Mechanics of Materials

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Chapter 5

Axial Loading Applications and Pressure Vessels

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Contents

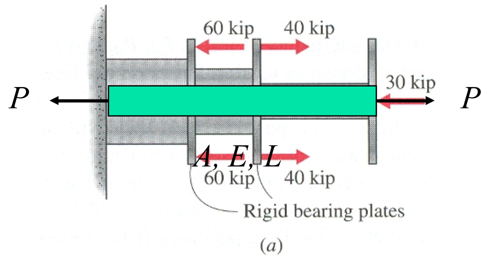
Axial Loading Applications and Pressure Vessels

p. 189

5-2 Deformation of Axially Loaded Members

- Uniform Member

$$\delta = \varepsilon L = \frac{\sigma L}{E} = \frac{PL}{AE}$$

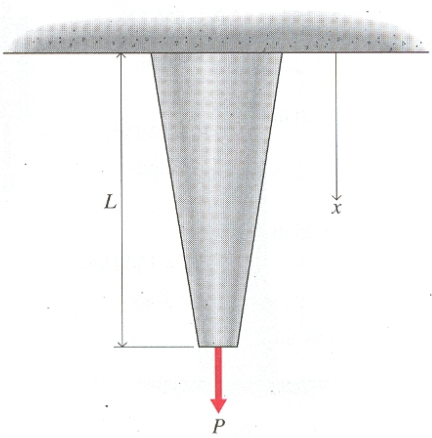


- Multiple Loads/Sizes

$$\delta = \sum_{i=1}^{n} \delta_i = \sum_{i=1}^{n} \frac{\sigma_i L_i}{E_i} = \sum_{i=1}^{n} \frac{P_i L_i}{E_i A_i}$$

p. 190

Nonuniform Deformation



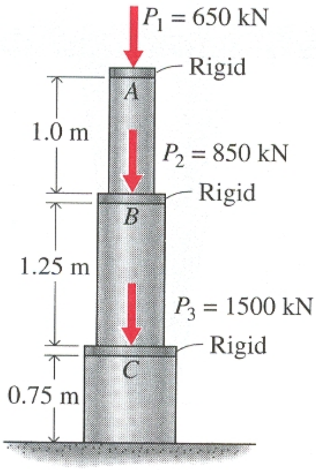
$$\varepsilon = \frac{d\delta}{dx}$$

$$d\delta = \varepsilon dx = \frac{\sigma}{E} dx = \frac{P_x}{EA_x} dx$$

$$\delta = \int_0^L d\delta = \int_0^L \frac{P_x}{EA_x} dx$$

p. 191

Example Problem 5-1 (I)

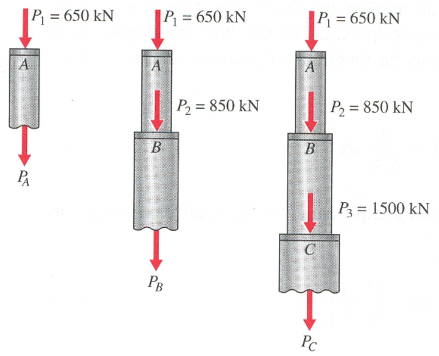


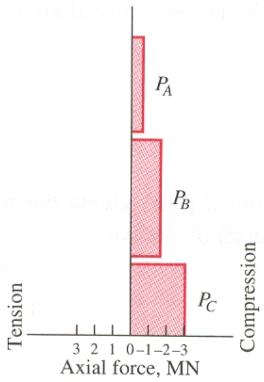
- Solid aluminum bar A
  - d = 100mm
  - E = 73 Gpa
- Brass tube B
  - d_o = 150mm, d_i = 100mm
  - E = 100 Gpa
- Steel pipe C
  - d_o = 200mm , d_i = 125mm
  - E = 210 Gpa

- Determine δ_{total} = ?

p. 191

Example Problem 5-1(II)





$$\sum F = -P_A - 650 = 0 \qquad P_A = -650 \text{ kN} = 650 \text{ kN (C)}$$

$$\sum F = -P_B - 650 - 850 = 0 \qquad P_B = -1500 \text{ kN} = 1500 \text{ kN (C)}$$

$$\sum F = -P_c - 650 - 850 - 1500 = 0 \qquad P_C = -3000 \text{ kN} = 3000 \text{ kN (C)}$$

p. 191

Example Problem 5-1 (III)

$$A_A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(100)^2 = 7584\,\text{mm}^2 = 0.007584\,\text{m}^2$$

$$A_B = \frac{\pi}{4}\left(d_o^2 - d_i^2\right)^2 = \frac{\pi}{4}\left(150^2 - 100^2\right) = 9817\,\text{mm}^2 = 0.009817\,\text{m}^2$$

$$A_C = \frac{\pi}{4}\left(d_o^2 - d_i^2\right)^2 = \frac{\pi}{4}\left(200^2 - 125^2\right) = 19{,}144\,\text{mm}^2 = 0.019144\,\text{m}^2$$

$$\delta_A = \frac{P_A L_A}{E_A A_A} = \frac{-650(10^3)(1.0)}{73(10^9)(0.007854)} = -1.1337(10^{-3})\,\text{m} = -1.1337\,\text{mm}$$

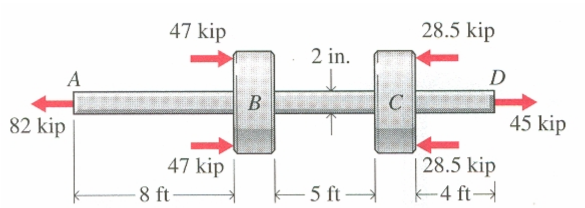
$$\delta_B = \frac{P_B L_B}{E_B A_B} = \frac{-1500(10^3)(1.25)}{100(10^9)(0.009817)} = -1.9100(10^{-3})\,\text{m} = -1.9100\,\text{mm}$$

$$\delta_C = \frac{P_C L_C}{E_C A_C} = \frac{-3000(10^3)(0.75)}{210(10^9)(0.019144)} = -0.5597(10^{-3})\,\text{m} = -0.5597\,\text{mm}$$

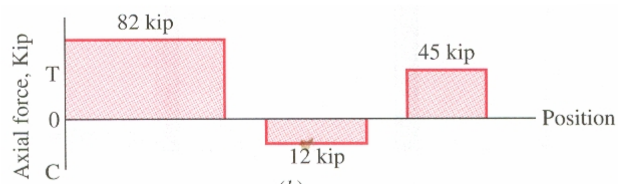
$$\delta_{\text{total}} = \delta_A + \delta_B + \delta_C = -3.6034\,\text{mm} \cong -3.60\,\text{mm}$$

p. 192

Example Problem 5-2 (I)



(a)



(b)

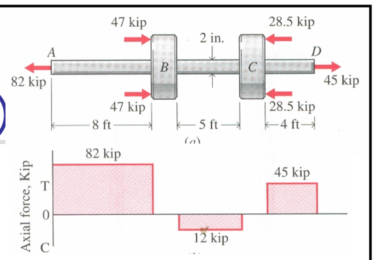
- Yokes (軛) B and C :
  - rigid
- Steel bar *AD*
  - *A* = 2 in × 2 in
  - *E* = 30,000 ksi

Determine

- $\sigma_{max} = ?$
- $\delta_{AB} = ?$
- $\delta_{BC} = ?$
- $\delta_{AD} = ?$

p. 193

Example Problem 5-2 (II)



$$\sigma_{max} = \frac{P_{max}}{A} = \frac{82}{4} = 20.5 \text{ ksi (T)}$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{E_{AB}A_{AB}} = \frac{+82(8)(12)}{30{,}000(4)} = +0.06560 \text{ in} \cong +0.0656 \text{ in}$$

$$\delta_{BC} = \frac{P_{BC}L_{BC}}{E_{BC}A_{BC}} = \frac{-12(5)(12)}{30{,}000(4)} = -0.00600 \text{ in}$$

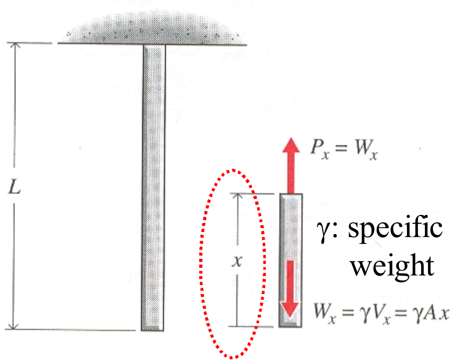
$$\delta_{CD} = \frac{P_{CD}L_{CD}}{E_{CD}A_{CD}} = \frac{+45(4)(12)}{30{,}000(4)} = +0.01800 \text{ in}$$

- Determine
  - $\sigma_{max} = ?$
  - $\delta_{AB} = ?$
  - $\delta_{BC} = ?$
  - $\delta_{AD} = ?$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$
$$= +0.06560 - 0.00600 + 0.01800 = +0.07760 \text{ in} \cong +0.0776 \text{ in}$$

p. 193

Example Problem 5-3



$$\delta = \int_0^L \frac{P_x}{EA_x} dx$$

$$= \int_0^L \frac{1}{EA} \gamma A x \, dx = \frac{\gamma}{E} \int_0^L x \, dx$$

$$= \left. \frac{\gamma x^2}{2E} \right|_0^L = \frac{\gamma L^2}{2E}$$

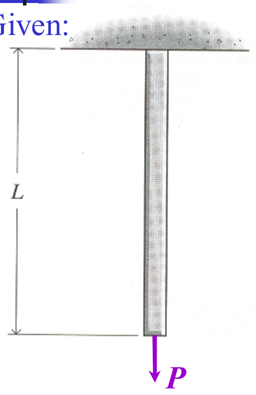
$$= \frac{W}{AL}\left[\frac{L^2}{2E}\right] = \frac{WL}{2AE}$$

- Homogeneous bar
  - W, L, A, E
- Determine $\delta = ?$

p. 194

Example Problem 5-3

Given:



Find: (b) δ = ? for an applied P at the end

Sol: Method of superposition is applicable for linear problems

$$\delta = \delta_W + \delta_P$$

$$= \frac{WL}{2EA} + \frac{PL}{EA} = \frac{L}{EA}\left(\frac{W}{2} + P\right)$$

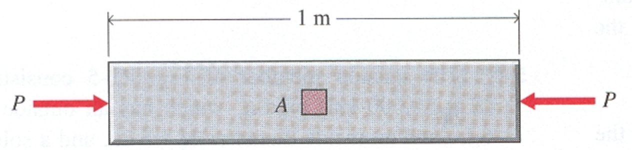
For your reference,

$$\boldsymbol{F = kx}$$

$$\boldsymbol{k = \frac{EA}{L}}$$

p. 194

Example Problem 5-4 (I)



=180 kN

- Steel bar
  - E = 200 Gpa
  - $Area$ = 30 × 30 mm

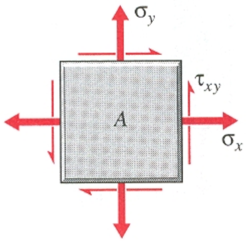
Determine

- δ = ?
- σ_x, σ_y, τ_{xy}, on element A = ?

p. 194

Example Problem 5-4 (II)

$$\delta = \frac{PL}{EA} = \frac{-180(10^3)(1)}{200(10^9)(0.030)^2} = -1.000(10^{-3})\,\text{m} = -1.000\ \text{mm}$$



$$\sigma_x = \frac{P}{A} = \frac{-180(10^3)}{(0.030)^2}$$

$$= -200(10^6)\,\text{N/m}^2 = 200\ \text{MPa (C)}$$

$$\sigma_y = 0$$

$$\tau_{xy} = 0$$

p. 195

Example Problem 5-4 (III)

The bar is subjected to an axial load.

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{-200\cdot 10^6}{200\cdot 10^9} = -0.00100 = -1000\ \mu\text{m/m}$$

$$\varepsilon_y = -\nu\varepsilon_x = -0.3\cdot(-1000) = 300\ \mu\text{m/m}$$

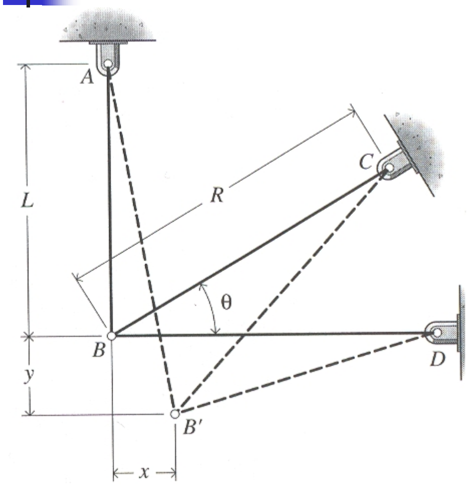
$$\varepsilon_z = -\nu\varepsilon_x = -0.3\cdot(-1000) = 300\ \mu\text{m/m}$$

The bar is subjected to an axial load only, no shearing force exists.

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0\ \mu\text{rad}$$

p. 201

5-3 Deformations in a System of Axially Loaded Bars



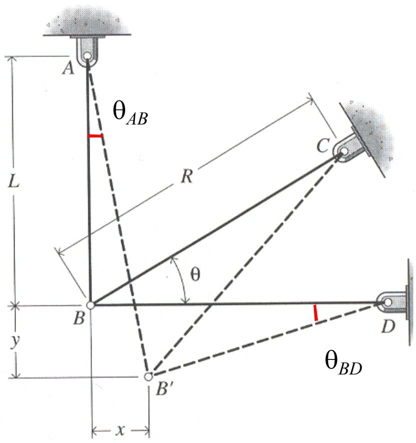
- Unknowns:
  - P_{AB}, P_{BC}, P_{BD}
- Equations
  - $\Sigma F_x = 0$
  - $\Sigma F_y = 0$

⇨ statically indeterminate

⇨ Another Eq. is need (compatibility)

p. 202

Compatibility of Strains



$$\delta_{AB} = \sqrt{(L+y)^2 + x^2} - L$$

$$\delta_{AB}^2 + 2L\delta_{AB} + L^2 = L^2 + 2Ly + y^2 + x^2$$

$$\delta_{AB} \cong y$$

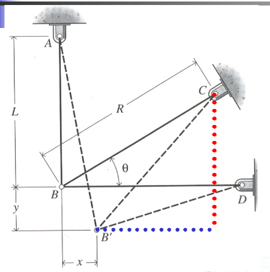
Similarly, $\delta_{BD} \cong x$

$$\theta_{AB} \cong \tan\theta_{AB} = \frac{x}{L_{AB}}$$

$$\theta_{BD} \cong \tan\theta_{BD} = \frac{y}{L_{BD}}$$

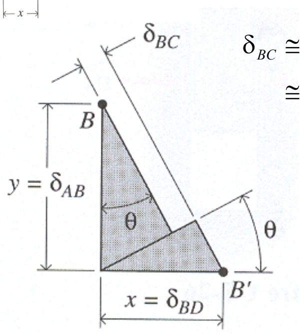
p. 203

Compatibility of Strains



$$\delta_{BC} = \sqrt{(R\cos\theta - x)^2 + (R\sin\theta + y)^2} - R$$

$$\delta_{BC}^2 + 2R\delta_{BC} + R^2 = R^2\cos^2\theta - 2Rx\cos\theta + x^2 + R^2\sin^2\theta + 2Ry\sin\theta + y^2$$

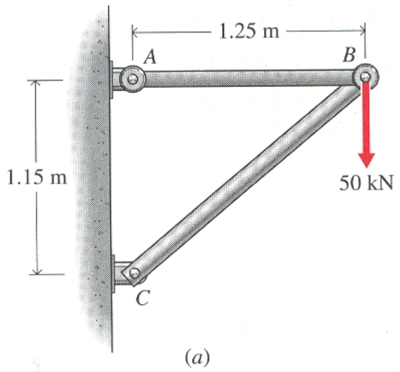


$$\delta_{BC} \cong y\sin\theta - x\cos\theta$$

$$\cong \delta_{AB}\sin\theta - \delta_{BD}\cos\theta$$

p. 203

Example Problem 5-5 (I)



(a)

- $A_{AB} = 650\ \text{mm}^2$
- $A_{BC} = 925\ \text{mm}^2$
- $E = 200$ GPa

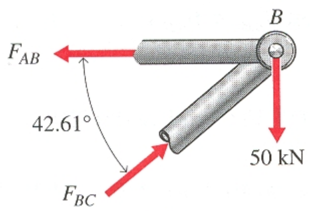
Determine

- σ_{AB} , σ_{BC} = ?
- δ_{AB} , δ_{BC} = ?
- δ_h , δ_v at B = ?
- θ_{AB} , θ_{BC} = ?

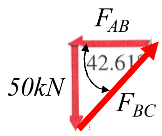
p. 203

Example Problem 5-5 (II)

- $A_{AB} = 650 \text{ mm}^2$
- $A_{BC} = 925 \text{ mm}^2$
- $E = 200 \text{ GPa}$



(b)



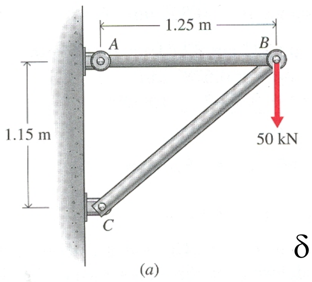
$$\boldsymbol{F_{BC}} = 50/\sin 42.61° = 73.85 \text{ kN (C)}$$
$$\boldsymbol{F_{AB}} = 50/\tan 42.61° = 54.36 \text{ kN (T)}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{+54.36(10^3)}{650(10^{-6})}$$
$$= +83.63(10^6)\text{ N/m}^2 \cong 83.6 \text{ MPa (T)}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{-73.85(10^3)}{925(10^{-6})}$$
$$= -79.84(10^6)\text{ N/m}^2 \cong 79.8 \text{ MPa (C)}$$

p. 204

Example Problem 5-5 (III)



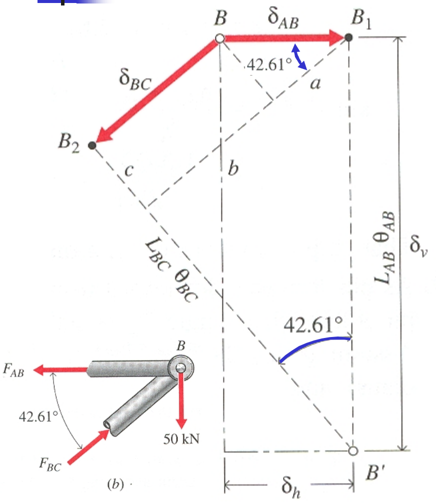
(a)

$$\delta_{AB} = \frac{\sigma_{AB} L_{AB}}{E} = \frac{+83.63(10^6)(1.25)}{200(10^9)}$$
$$= +0.5227(10^{-3})\text{ m} \cong +0.523 \text{ mm}$$

$$\delta_{BC} = \frac{\sigma_{BC} L_{BC}}{E} = \frac{-79.84(10^6)(1.699)}{200(10^9)}$$
$$= -0.6782(10^{-3})\text{ m} \cong -0.678 \text{ mm}$$

p. 204

Example Problem 5-5 (IV)



$$\delta_h = \delta_{AB} = +0.5227\ \text{mm} \cong 0.523\ \text{mm}$$

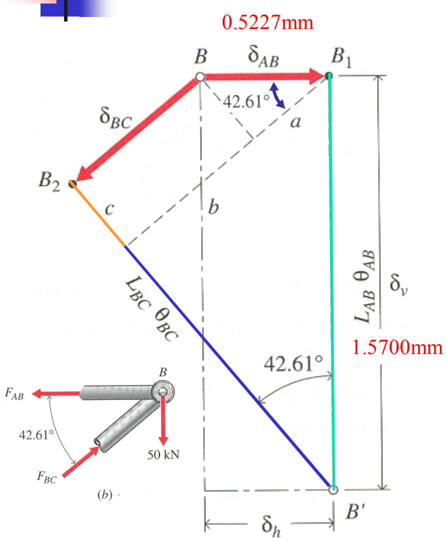
$$a = \delta_{AB} \cos 42.61° = 0.5227 \cos 42.61°$$
$$= 0.3847\ \text{mm}$$

$$\sin 42.61° = \frac{b+a}{\delta_v} = \frac{\delta_{BC}+a}{\delta_v}$$
$$= \frac{0.6782+0.3847}{\delta_v} = \frac{1.0629}{\delta_v}$$

$$\delta_v = \frac{1.0629}{\sin 42.61°} = 1.5700\ \text{mm}$$
$$\cong 1.570\ \text{mm}$$

p. 205

Example Problem 5-5 (V)



$$\theta_{AB} \cong \tan\theta_{AB} = \frac{\delta_v}{L_{AB}} = \frac{1.570}{1250}$$
$$= 0.001256\ \text{rad} \cong 0.0720°$$

$$\theta_{BC} \cong \tan\theta_{BC} = \frac{c+\delta_v \cos 42.61°}{L_{BC}}$$
$$= \frac{0.5227 \sin 42.61° + 1.5700 \cos 42.61°}{\sqrt{1150^2+1250^2}}$$
$$= 0.000889\ \text{rad} \cong 0.0509°$$

p. 209

5-4 Statically Indeterminate Axially Loaded Members

- Statically determinate (靜定)

  Equilibrium eqs. = unknowns (no. of member force + reactions)

- Statically indeterminate (靜不定)

  No. of equilibrium eqs. < unknowns (no. of member force + reactions)

p. 209

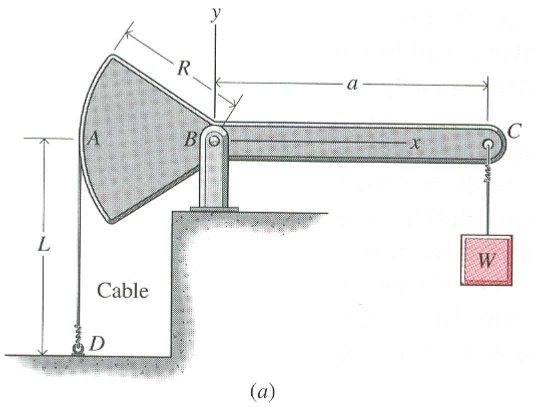
5-4 Statically Indeterminate Axially Loaded Members

- Procedure
  - Draw a free-body diagram.
  - Note the number of unknowns involved.
  - Note the number of independent equations.
  - If no. unknowns > no. equil. eqs. ⇒ stat. indeterminate
    ⇒ Write deformation equations.
  - Solve equilibrium (and deformation if needed) equations.

- Assumption
  - The body is rigid when solving the equilibrium equations

p. 210

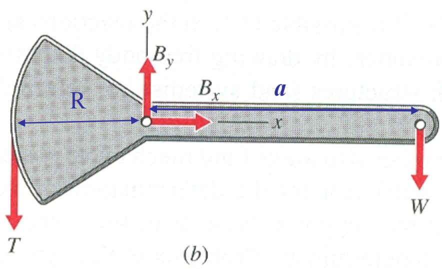
Example



- Condition 1
  Cable rigid
- Condition 2
  Cable deformable

p. 210

Condition 1 – Cable Rigid

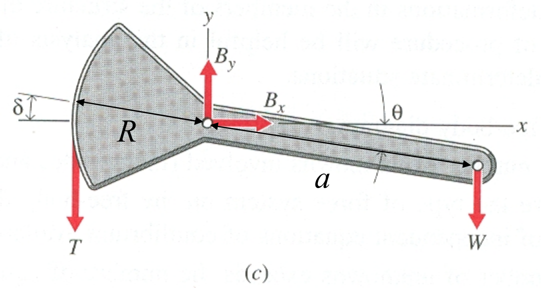


$$\sum M_B = TR - Wa = 0$$

$$T = \frac{Wa}{R}$$

p. 210

Condition 2 - Cable Deformable



$$\delta = \frac{TL}{EA}$$

$$\frac{\delta EA}{L} = T = \frac{Wa}{R}\cos\theta$$

$$\delta = R\theta$$

$$\sum M_B = TR - Wa\cos\theta = 0$$

$$T = \frac{Wa}{R}\cos\theta$$

$$\boldsymbol{R^2 EA\theta = WaL\cos\theta}$$

Solved by iterations

p. 211

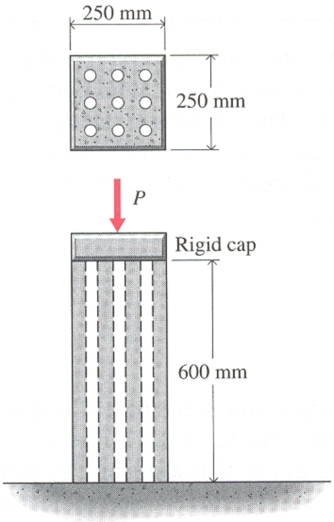
Example – W = 100 lb, a = 30 in, R = 15 in

Case	θ (°)	T (lb)	$\%D = 100\,\% \times (T_{rigid} - T)/T$
rigid cable	0	200	0
steel cable d = 3/32 in. E = 29,000 ksi	0.1717	199.999	0.0005%
aluminum cable d = 3/32 in. E = 10,600 ksi	0.4698	199.993	0.0035%

error acceptable, cable can be assumed rigid

p. 212

Example Problem 5-7 (I)



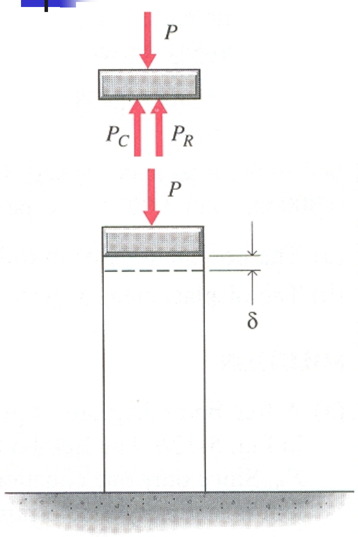
- Concrete pier
  - E = 30 GPa
- Steel bars (25mm ϕ)
  - E = 200 GPa
- P = 650 kN

Determine

- σ_C (concrete) = ?
- σ_R (steel) = ?
- δ (pier) = ?

p. 212

Example Problem 5-7 (II)



$$\sum F_y = P_R + P_C - P = 0$$ equilibrium

$$\delta_R = \delta_C \Rightarrow \frac{P_R L_R}{E_R A_R} = \frac{P_C L_C}{E_C A_C}$$ deformation

$$A_R = 9\left(\pi/4\right)(25)^2 = 4418 \text{ mm}^2$$

$$A_C = (250)^2 - 4418 = 58{,}080 \text{ mm}^2$$

$$\frac{P_R(0.600)}{200(10^9)(4418)(10^{-6})} = \frac{P_C(0.600)}{30(10^9)(58{,}080)(10^{-6})}$$

$$P_R = 0.5071 P_C$$

p. 213

Example Problem 5-7 (III)

$$\begin{cases} P_R + P_C - 650\left(10^3\right) = 0 \\ P_R = 0.5071 P_C \end{cases} \Rightarrow \quad \begin{aligned} P_R &= 218.7\left(10^3\right)\text{N} \cong 219\text{ kN (C)} \\ P_C &= 431.3\left(10^3\right)\text{N} \cong 431\text{ kN (C)} \end{aligned}$$

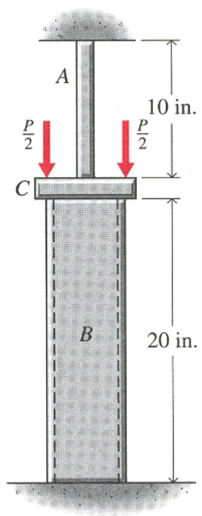
$$\Rightarrow \quad \sigma_R = \frac{P_R}{A_R} = \frac{218.7\left(10^3\right)}{4418\left(10^{-6}\right)} \cong 49.5\text{ MPa (C)}$$

$$\sigma_C = \frac{P_R}{A_R} = \frac{431.3\left(10^3\right)}{58{,}080\left(10^{-6}\right)} \cong 7.43\text{ MPa (C)}$$

$$\delta = \delta_C = \delta_R = \frac{\sigma_R L_R}{E_R} = \frac{49.5\left(10^6\right)(0.600)}{200\left(10^9\right)} \cong 0.1485\text{ mm}$$

p. 214

Example Problem 5-8 (I)



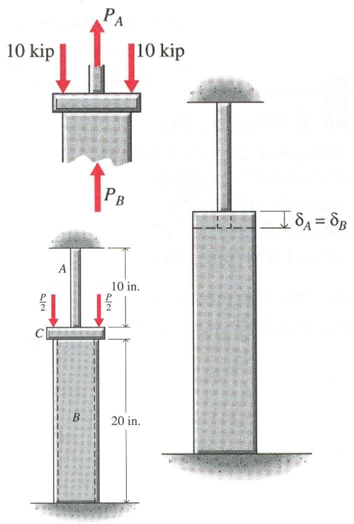
- Plate C : rigid
- Steel Rod A
  - E = 30,000 ksi
  - A = 0.800 in^2
- Aluminum alloy pipe B
  - E = 10,000 ksi
  - A = 3.00 in^2
- P = 20 kip

Determine:

- σ_A = ?
- σ_B = ?
- δ_C = ?

p. 214

Example Problem 5-8 (II)



$$\sum F_y = P_A + P_B - 20 = 0$$ equilibrium

$$\Rightarrow \quad 0.800\sigma_A + 3.00\sigma_B = 20$$

$$\delta_A = \delta_B \quad \Rightarrow \quad \frac{\sigma_A L_A}{E_R} = \frac{\sigma_B L_B}{E_B}$$ deformation

$$\frac{\sigma_A(10)}{30{,}000} = \frac{\sigma_B(20)}{10{,}000}$$

$$\Rightarrow \quad \sigma_A = 6\sigma_B$$

p. 215

Example Problem 5-8 (III)

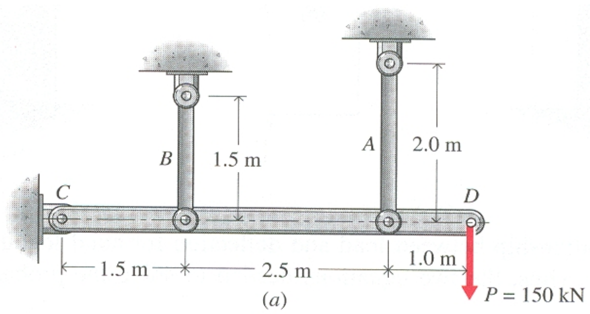
$$\begin{cases} 0.800\sigma_A + 3.00\sigma_B = 20 \\ \sigma_A = 6\sigma_B \end{cases}$$

$$\Rightarrow \quad \begin{aligned} \sigma_A &= 15.384 \text{ ksi} \cong 15.38 \text{ ksi (T)} \\ \sigma_B &= 2.564 \text{ ksi} \cong 2.56 \text{ ksi (C)} \end{aligned}$$

$$\delta = \delta_A = \delta_B = \frac{\sigma_A L_A}{E_A} = \frac{15.384(10)}{30{,}000} \cong 0.00513 \text{ in } (\downarrow)$$

p. 216

Example Problem 5-9 (I)



(a)

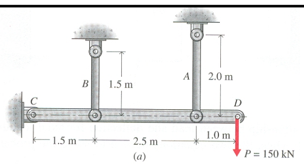
- Member *CD* : rigid
- Aluminum alloy bar *A*
  - $E = 75$ GPa
  - $A = 1000$ mm^2
- Steel bar *B*
  - $E = 200$ GPa
  - $A = 500$ mm^2

Determine:

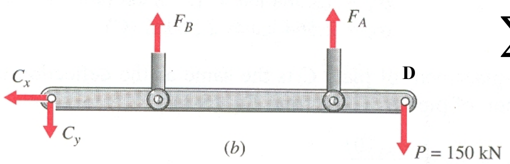
- $\sigma_A = ?$
- $\sigma_B = ?$
- $\delta_D = ?$

p. 216

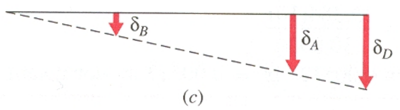
Example Problem 5-9 (II)



(a)



(b)



(c)

$$\sum M_C = -5P + 4F_A + 1.5F_B = 0$$

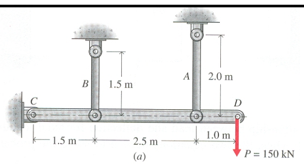
$$\Rightarrow \quad 4F_A + 1.5F_B = 750\left(10^3\right)$$

$$\frac{\delta_A}{4} = \frac{\delta_B}{1.5} \quad \Rightarrow \quad \frac{F_A L_A}{4E_A A_A} = \frac{F_B L_B}{1.5E_B A_B}$$

$$\frac{F_A(2)}{4(75)\left(10^9\right)(1000)\left(10^{-6}\right)} = \frac{F_B(1.5)}{1.5(200)\left(10^9\right)(500)\left(10^{-6}\right)} \quad \Rightarrow \quad F_A = 1.5F_B$$

p. 217

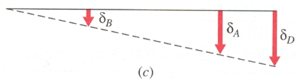
Example Problem 5-9 (III)



$$\begin{cases} 4F_A + 1.5F_B = 750\left(10^3\right) \\ F_A = 1.5F_B \end{cases} \Rightarrow \begin{aligned} F_A &= 150.0\left(10^3\right)\text{N} = 150.0\text{ kN} \\ F_B &= 100.0\left(10^3\right)\text{N} = 100.0\text{ kN} \end{aligned}$$

$$\Rightarrow \quad \sigma_A = \frac{F_A}{A_A} = \frac{150.0\left(10^3\right)}{1000\left(10^{-6}\right)} = 150.0\text{ MPa (T)}$$

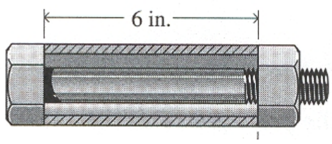
$$\sigma_B = \frac{F_B}{A_B} = \frac{100.0\left(10^3\right)}{500\left(10^{-6}\right)} = 200\text{ MPa (T)}$$



$$\delta_D = \frac{5}{4}\delta_A = \frac{5\sigma_A L_A}{4E_A} = \frac{5(150.0)\left(10^6\right)(2)}{4(75)} = 5.000\left(10^{-3}\right)\text{m} = 5.00\text{ mm}\,(\downarrow)$$

p. 217

Example Problem 5-10 (I)



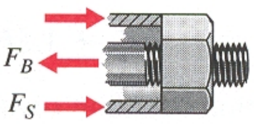
- Alloy-steel bolt
  - E = 30,000 ksi
  - d = 1/2 in
- Cold-rolled brass sleeve
  - E = 15,000 ksi
  - A = 0.375 in^2
- Tightening nut ¼ turn (0.02 in.)

Determine:

- σ_B = ?
- σ_S = ?

p. 218

Example Problem 5-10 (II)



- Alloy-steel bolt
  - $d = 1/2$ in
- Cold-rolled brass sleeve
  - $A = 0.375$ in^2

$$\sum F_x = F_S - F_B = 0$$

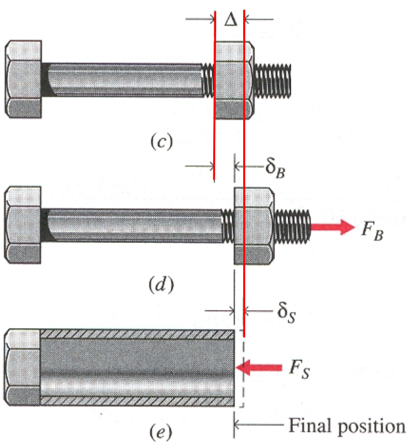
⇨ $$0.375\sigma_S = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 \sigma_B$$

➡ $$\sigma_S = 0.523\sigma_B$$

p. 218

Example Problem 5-10 (III)

$$\delta_S + \delta_B = \Delta$$ deformation



⇨ $$\frac{\sigma_B L_B}{E_B} + \frac{\sigma_S L_S}{E_S} = \Delta$$

$$\frac{\sigma_B(6)}{30{,}000} + \frac{\sigma_S(6)}{15{,}000} = 0.020$$

⇨ $$\sigma_B + 2\sigma_S = 100$$

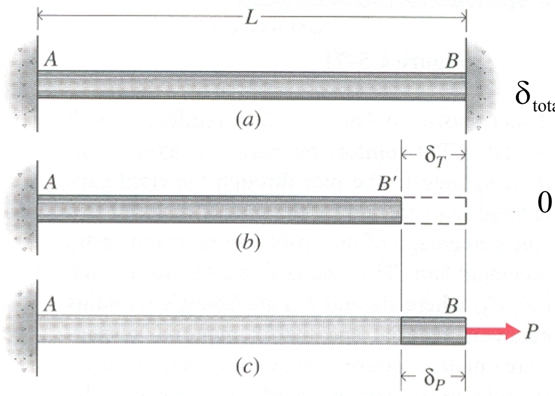
➡ $$\sigma_S = 0.523\sigma_B$$

⇨ $$\sigma_B = 48.84 \text{ ksi} \cong 48.8 \text{ ksi (T)}$$

$$\sigma_S = 25.58 \text{ ksi} \cong 25.6 \text{ ksi (C)}$$

p. 225

5-5 Thermal Effects



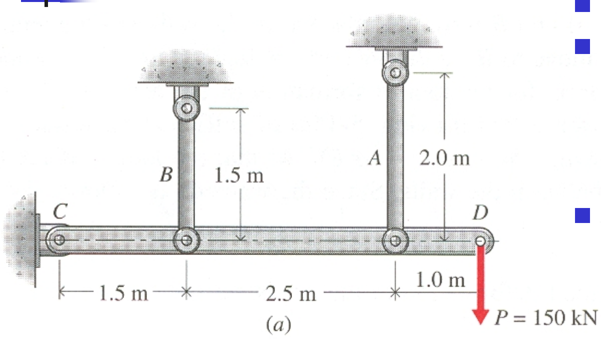
$$\delta_{\text{total}} = \delta_T + \delta_\sigma = \varepsilon_T L + \varepsilon_\sigma L$$

$$0 = \alpha\, \Delta T\, L + \frac{\sigma}{E} L$$

for fixed ends.

p. 226

Example Problem 5-12 (I)



$\Delta T = 100°C$

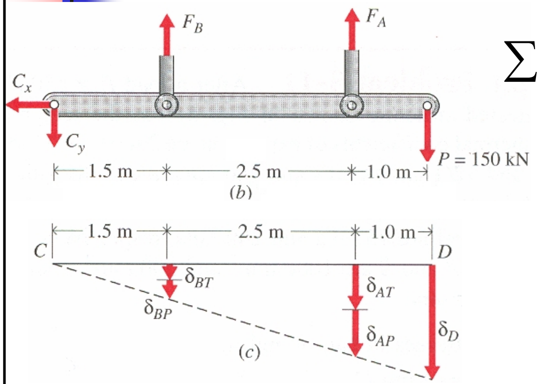
- Member *CD* : rigid
- Aluminum alloy bar *A*
  - $E = 75$ GPa
  - $A = 1000\ \text{mm}^2$
  - $\alpha = 22(10^{-6})/°C$
- Steel bar *B*
  - $E = 200$ GPa
  - $A = 500\ \text{mm}^2$
  - $\alpha = 12(10^{-6})/°C$

Determine

- σ_A , σ_B = ?
- δ_D = ?

p. 227

Example Problem 5-12 (II)



$$\sum \boldsymbol{M}_C = -5\boldsymbol{P} + 4\boldsymbol{F}_A + 1.5\boldsymbol{F}_B = 0$$

⇨ $4F_A + 1.5F_B = 750\left(10^3\right)$

$$\frac{\delta_A}{4} = \frac{\delta_B}{1.5}$$

$$\frac{F_A L_A}{4E_A A_A} + \frac{\alpha_A L_A \Delta T}{4} = \frac{F_B L_B}{1.5E_B A_B} + \frac{\alpha_B L_B \Delta T}{1.5}$$

p. 227

Example Problem 5-12 (III)

$$\frac{F_A L_A}{4E_A A_A} + \frac{\alpha_A L_A \Delta T}{4} = \frac{F_B L_B}{1.5E_B A_B} + \frac{\alpha_B L_B \Delta T}{1.5}$$

⇨
$$\frac{F_A(2.0)}{4(75)\left(10^9\right)(1000)\left(10^{-6}\right)} + \frac{22\left(10^{-6}\right)(2.0)(100)}{4}$$
$$= \frac{F_B(1.5)}{1.5(200)\left(10^9\right)(500)\left(10^{-6}\right)} + \frac{12\left(10^{-6}\right)(1.5)(100)}{1.5}$$

⇨ $F_A = 1.5F_B + 15\left(10^3\right)$

p. 227

Example Problem 5-12 (IV)

$$4F_A + 1.5F_B = 750(10^3)$$
$$F_A = 1.5F_B + 15(10^3)$$
⇨
$$F_A = 153.00(10^3)\,\text{N} = 153.00\,\text{kN}$$
$$F_B = 92.00(10^3)\,\text{N} = 92.00\,\text{kN}$$

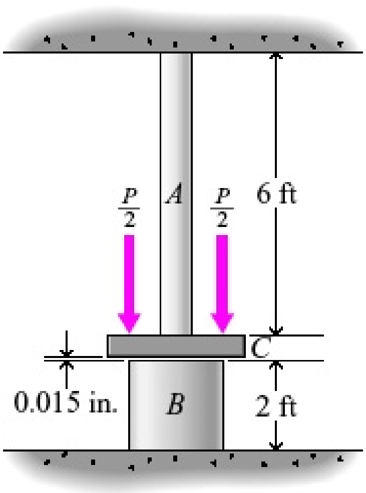
⇨

$$\sigma_A = \frac{F_A}{A_A} = \frac{153.00(10^3)}{1000(10^{-6})} = 153.0\,\text{MPa (T)}$$
$$\sigma_B = \frac{F_B}{A_B} = \frac{92.00(10^3)}{500(10^{-6})} = 184.0\,\text{MPa (T)}$$

$$\delta_D = \frac{5}{4}\delta_A = \frac{5}{4}\left[\frac{\sigma_A L_A}{E_A} + \alpha_A L_A \Delta T\right] = \frac{5}{4}\left[\frac{(150.0)(10^6)(2.0)}{(75)} + (22)(10^{-6})(2.0)(100)\right]$$
$$= 10.60(10^{-3})\ \text{m} = 10.60\ \text{mm}\ (\downarrow)$$

p. 228

Example Problem 5-13



Given:
- Steel rod A
  - E_A = 30,000 ksi
  - A_A = 2.50 in^2
  - α_A = 6.6(10^{-6})/°F
- Member C : rigid
- Bronze bar B
  - E_B = 15,000 ksi
  - A_B = 3.75 in^2
  - α_B = 9.4(10^{-6})/°F
- δ_{BC} = 0.015 in.
- P = 5kip

Find:

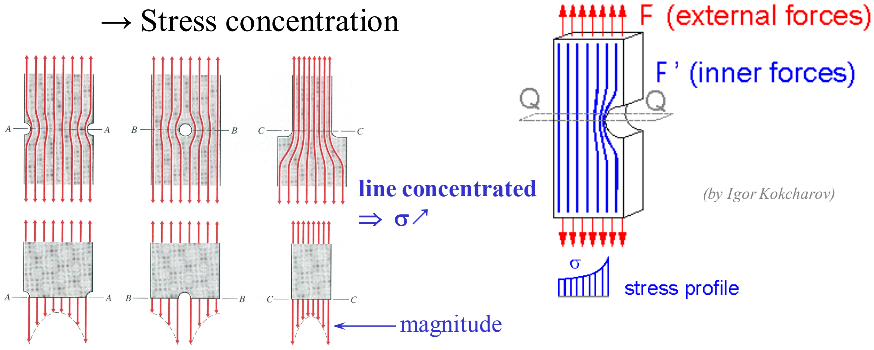
σ_A , σ_B as a function of temperature increase for $0°F < \Delta T < 50°F$

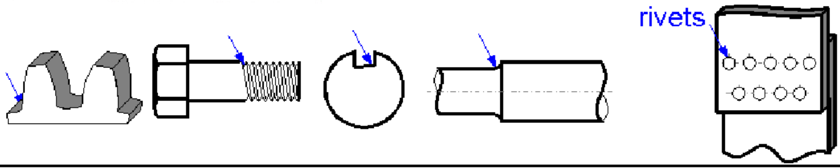
Please do by yourself!!

p. 234

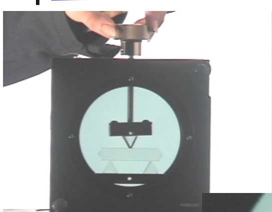
5-6 Stress Concentrations

- Discontinuity that interrupts the stress path (*stress trajectory*)
  → Stress concentration

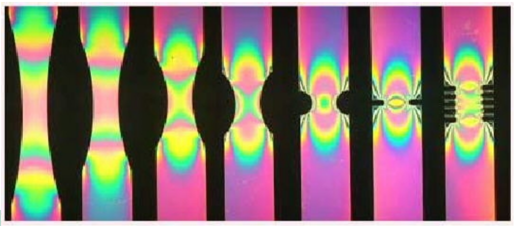




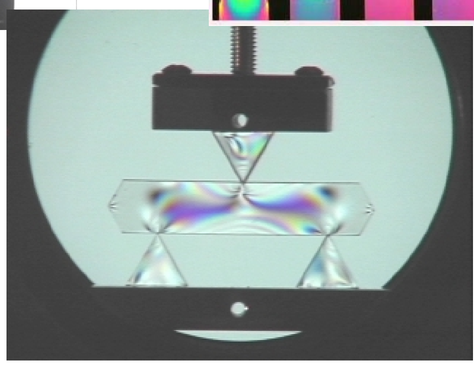
Stress Concentrations by Photoelastic experiments



www.physics.montana.edu



www.ndt.net/article/v08n09/asundi/fig15.jpg



p. 235

Stress Concentration factor

- Stress Concentration Factor K

$$\sigma = K\frac{P}{A}$$

maximum stress (σ) nominal stress ($\frac{P}{A}$)

- Ascertaining K is based on

$A = A_t$ (net section) for $K = K_t$

$A = A_g$ (gross section) for $K = K_g$

p. 236

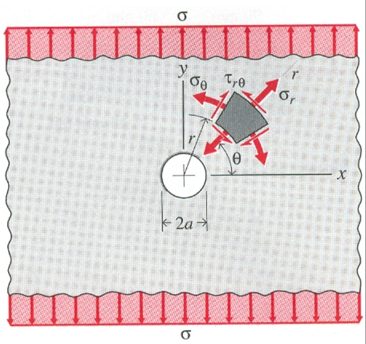
A wide plate under uniform unidirectional tension

From elasticity theory:

$$\sigma_r = \frac{\sigma}{2}\left(1-\frac{a^2}{r^2}\right)-\frac{\sigma}{2}\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{\sigma}{2}\left(1+\frac{a^2}{r^2}\right)+\frac{\sigma}{2}\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\tau_{r\theta} = \frac{\sigma}{2}\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta$$



At $r = a$

$\sigma_r = 0$

$\sigma_\theta = \sigma(1+2\cos 2\theta)$ ⇨ ***at $\theta = 0$***, max. $\sigma = 3\sigma$ ⇨ ***$K = 3$***

$\tau_{r\theta} = 0$

p. 236

A wide plate under uniform unidirectional tension (cont.)

- At $r = 3a$ *(1 diameter away from hole)*, $\sigma_\theta = 1.074\sigma$
  $\Rightarrow$ rapid decay
- Stress concentration factor is **significant** for
  - **brittle** material under **static** loading
  - any material under **impact** or **repeated loading**

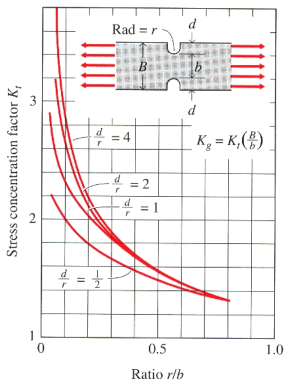
- **Saint-Venant's principal**

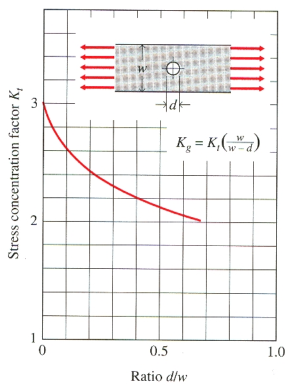
  In the regions of support and load application, the stress distribution varies from the nominal value. However, these localized effects disappear at a short distance from such locations.

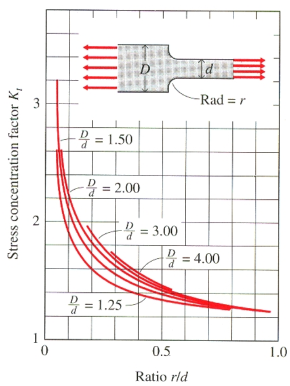
  "... the difference between the effects of two different but statically equivalent load becomes very small at sufficiently large distances from load." (From Wikipedia)

p. 236

Stress Concentration factors for grooves, holes and fillets



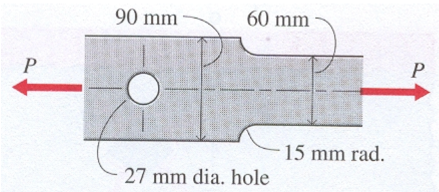




强化玻璃怎麼打也打不破？流言追追追-【實驗精華片段】

p. 237

Example Problem 5-14 (I)

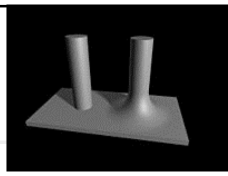


- 0.4 percent carbon hot-rolled steel
  - Yield strength = 360 MPa (from Appendix B, A43)
- Thickness = 20mm
- Factor of safety = 2.5 ⇨ $\sigma_{all} = 360/2.5 = 144$ MPa

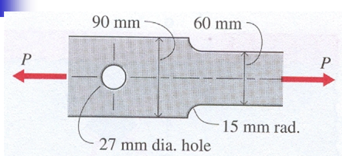
- Determine P_{max} = ?

p. 237

Example Problem 5-14 (II)



[Wikipedia]non-filleted pole (left) and a filleted pole (right)



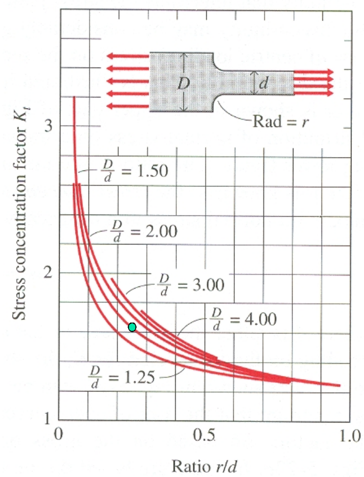
- Fillet

$$\frac{D}{d} = \frac{90}{60} = 1.5 \qquad \frac{r}{d} = \frac{15}{60} = 0.25$$

$$K_t = 1.62$$

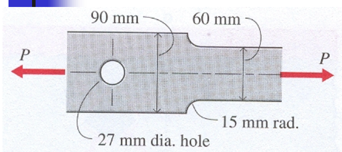
$$P = \frac{\sigma_{all} A_t}{K_t} = \frac{144(10^6)(60)(20)(10^{-6})}{1.62}$$

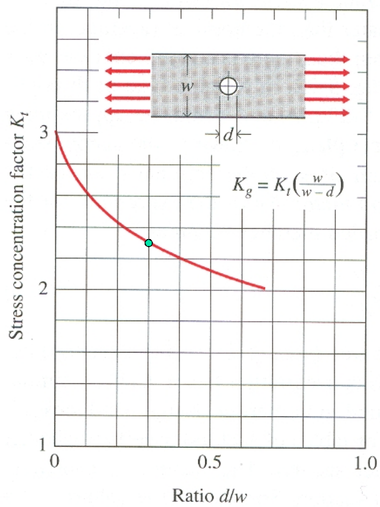
$$\cong 106.7 \text{ kN}$$



p. 237

Example Problem 5-14 (III)





- hole

$$\frac{d}{w} = \frac{27}{90} = 0.3$$

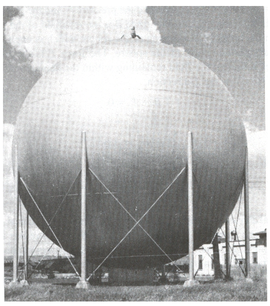
$$K_t = 2.30$$

$$P = \frac{\sigma_{all} A_t}{K_t} = \frac{144(10^6)(90-27)(20)(10^{-6})}{2.30}$$

$$\cong 78.9 \text{ kN} \longleftarrow P_{\max}$$

p. 246

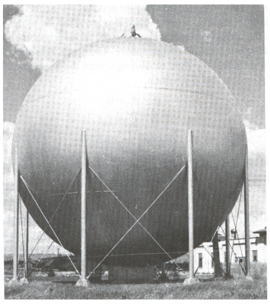
5-9 Thin-Walled Pressure Vessels



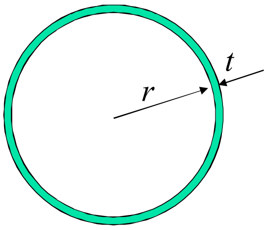
- Boilers
- Gas storage tanks
- Pipelines
- Metal tires

Why spherical shape?

5-9 Thin-Walled Pressure Vessels



- Boilers
- Gas storage tanks
- Pipelines
- Metal tires

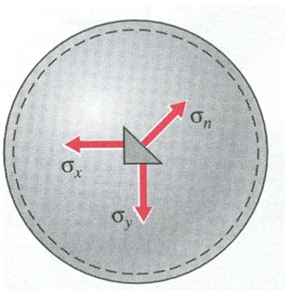


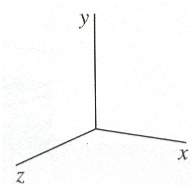
- If $t/r \ll 1$

almost uniform

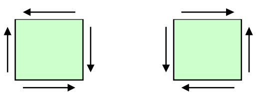
- If $t/r < 0.1$, $\sigma_{max} < 1.05\sigma_{avg}$

Spherical Pressure Vessels





- Weights of gas and vessel negligible
- symmetry $\sigma_x = \sigma_y = \sigma_n, \quad \tau_{nt} = 0$



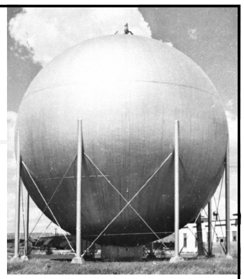
∵ symmetry

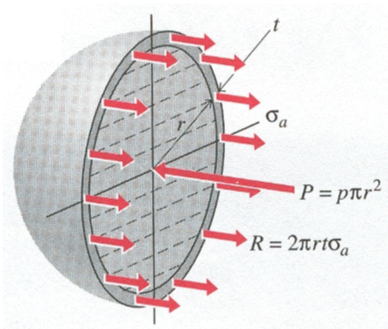
- σ_n : meridional (子午線的) or axial stress

σ_m σ_a

p. 247

Spherical Pressure Vessels





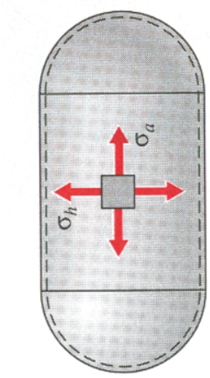
$$R - P = 0$$

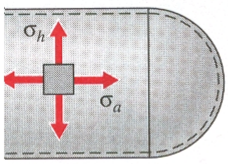
$$2\pi r t \sigma_a = p\pi r^2$$

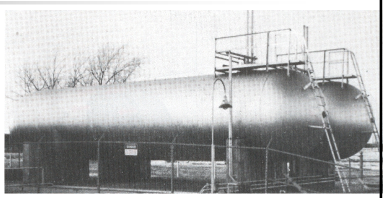
$$\sigma_a = \frac{pr}{2t}$$

p. 248

Cylindrical Pressure Vessels







- Meridional (子午線的) or axial stress
  σ_m or σ_a
  hoop(箍), tangential, or circumferential stress
  (σ_h, σ_t or σ_c)

p. 248

Cylindrical Pressure Vessels

- From FBD of hemisphere

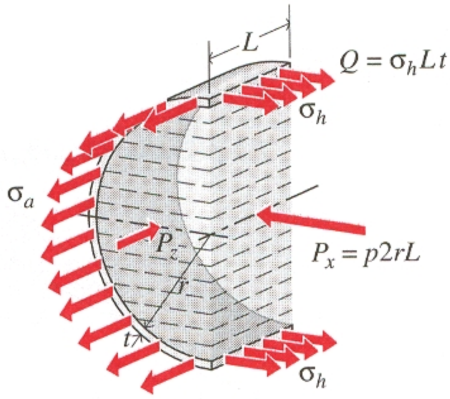
$$\sigma_a = \frac{pr}{2t}$$

- From FBD of a cylindrical section

$$P_x - 2Q = 0$$

$$p2rL = 2\sigma_h Lt$$

$$\sigma_h = \frac{pr}{t} (= 2\sigma_a)$$



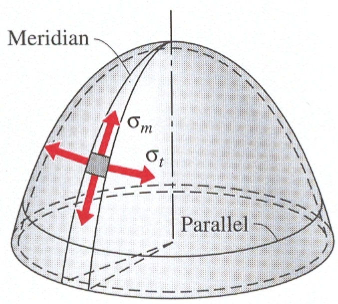
p. 249

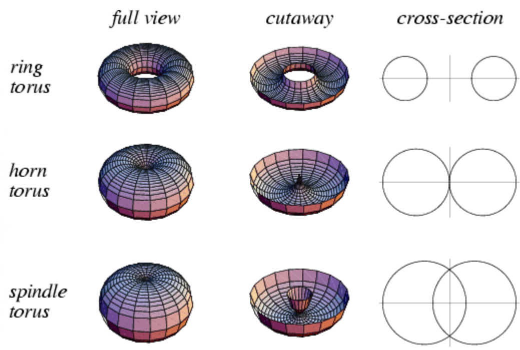
Thin Shells of Revolution

- Thin shell of revolution

  generated by rotating a plane curve, called the meridian, about an axis lying in the plane of the curve

  - sphere, hemisphere, torus, cylinder, cone, ellipsoid

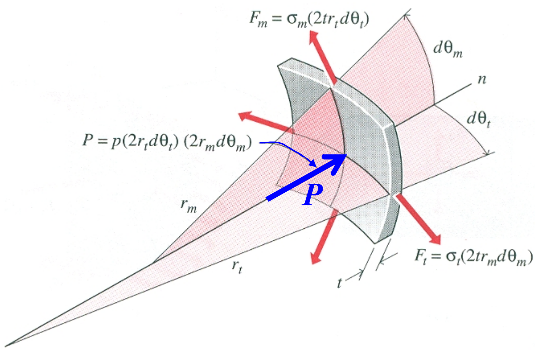




p. 249

Thin Shells of Revolution

- equilibrium in the *n*-direction



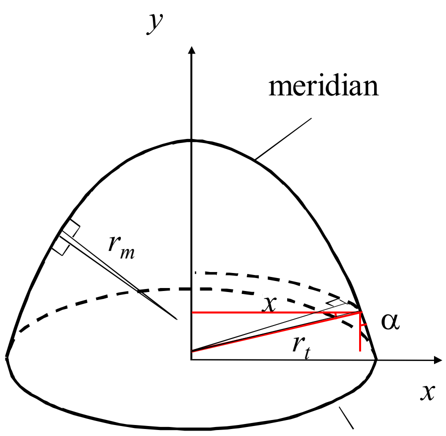
$$P = 2F_m \sin d\theta_m + 2F_t \sin d\theta_t$$

$$p(2r_t d\theta_t)(2r_m d\theta_m) = 2\sigma_m(2tr_t d\theta_t)\sin d\theta_m + 2\sigma_t(2tr_m d\theta_m)\sin d\theta_t$$

$$\frac{\sigma_m}{r_m} + \frac{\sigma_t}{r_t} = \frac{p}{t}$$

p. 249

Thin Shells of Revolution



$$y = y(x)$$

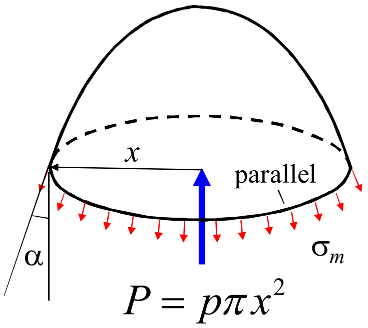
$$r_m = \frac{\left[1 + \left(dy/dx\right)^2\right]^{1.5}}{d^2y/dx^2}$$

$$r_t = \frac{x}{\cos\alpha}$$

p. 249

Thin Shells of Revolution

- equilibrium of a portion of the vessel above a paraboloid



$$R - P = 0$$

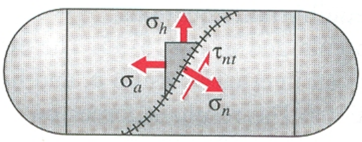
$$p\pi x^2 = 2\pi x t \sigma_m \cos\alpha$$

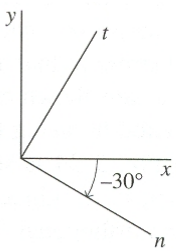
$$\sigma_m = \frac{px}{2t\cos\alpha}$$

Subst. into $\frac{\sigma_m}{r_m} + \frac{\sigma_t}{r_t} = \frac{p}{t}$ ⇨ σ_t

p. 250

Example Problem 5-16 (I)





- d_i = 1.50 m
- t = 15 mm
- P = 1500 kPa

Determine

- σ_n , τ_{nt} = ? along the weld

p. 250

Example Problem 5-16 (II)

- d_i = 1.50 m
- t = 15 mm
- P = 1500 kPa

$$\sigma_h = \frac{pr}{t} = \frac{1500(10^3)(0.75)}{0.015} = 75.0(10^6)\ \text{N/m}^2 = 75.0\ \text{MPa}$$

$$\sigma_a = \frac{pr}{2t} = \frac{1500(10^3)(0.75)}{2(0.015)} = 37.5(10^6)\ \text{N/m}^2 = 37.5\ \text{MPa}$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= 37.5\cos^2(-30°) + 75.0\sin^2(-30°) \cong 46.9\ \text{MPa}$$

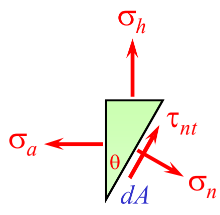
$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -(37.5 - 75.0)\sin(-30°)\cos(-30°) \cong -16.24\ \text{MPa}$$

p. 250

Example Problem 5-16 (III)

Alternatively,



$$\sum F_n = 0: \quad \sigma_n dA - \sigma_a dA\cos\theta\cos\theta - \sigma_h dA\sin\theta\sin\theta = 0$$

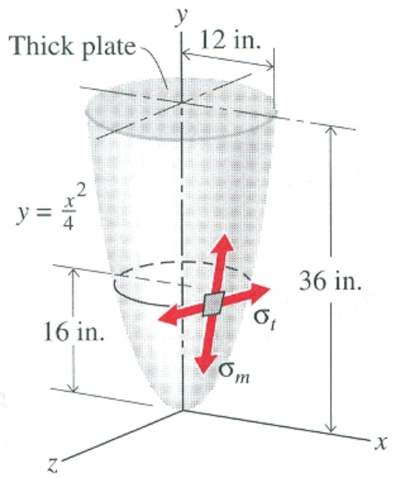
$$\Rightarrow \quad \sigma_n = 46.88\ \text{MPa} = 46.9\ \text{MPa}$$

$$\sum F_t = 0: \quad \tau_{nt} dA - \sigma_a dA\cos\theta\sin\theta + \sigma_h dA\sin\theta\cos\theta = 0$$

$$\Rightarrow \quad \tau_{nt} = -16.238\ \text{MPa} = -16.24\ \text{MPa}$$

p. 251

Example Problem 5-17 (I)



A pressure vessel

- $t = 1/4$ in.
- $y = x^2/4$
- $p = 250$ psi

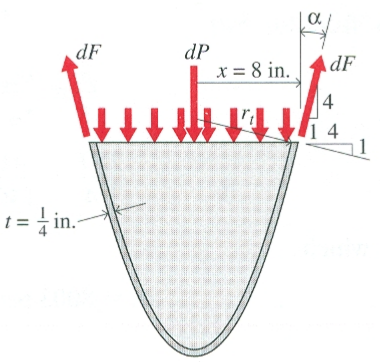
Determine

- σ_m , σ_t = ? at $y = 16$ in.

p. 251

- $t = 1/4$ in.
- $y = x^2/4$
- $p = 250$ psi

Example Problem 5-17 (II)



- At $y = 16$ in.

$$x = \sqrt{4y} = \sqrt{64} = 8 \text{ in.}$$

$$\cot\alpha = \frac{dy}{dx} = \frac{x}{2}\bigg|_{x=8} = 4 \Rightarrow \cos\alpha = \frac{4}{\sqrt{17}}$$

$$\int_{A_p} dP = \int_{A_p} dF \cos\alpha$$

$$p\pi x^2 = 2\pi x t \sigma_m \cos\alpha$$

$$\sigma_m = \frac{250(8)}{2(1/4)(4/\sqrt{17})} \cong 4120 \text{ psi (T)}$$

p. 251

Example Problem 5-17 (III)

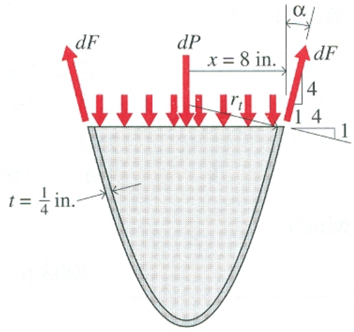
- At $y = 16$ in. $\quad y = \dfrac{x^2}{4} \;\Rightarrow\; \dfrac{dy}{dx} = \dfrac{x}{2}$ and $\dfrac{d^2y}{dx^2} = \dfrac{1}{2}$

$$\Rightarrow \quad r_m = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1.5}}{\frac{d^2y}{dx^2}} = \frac{\left(1+4^2\right)}{\frac{1}{2}} = 140.19 \text{ in}$$

$$r_t = \frac{x}{\cos\alpha} = \frac{8}{\frac{4}{\sqrt{17}}} = 8.246 \quad (\quad)$$

$$\frac{\sigma_m}{r_m} + \frac{\sigma_t}{r_t} = \frac{p}{t} \qquad \frac{4123}{140.19} + \frac{\sigma_t}{8.246} = \frac{250}{1/4}$$

$$\sigma_t = 8003 \text{ psi} \cong 8000 \text{ psi (T)}$$



p. 264

5-11 Design (Modes of failure and Factor of Safety)

- Failure: a member or structure no longer functions as intended
- Modes of Failures depends on **material, load conditions**, …
  - **Elastic failure** – excessive elastic deformation
    - Significant property: E, ν
  - **Yielding (slip failure)** – excessive plastic deformation
    - Significant property: yield strength, yield point

p. 264

5-11 Design

- Modes of Failures
  - **Creep failure** – excessive plastic deformation over a long time under constant stress
    - E.g. machine under high σ, T
    - Significant property: **creep limit**
  - **Fracture** – complete separation of the material
    - E.g. brittle material, crack, flaw, repeated loading
    - Significant property: **ultimate strength**
- Mathematical Analysis
  - Allowable stress design:
    - **Strength ≥ Stress**

p. 265

5-11 Design

- Uncertainties
  - Loads – varied, future time
  - Material properties – only test specimen, local defect
  - Stress – model error, non-uniformity
- **Factor of Safety**; allowable stress
  - Strength ≥ (Factor of Safety) · (Stress)

$$FS = \frac{\text{failure load}}{\text{actual computed load}} \quad \text{or} \quad = \frac{\text{failure stress}}{\text{actural computed stress}}$$

$$\text{Allowable stress} = \frac{\text{failure stress}}{FS}$$

p. 265

5-11 Design

- Example: axially loaded rod
  - Given: F, σ_y, FS
  - Find: d

$$\left(\sigma_{y(\text{actual})} = \frac{F}{A} = \frac{F}{\pi d^2 / 4}\right) \cdot (\text{FS}) \le \sigma_{y(\text{allowable})}$$

$$d \ge \sqrt{4 \cdot (\text{FS}) \cdot F / (\pi \sigma_y)} = d_{\min}$$

Please Study Example Problems 5-20 ~ 5-23 by yourself

8 Exercises

- **5-32, 5-66, 5-71, 5-89, 5-100, 5-104, 5-107 5-142**