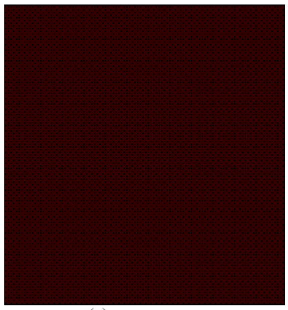
Strain Rosettes (II)



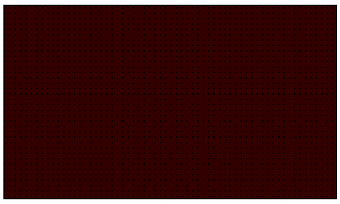
(a)



(b)



(c)



(d)

p.145

Example Problem 3-7 (I)

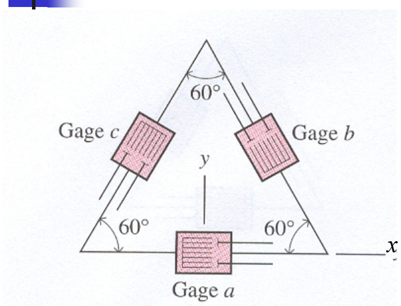


Figure 3-19

$\varepsilon_a = +1000\mu$

$\varepsilon_b = +750\mu$

$\varepsilon_c = -650\mu$

- $\varepsilon_{p1},\ \varepsilon_{p2}, \gamma_{max} = ?$

$$\varepsilon_a = \varepsilon_0 = +1000\mu \text{ (measured)} = \varepsilon_x$$

$$\varepsilon_c = \varepsilon_{60} = -650\mu = 1000\mu \cos^2 60° + \varepsilon_y \sin^2 60° + \gamma_{xy} \sin 60° \cos 60°$$

$$\varepsilon_b = \varepsilon_{120} = +750\mu = 1000\mu \cos^2 120° + \varepsilon_y \sin^2 120° + \gamma_{xy} \sin 120° \cos 120°$$

p.145

Example Problem 3-7 (II)

$\varepsilon_y = -266.7\mu \qquad \gamma_{xy} = -1616.6\mu$

$$\varepsilon_{p1}, \varepsilon_{p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{1000 - 266.7}{2} \pm \sqrt{\left(\frac{1000 + 266.7}{2}\right)^2 + \left(\frac{-1616.6}{2}\right)^2}\right] \times 10^{-6}$$

$$= [366.7 \pm 1026.9] \times 10^{-6}$$

$\varepsilon_{p1} \cong 1394\mu \quad \varepsilon_{p2} \cong -660\mu$

$$\varepsilon_z = \varepsilon_{p3} = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{1-1/3}(1000\mu - 266.7\mu) \cong -367\mu$$

p.146

Example Problem 3-7 (III)

$\because \ \varepsilon_{p2} = -660\mu < \varepsilon_{p3} = -367\mu < \varepsilon_{p1} = 1394\mu$

$$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1393.6\mu + 660.2\mu = 2053.8\mu \cong 2050\mu$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-1616.6}{1000 + 266.7} = -1.2763$$

$\theta_p = -25.96° \cong -26.0°$

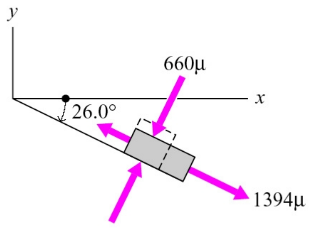


Figure 3-21

8 Exercises

3-10,	**3-26,**	**3-36,**	**3-45,**
3-53,	**3-57,**	**3-66,**	**3-75**