Mechanics of Materials

(http://bernoulli.iam.ntu.edu.tw/)

Chapter 4

Material Properties and Stress-Strain Relationships

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Contents

Material Properties and Stress-Strain Relationships

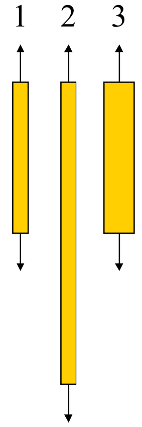
p. 153

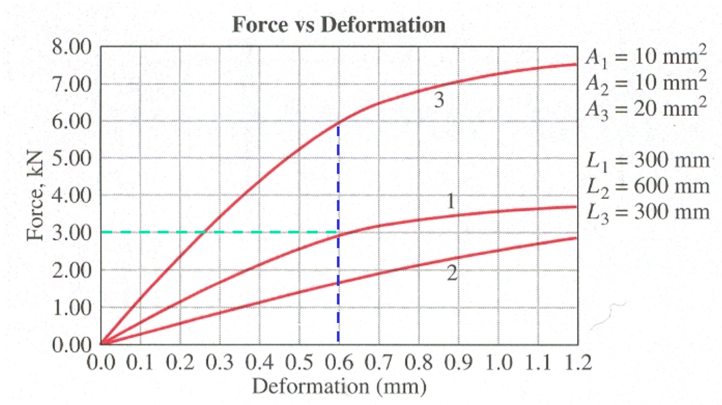
4-1 Introduction

- Deformation or Strain
  – due to applied loads or reactions
  – temperature changes, i.e., thermal expansions
- Decouple the effects caused by loads and temperature changes
  – Superposition for linear problem
  – Compute separately and add together for total deformation

p. 154

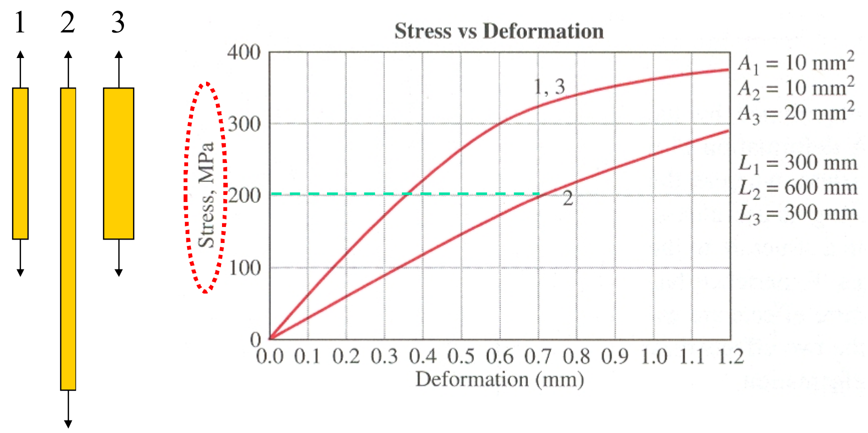
4-2 Stress-Strain Diagrams





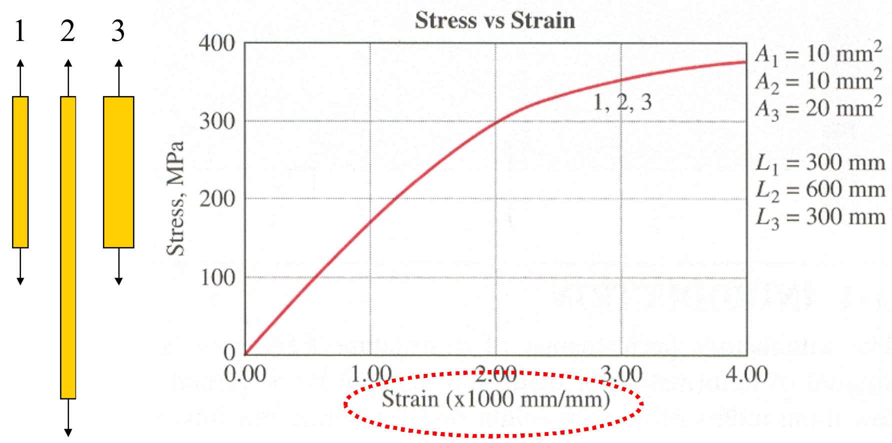
p. 154

4-2 Stress-Strain Diagrams



p. 154

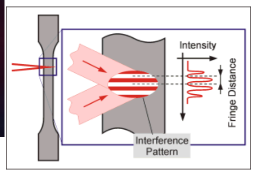
4-2 Stress-Strain Diagrams



p. 154

Tensile Test

- Stress = load / initial area
  (most commonly used)
- True stress = load / actual area
- Strain Measurement
  - measured by strain gages or extensometers (or compressometers)
  - In engineering structures, usually $\varepsilon < 0.001$



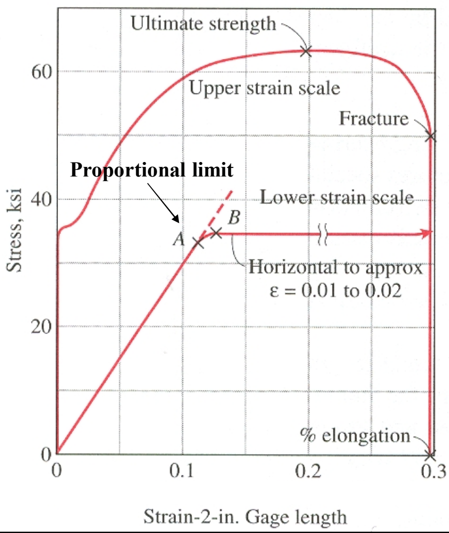
Laser Doppler Extensometer

(From www.fiedler-oe.de/ images/diaz.gif)

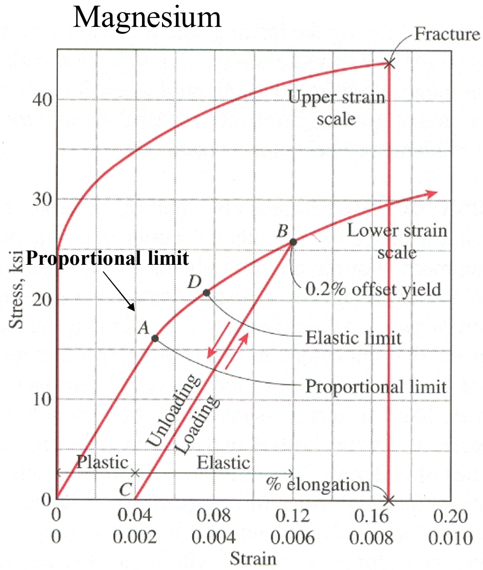
p. 156

Example Stress–Strain Diagrams

Low carbon steel

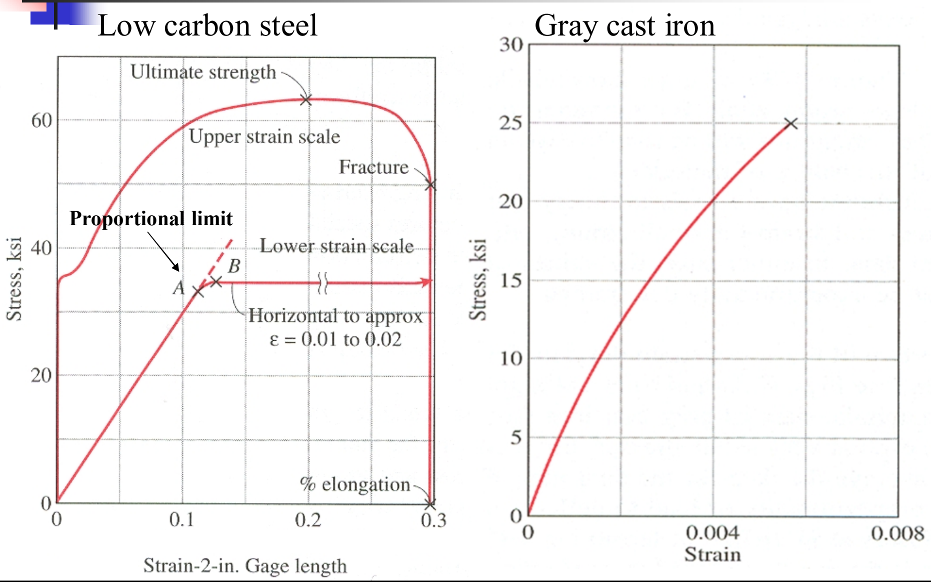


Magnesium



p. 156

Example Stress–Strain Diagrams



p. 155

Modulus of Elasticity

- Hooke's law (1678 by Robert Hooke)

  $\sigma = E\varepsilon$

- Young's modulus or **modulus of elasticity** (1807 by Thomas Young) or "***stiffness***"

  $E = \sigma/\varepsilon$ (normal stress-strain)

- Shear modulus **(modulus of rigidity)**

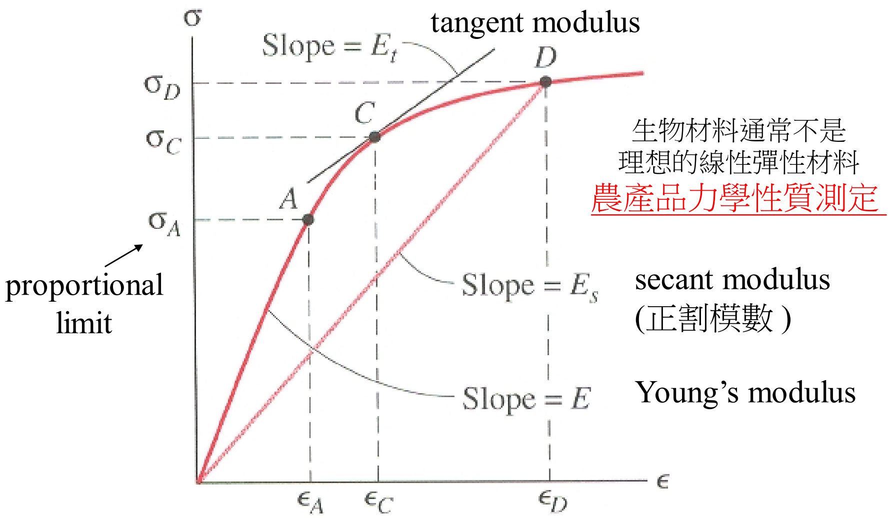
  $G = \tau/\gamma$ (shear stress-strain)

- Proportional limit

  max stress for which stress and strain are proportional

p. 155

Modulus of Elasticity



p. 157

Elastic Limit

- **Elastic**

  the strain resulting from loading **disappears** when the load is removed.

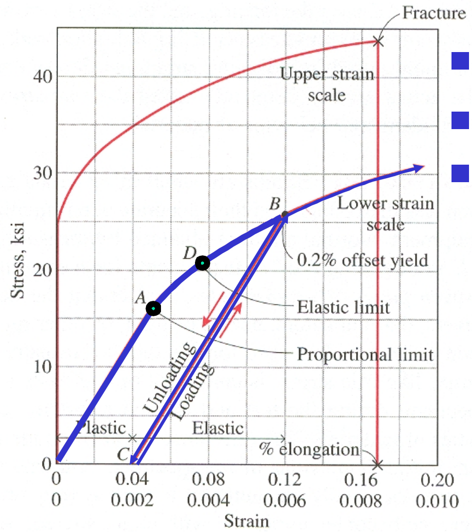
- Elastic limit

  the max stress for which the material acts elastically.

  If σ > elastic limit ⇒ **plastic** deformation

p. 157

Elastic/Plastic deformation: Elastic Limit



- loading/unloading
- strain/work hardening
- plastic deformation
  - **(Crystals) Slip:**

    depends on load, independent of time
  - **Creep（潛變）：**

    continues to increase under **long-term** constant load (**time-dependent** permanent deformation)

    often occurs at high T

p. 158

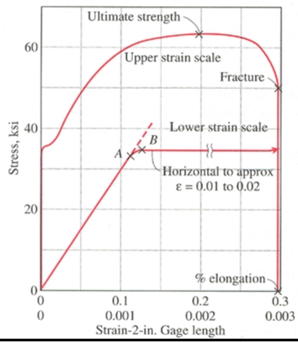
Yield Point（降伏點）

- The stress at which there is an **appreciable** increase in strain with **no** increase in stress.

  ⇒ the load of the testing machine ceases to rise

  If straining is continued, the stress will again increase.

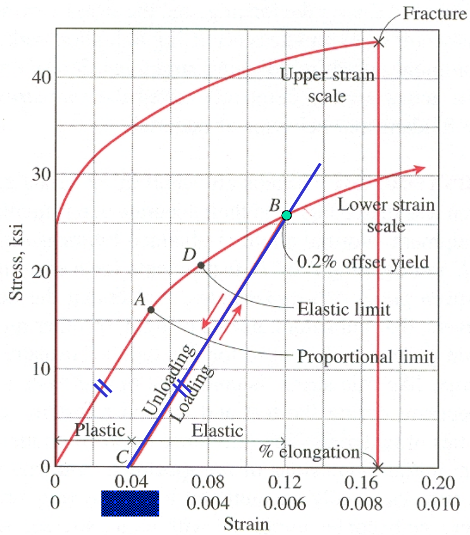
  ⇒ kink or knee in the σ–ε diagram.
- Used as the proportional limit
- Example: low-carbon steel



p. 158

Yield Strength (降伏強度)

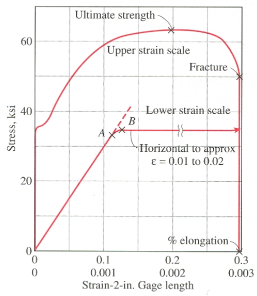
- The stress that will induce a specified permanent set usually 0.05 to 0.3% **(0.2% is most commonly used)**
- Used as the proportional limit

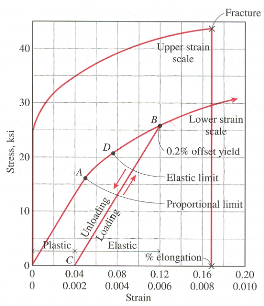


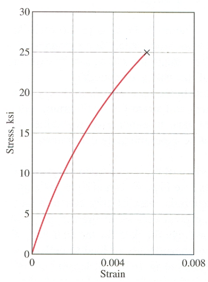
p. 158

Ultimate Strength (I)

- The maximum stress (nominal stress) developed in a material before rupture. (may be tensile, compressive or shearing strength)
- **Ductile materials** undergo considerable plastic deformation before rupture.

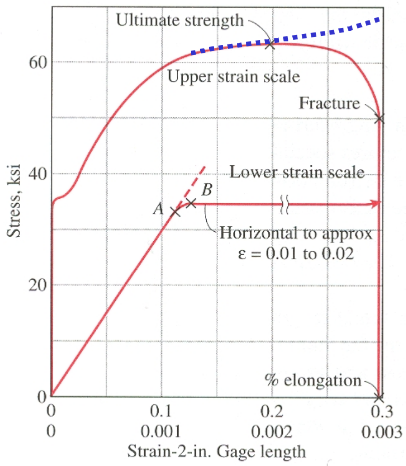




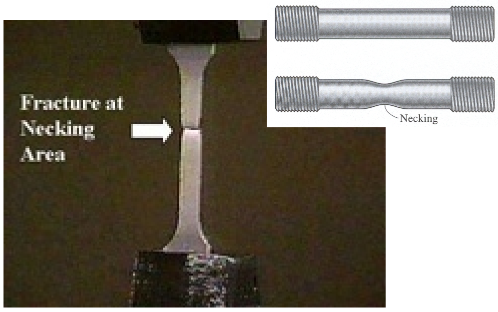


p. 158

Ultimate Strength (II)



- Necking phenomenon (ductile material) nominal stress ↓ beyond the ultimate strength, but true stress continues ↑ until rupture.



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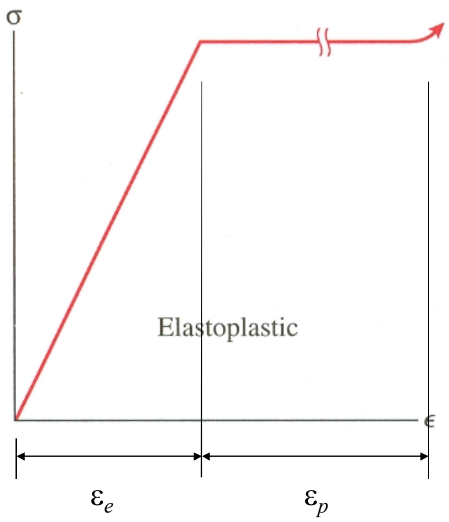
p. 158

Elasto-Plastic Materials



- mild steel

  $\varepsilon_p \cong 16 \sim 20\, \varepsilon_e$

p. 158

Ductility (延展性)

- Capacity of plastic deformation in tension or shear
- Indices:
  - ultimate **elongation** (expressed as a percent elongation of the gage length at rupture)
  - the **reduction** (in %) **of cross-sectional area** at the section where rupture occurs.

p. 159

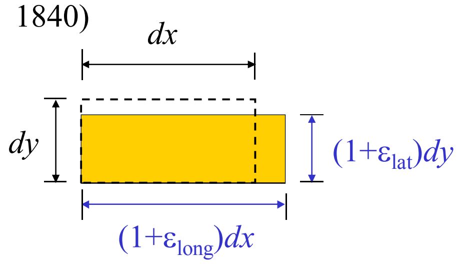
Creep Limit

- **Creep Limit**: The maximum stress for which the plastic strain will not exceed a specified amount during a specified time interval at a specified temperature.
- The resistance of a material to failure by creep
- Lead, rubber and some plastics may creep at ordinary temperature
- Important for polymeric materials or metal parts subjected to high temperatures (35~50% of melting temperature).
- **Creep strength**: the stress which, at a given temperature, will result in a creep rate of **1% deformation within a specified time** (e.g. 1000 ~ 100,000 hours).

p. 159

Poisson's Ratio

- Simeon D. Poisson, 1811 (French mathematician (1781-1840)



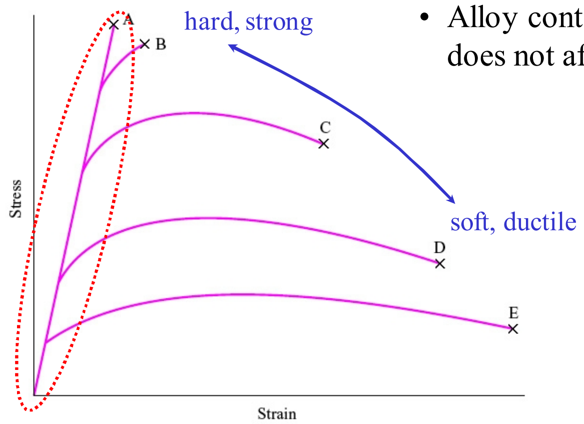
$$\nu = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = -\frac{\varepsilon_t}{\varepsilon_a}$$

- $E = 2(1+\nu)G$ (shown in section 4-3)
- **ν = constant for σ < proportional limit**
- **ν = 1/4 ~ 1/3 for most metals**

p. 159

Effect of Composition

- Various alloy contents for steels



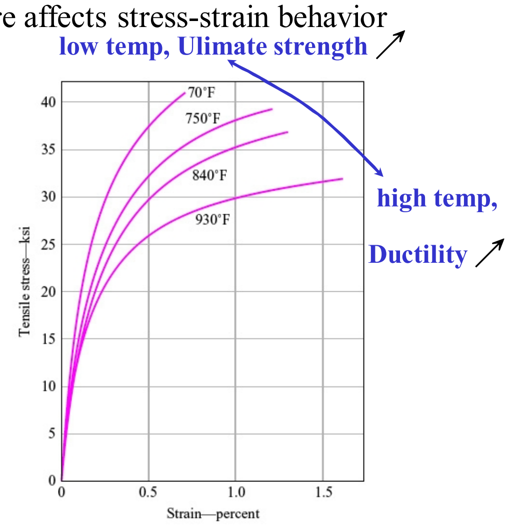
- Alloy content does not affect E

Figure 4-7

p. 160

Effect of Temperature

- Temperature affects stress-strain behavior

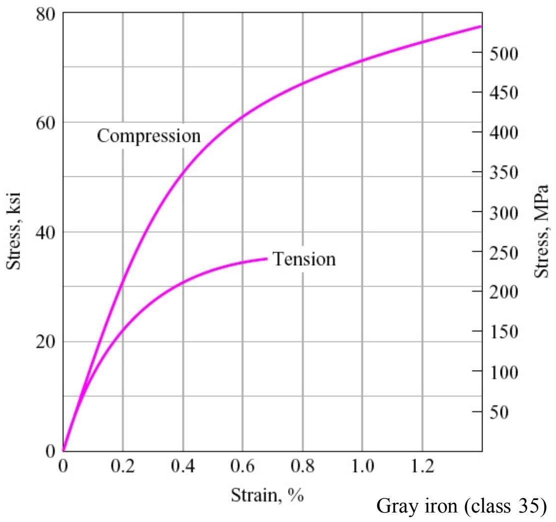


Gray iron (class 40)

p. 161

Effect of Tension or Compression

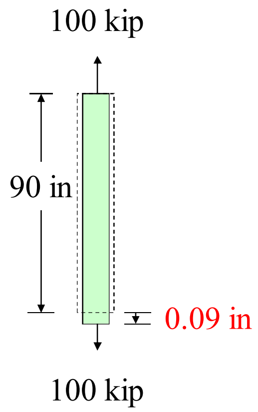
- For ***ductile material***, the tension and compression behavior are usually assumed to be ***the same***.
- ***Brittle*** materials , e.g., gray iron, behave differently on load conditions
- Material properties: Appendix B (A42/43)

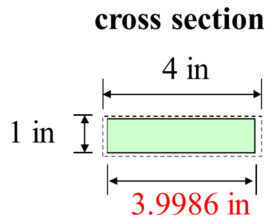


Gray iron (class 35)

p. 161

Example Problem 4-1



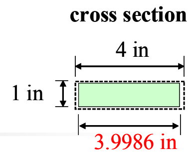


- Determine

  $\nu,\ E,\ G = ?$

p. 161

Example Problem 4-1



$$\delta_{lat} = 3.9986 - 4 = -0.0014\,\text{in} \qquad \varepsilon_{lat} = \frac{\delta_{lat}}{L} = \frac{-0.0014}{4} = -0.00035$$

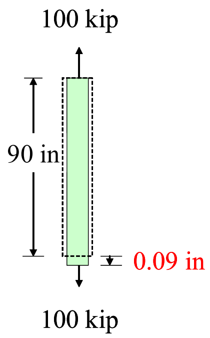
$$\varepsilon_{long} = \frac{\delta_{long}}{L} = \frac{0.09}{90} = 0.00100$$

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{0.00035}{0.00100} = 0.35$$

$$\sigma = \frac{P}{A} = \frac{100}{4(1)} = 25\,\text{ksi}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{25}{0.00100} = 25,000\,\text{ksi}$$

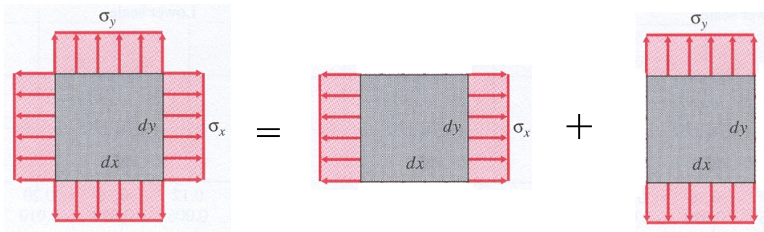
$$G = \frac{E}{2(1+\nu)} = \frac{25,000}{2(1+0.35)} = 9260\,\text{ksi}$$



p. 164

4-3 Generalized Hooke's Law

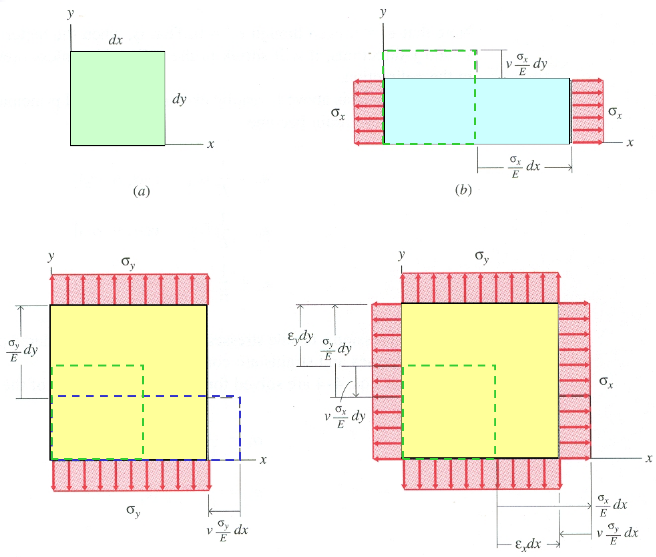
Principle of **superposition**



- Valid under conditions of
  - σ ~ ε linear (σ < proportional limit)
  - effect σ_x does not significantly change the effect of σ_y (if ε small → ΔA small)

p. 165

4-3 Generalized Hooke's Law



p. 165

$\sigma_z = 0$

Generalized Hooke's Law – Biaxial Stress

- Isotropic material: material properties **independent of direction**

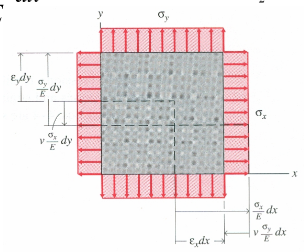
$$d\delta_x = \varepsilon_x dx = \frac{\sigma_x}{E}dx - \nu\frac{\sigma_y}{E}dx \qquad \varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$$

$$d\delta_y = \varepsilon_y dx = \frac{\sigma_y}{E}dx - \nu\frac{\sigma_x}{E}dx \quad \Rightarrow \quad \varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right)$$

$$d\delta_z = \varepsilon_z dx = -\nu\frac{\sigma_x}{E}dx - \nu\frac{\sigma_y}{E}dx \qquad \varepsilon_z = -\frac{\nu}{E}\left(\sigma_x + \sigma_y\right)$$

$$\Rightarrow \quad \sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right)$$

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right)$$



p. 166

Generalized Hooke's Law – **Triaxial** Stress

For $\sigma_z = 0$, $\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$ $\qquad \sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right)$

third component

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] \quad \Rightarrow$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu\left(\varepsilon_y + \varepsilon_z\right)\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_y + \nu\left(\varepsilon_z + \varepsilon_x\right)\right]$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_z + \nu\left(\varepsilon_x + \varepsilon_y\right)\right]$$

p. 166

Hooke's Law for **Shear Stress**

$$\tau = G\gamma$$

$$\tau_{xy} = G\gamma_{xy}$$

$$\tau_{yz} = G\gamma_{yz}$$

$$\tau_{zx} = G\gamma_{zx}$$

p. 167

Relationship between E, ν, and G

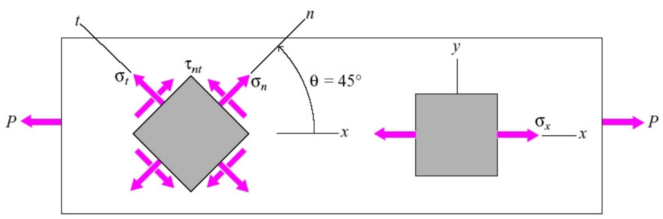


Figure 4-12

(from Eq. 2-13a)

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -(\sigma_x - 0)\sin(45°)\cos(45°) + 0$$

$$= -\frac{\sigma_x}{2}$$

(from Eq. 3-8a)

$$\gamma_{nt} = -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -2(\varepsilon_x - \varepsilon_y)\sin(45°)\cos(45°) + 0$$

$$= -(\varepsilon_x - \varepsilon_y)$$

p. 166

ε_z in Plane Stress ($\sigma_z = 0$)

$$\varepsilon_z = -\frac{\nu}{E}\left(\sigma_x + \sigma_y\right)$$

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) \qquad \sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right)$$

$$\varepsilon_z = -\frac{\nu}{E}\left[\frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) + \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right)\right]$$

$$= -\frac{\nu}{E}\left(\frac{E}{1-\nu^2}\right)\left[\left(\varepsilon_x + \nu\varepsilon_y\right) + \left(\varepsilon_y + \nu\varepsilon_x\right)\right]$$

$$= -\frac{\nu}{1-\nu}\left(\varepsilon_x + \varepsilon_y\right)$$

see Chapter 3 Eq 3-13.

For $\nu = 1/3 \rightarrow \nu/(1-\nu) = 1/2$,
ε_z is simply the average of ε_x and ε_y.

p. 167

Relationship between E, ν, and G

For the element subjected to an axial load,

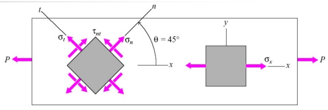


Figure 4-12

$$\nu = \frac{-\varepsilon_t}{\varepsilon_a} = \frac{-\varepsilon_y}{\varepsilon_x} \quad \Rightarrow \quad \varepsilon_y = -\nu\varepsilon_x$$

$$\gamma_{nt} = -(\varepsilon_x - \varepsilon_y) \quad \Rightarrow \quad \gamma_{nt} = -\varepsilon_x(1+\nu)$$

$$\tau_{nt} = G\gamma_{nt} = -G\varepsilon_x(1+\nu)$$

$$= -G\frac{\sigma_x}{E}(1+\nu) \quad \Rightarrow \quad G = \frac{E}{2(1+\nu)}$$

$$(\because \tau_{nt} = -\frac{\sigma_x}{2})$$

$$= -G\frac{-2\tau_{nt}}{E}(1+\nu)$$

p. 167

Relationship between E, ν, and G

$$\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$$

$$\Rightarrow \quad \tau_{yz} = G\gamma_{yz} = \frac{E}{2(1+\nu)}\gamma_{yz}$$

$$\tau_{zx} = G\gamma_{zx} = \frac{E}{2(1+\nu)}\gamma_{zx}$$

Biaxial Stress for isotropic materials

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\sigma_z = 0$$

$$\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$$

$$\tau_{yz} = G\gamma_{yz} = \frac{E}{2(1+\nu)}\gamma_{yz}$$

$$\tau_{zx} = G\gamma_{zx} = \frac{E}{2(1+\nu)}\gamma_{zx}$$

$$G = \frac{E}{2(1+\nu)}$$

p. 168

Example Problem 4-2 (I)

- Biaxial stress
  - $E = 210$ GPa, $\nu = 0.30$
  - $\varepsilon_x = +1394$ μm/m
    $\varepsilon_y = -660$ μm/m
    $\gamma_{xy} = 2054$ μrad

Determine

- σ_x , σ_y , $\tau_{xy} = ?$
- σ_p , $\tau_{max} = ?$

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) = \frac{210\left(10^9\right)}{1-0.3^2}\left[1394+0.3(-660)\right]\left(10^{-6}\right)$$
$$= +276.0\left(10^6\right)\text{N/m}^2 \cong 276\text{ MPa (T)}$$

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) = \frac{210\left(10^9\right)}{1-0.3^2}\left[-660+0.3(1394)\right]\left(10^{-6}\right)$$
$$= -55.80\left(10^6\right)\text{N/m}^2 \cong 55.8\text{ MPa (C)}$$

p. 169

Example Problem 4-2 (II)

$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} = \frac{210\left(10^9\right)}{2(1+0.30)}(2054)\left(10^{-6}\right) = 165.90\left(10^6\right)\text{N/m}^2 \cong 165.9\text{ MPa}$$

$$\sigma_{p1,p2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{276.0-55.80}{2} \pm \sqrt{\left(\frac{276.0+55.80}{2}\right)^2 + 165.90^2}$$

$$\sigma_{p1} = 110.10 + 234.62 = 344.72\text{ MPa} \cong 345\text{ MPa (T)}$$

$$\sigma_{p2} = 110.10 - 234.62 = -124.52\text{ MPa} \cong 124.5\text{ MPa (C)}$$

$$\sigma_{p3} = \sigma_z = 0$$

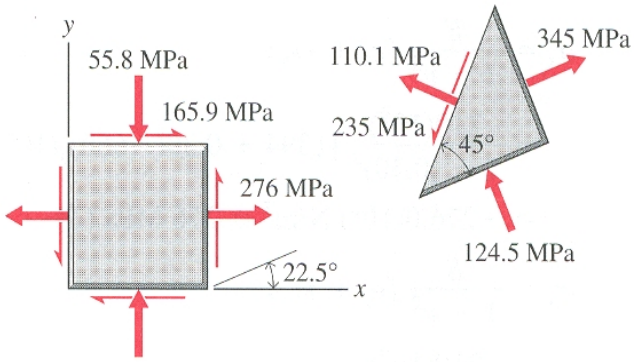
$$\tau_{max} = \tau_p = \frac{\sigma_{p1}-\sigma_{p2}}{2} = \frac{344.72+124.52}{2} \cong 235\text{ MPa}$$

p. 170

Example Problem 4-2 (III)

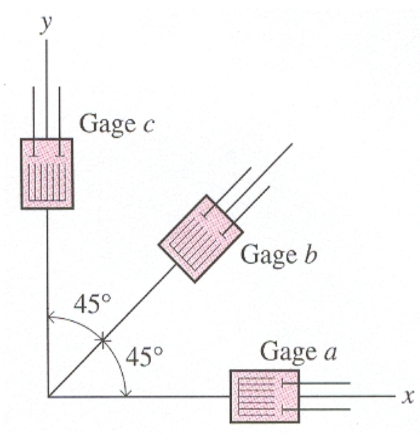
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(165.90)}{276.0 + 55.80} = 1.000$$

$$2\theta_p = 45.00° \qquad \theta_p = 22.5°$$



p. 170

Example Problem 4-3 (I)



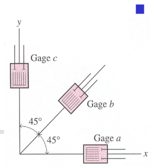
- Plane stress
  - E = 30,000 ksi
    ν = 0.30
  - ε_a = +650 μin/in
    ε_b = +475 μin/in
    ε_c = –250 μin/in

Determine

- σ_x , σ_y , τ_{xy} = ?
- ε_p , γ_{max} = ?
- σ_p , τ_{max} = ?

p. 171

Example Problem 4-3 (II)



- Plane stress
  - $E = 30{,}000$ ksi
    $\nu = 0.30$
  - $\varepsilon_a = +650$ μin/in
    $\varepsilon_b = +475$ μin/in
    $\varepsilon_c = -250$ μin/in

$\varepsilon_a = \varepsilon_x = +650\mu$ (measured) $\qquad \varepsilon_c = \varepsilon_y = -250\mu$ (measured)

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$$

$$= 650\mu\cos^2 45° - 250\mu\sin^2 45° + \gamma_{xy}\sin 45°\cos 45° = +475\mu$$

$$\Rightarrow \quad \gamma_{xy} = 550\mu$$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{30{,}000}{1-0.3^2}[650 + 0.3(-250)](10^{-6})$$

$$= +18.956\,\text{ksi} \cong 18.96\,\text{ksi (T)}$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{30{,}000}{1-0.3^2}[-250 + 0.3(650)](10^{-6})$$

$$= -1.8132\,\text{ksi} \cong 1.813\,\text{ksi (C)}$$

$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} = \frac{30{,}000}{2(1+0.30)}(550)(10^{-6}) = 6.346\,\text{ksi} \cong 6.35\,\text{ksi}$$

p. 171

Example Problem 4-3 (III)

$$\varepsilon_{p1}, \varepsilon_{p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{650\mu - 250\mu}{2} \pm \sqrt{\left(\frac{650\mu + 250\mu}{2}\right)^2 + \left(\frac{550\mu}{2}\right)^2}$$

$$= 200\mu \pm 527.4\mu$$

$\varepsilon_{p1} \cong 727\mu \qquad \varepsilon_{p2} \cong -327\mu$

$$\varepsilon_{p3} = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{0.3}{1-0.3}(650\mu - 250\mu) = -171.4\mu$$

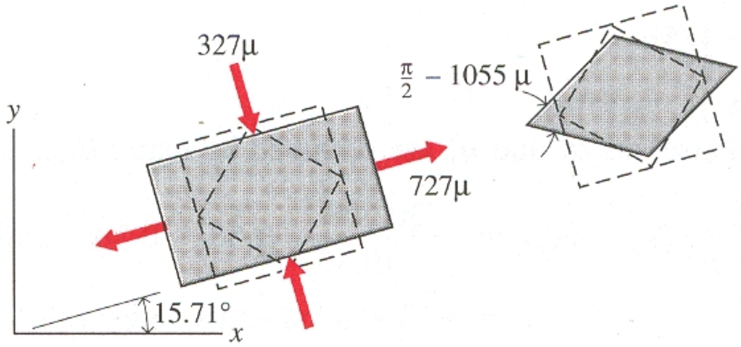
$$\gamma_{\max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 724.4\mu + 327.4\mu \cong 1055\mu$$

p. 171

Example Problem 4-3 (IV)

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{550\mu}{650\mu + 250\mu} = 0.61111$$

$$2\theta_p = 31.429° \qquad \theta_p \cong 15.71°$$



p. 172

Example Problem 4-3 (V)

$$\sigma_{p1} = \frac{E}{1-\nu^2}\left(\varepsilon_{p1} + \nu\varepsilon_{p2}\right) = \frac{30,000}{1-0.3^2}\left[727.4 + 0.3(-327.4)\right]\left(10^{-6}\right)$$
$$= +20.742 \text{ ksi} \cong 20.76 \text{ ksi (T)}$$

$$\sigma_{p2} = \frac{E}{1-\nu^2}\left(\varepsilon_{p2} + \nu\varepsilon_{p1}\right) = \frac{30,000}{1-0.3^2}\left[-327.4 + 0.3(727.4)\right]\left(10^{-6}\right)$$
$$= -3.599 \text{ ksi} \cong 3.60 \text{ ksi (C)}$$

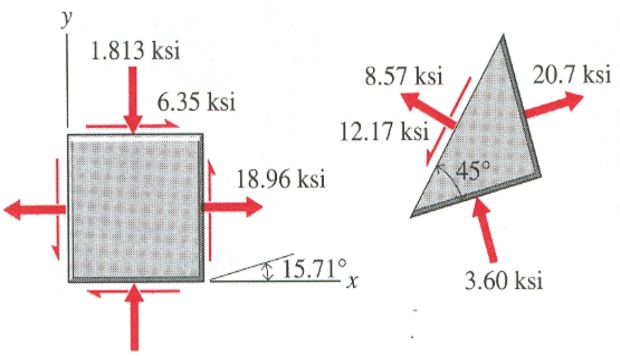
$$\sigma_{p3} = \sigma_z = 0$$

$$\tau_{\max} = \frac{E}{2(1+\nu)}\gamma_{\max} = \frac{30,000}{2(1+0.30)}(1054.8)\left(10^{-6}\right) = 12.171 \text{ ksi} \cong 12.17 \text{ ksi}$$

$$\left(\tau_{max} = \tau_p = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \frac{20.742 + 3.599}{2} \cong 12.17\ MPa\ \right)$$

p. 172

Example Problem 4-3 (VI)



- Note:
  The principal directions for stress and strain are the same for isotropic material.

p. 176

4-4 Thermal Strain

$\varepsilon_T = \alpha \Delta T$ α: coefficient of thermal expansion

normal strain

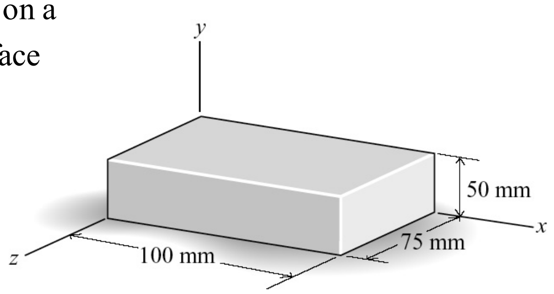
$$\varepsilon_{\text{total}} = \varepsilon_\sigma + \varepsilon_T = \frac{\sigma}{E} + \alpha \Delta T$$

- Homogeneous, isotropic materials expand uniformly in all directions when heated. NO **shear strain**.
- α is constant for a large range of temperature.

p. 177

Example Problem 4-6

- Aluminum block rests on a smooth horizontal surface
  - $E = 70$ GPa
    $\alpha = 22.5\times10^{-6}$ /ºC



Find:

(a) ε_{Tx}, ε_{Ty} =? for $\Delta T = +20$ºC

(b) δ_x, δ_y, δ_z = ?

(c) γ_{xy} = ?

p. 177

Example Problem 4-6

Sol: (a) Thermal strain $\varepsilon_T = \alpha\,\Delta T$

$$\varepsilon_T = \varepsilon_{Tx} = \varepsilon_{Ty} = \varepsilon_{Tz} = \alpha\Delta T$$
$$= 22.5\cdot10^{-6}\cdot20$$
$$= 450\cdot10^{-6}\ \text{m/m} = 450\ \mu\text{m/m}$$

(b) Deformation $\delta = \varepsilon_T L = \alpha\cdot\Delta T\cdot L$

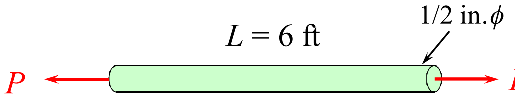
$$\delta_x = \varepsilon_{Tx}L_x = 450\cdot10^{-6}\cdot0.1 = 45\cdot10^{-6}\ \text{m}$$
$$\delta_y = \varepsilon_{Ty}L_y = 450\cdot10^{-6}\cdot0.05 = 22.5\cdot10^{-6}\ \text{m}$$
$$\delta_z = \varepsilon_{Tz}L_z = 450\cdot10^{-6}\cdot0.075 = 33.8\cdot10^{-6}\ \text{m}$$

(c) Shearing strain $\gamma_{xy} = 0$, since the block remains a rectangular parallelepiped when unconstrained.

p. 178

Example Problem 4-7

Given:



steel: $E = 30{,}000$ ksi, $\alpha = 6.5(10^{-6})$/°F

$P = 5000$ lb, $\Delta T = -50$ °F

Find $\delta = ?$

Sol: $\varepsilon = \frac{\delta}{L} = \frac{\sigma}{E} + \alpha\Delta T = \frac{P}{EA} + \alpha\Delta T$

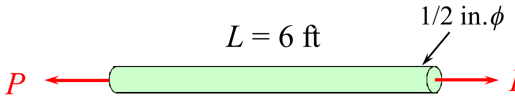
$$\delta = \left(\frac{P}{EA} + \alpha\Delta T\right)L = \left[\frac{5000}{30(10^6)(\pi/4)(1/2)^2} + 6.5(10^{-6})(-50)\right](6)$$

$= 0.003143\,\text{ft} \cong 0.0377\,\text{in}$ $\delta = 0.0845$ in for $\Delta T = +50$°F

p. 179

Example Problem 4-7

Given:



steel: $E = 30{,}000$ ksi, $\alpha = 6.5(10^{-6})$/°F

$P = 5000$ lb, $\Delta T = -50$ °F

Find: $\delta_d = ?$ (change in diameter)

Sol: $\varepsilon_{dP} = -\nu\varepsilon_P$; $\delta_{dP} = -\nu\delta_P$

$$\delta_d = \delta_{dP} + \delta_{dT} = -\nu\frac{Pd}{AE} + \alpha d\Delta T$$

Note: multiply by d

$$= -\frac{0.3\cdot 5000\cdot 0.5}{(\pi 0.5^2/4)\cdot 30\cdot 10^6} + 6.5\cdot 10^{-6}\cdot 0.5\cdot(-50)$$

$= -0.000290\,in$

8 Exercises

4-5,	4-6,	4-8,	4-16,
4-23,	4-32,	4-49,	4-51