Mechanics of Materials

(http://bernoulli.iam.ntu.edu.tw/)

Chapter 6

Torsional Loading of Shafts

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Torsional Loading

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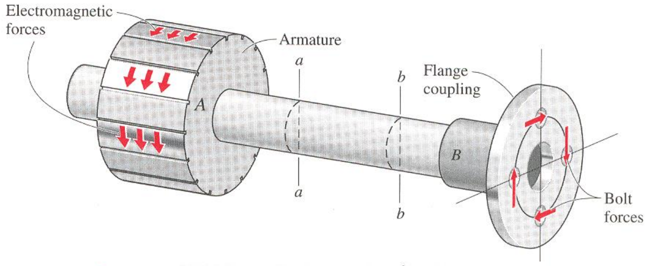
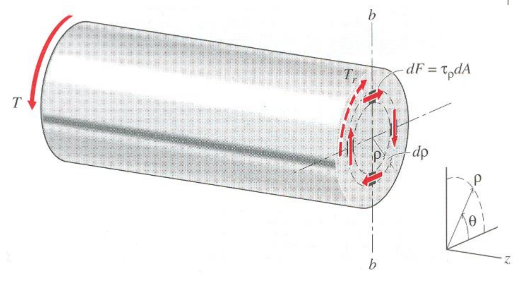
6-1 Introduction





$$T = T_r = \int_{\text{area}} \rho \, dF$$

$$= \int_{\text{area}} \rho \tau_\rho \, dA$$

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6-1 Introduction

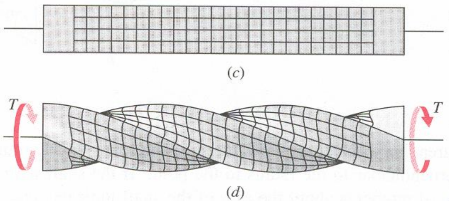
- 1784, Charles-Augustin Coulomb
  - **$\theta \sim T$ (experiment)**
- 1820, A. Duleau (French)
  - **$\theta \sim T$ (analytical)** by assuming:
  - **plane remains plane**
  - **diameter remains straight**

○ for circular shaft

× for other shaft,

warping (翹曲) occurs

1. Englander
2. German
3. French
4. American



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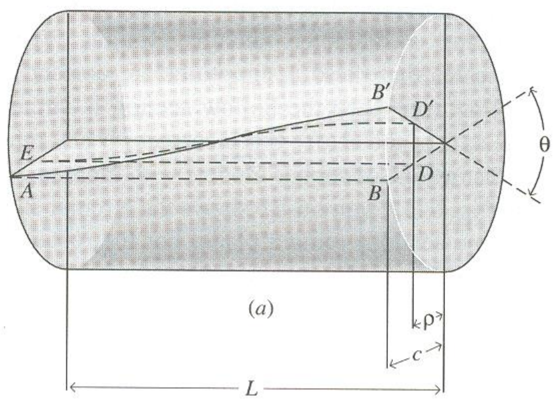
6-2 Torsional Shearing Strain



straight shaft of constant diameter

- plane remains plane
- diameter remains straight
- **θ** : angle of twist
- *B* → *B'*, *D* → *D'*

  BB', DD' in the same plane

  B'& B and D & D' on the same radius

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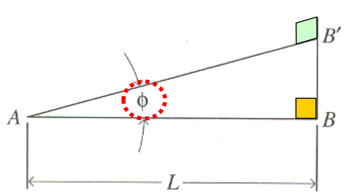
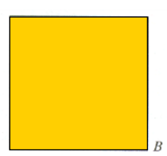
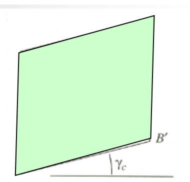
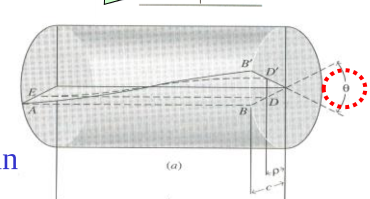
6-2 Torsional Shearing Strain









Two torsional angles: ϕ and θ

small strain

$$\tan\gamma_c = \tan\phi = \frac{BB'}{AB} = \frac{c\theta}{L} \cong \gamma_c$$

$$\tan\gamma_\rho = \frac{DD'}{ED} = \frac{\rho\theta}{L} \cong \gamma_\rho$$

$$\theta = \frac{\gamma_c L}{c} = \frac{\gamma_\rho L}{\rho} \Rightarrow \gamma_\rho = \frac{\rho}{c}\gamma_c$$

valid for elastic **or inelastic** homogeneous **or heterogeneous** materials

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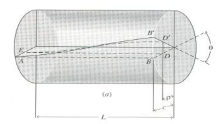
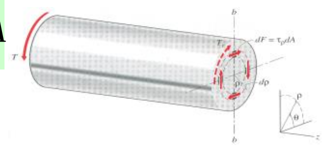
6-3 Torsional Shearing Stress – The Elastic Torsion Formula

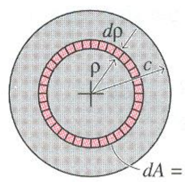
$$\gamma_\rho = \frac{\rho}{c}\gamma_c \quad \Rightarrow \quad \boxed{\tau_\rho = \frac{\rho}{c}\tau_c}$$



$$T = T_r = \int_A \rho\tau_\rho dA = \frac{\tau_c}{c}\int_A \rho^2 dA = \frac{\tau_\rho}{\rho}\int_A \rho^2 dA$$



$$J = \int_A \rho^2 dA$$ polar second moment of area



$$J = \int_0^c \rho^2(2\pi\rho)d\rho = \frac{\pi c^4}{2}$$

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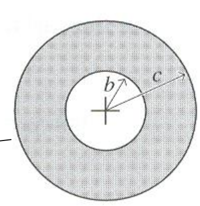
6-3 Torsinal Shearing Stress – The Elastic Torsion Formula

$$T = T_r = \frac{\tau_c J}{c} = \frac{\tau_\rho J}{\rho}$$

$$\boxed{\tau_c = \frac{Tc}{J} \qquad \tau_\rho = \frac{T\rho}{J}}$$

- valid for linearly elastic, homogeneous, isotropic material ($\tau = G\gamma$)
- $\tau_{max} = \tau_c$
- $\boldsymbol{\tau_\rho = 0}$ at center

$$J = \int_b^c \rho^2(2\pi\rho)d\rho = \frac{\pi c^4}{2} - \frac{\pi b^4}{2}$$



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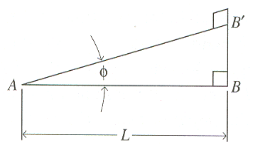
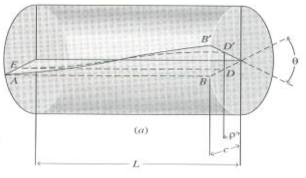
6-4 Torsional Displacements

$$\begin{cases} \theta = \dfrac{\gamma_\rho L}{\rho} \\ \tau_\rho = \dfrac{T\rho}{J} \end{cases}$$

(or $\gamma_\rho = \rho \dfrac{d\theta}{dL}$, if T or J varies along L)





⇨ $\theta = \dfrac{\gamma_\rho L}{\rho} = \dfrac{\tau_\rho L}{G\rho}$

⇨ $\theta = \dfrac{TL}{GJ}$

Comparison: $\delta = \dfrac{PL}{EA}$

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6-4 Torsional Displacements

- $T = T_r$ must be obtained from a free-body diagram.
- If T, G , or J is not constant along the length of the shaft,

$$\theta = \sum_{i=1}^{n} \frac{T_i L_i}{G_i J_i}$$

or

$$\theta = \int_0^L \frac{T_r(x)}{G(x)J(x)} dx$$

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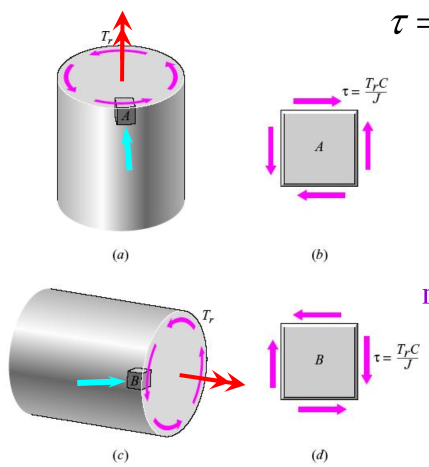
6-4 Torsional Displacements

positive outward for T and θ

- Sign convention

$$\tau = \frac{T\rho}{J}$$

Direction of τ from T



positive shear?

Not always consistent!

negative shear?

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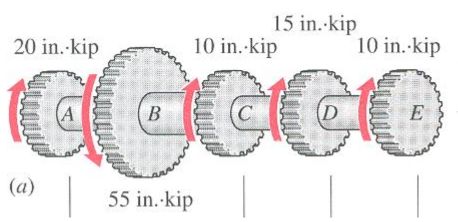
Example Problem 6-1 (I)



Determine

- $T_{AB}, T_{BC}, T_{CD}, T_{DE} = ?$
- Draw a **torque diagram**.

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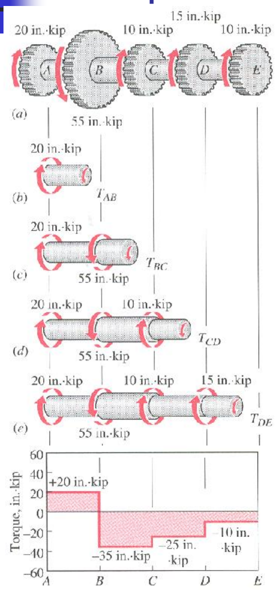
Example Problem 6-1 (II)



$$\sum M = T_{AB} - 20 = 0$$

$$\sum M = T_{BC} - 20 + 55 = 0$$

$$\sum M = T_{CD} - 20 + 55 - 10 = 0$$

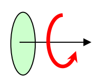
$$\sum M = T_{AB} - 20 + 55 - 10 - 15 = 0$$

$$T_{AB} = +20.0\ \text{in} \cdot \text{kip}$$

$$T_{BC} = -35.0\ \text{in} \cdot \text{kip}$$

$$T_{CD} = -25.0\ \text{in} \cdot \text{kip}$$

$$T_{DE} = -10.0\ \text{in} \cdot \text{kip}$$



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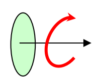

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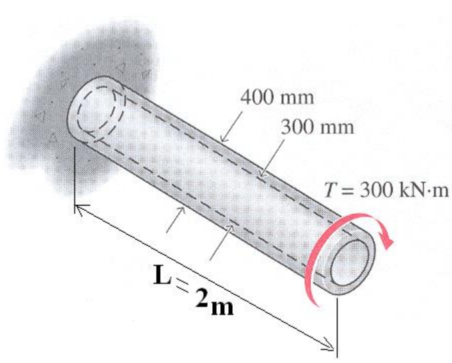
Example Problem 6-2 (I)



- r_i = 150 mm
- r_o = 200 mm
- G = 80 Gpa
- L = 2 m

Determine

- τ_{max} = ?
- τ_{ρ}(inside surface)= ?
- θ = ?

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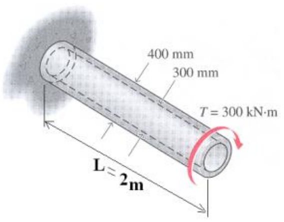
Example Problem 6-2 (II)



$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = \frac{\pi}{2}\left(200^4 - 150^4\right) \cong 17181\left(10^{-6}\right)\text{m}^4$$

$$\tau_{max} = \tau_c = \frac{Tc}{J} = \frac{300\left(10^3\right)\left(200\right)\left(10^{-3}\right)}{17181\left(10^{-6}\right)} \cong 34.9\ \text{MPa}$$

$$\tau_\rho = \frac{T\rho}{J} = \frac{300\left(10^3\right)\left(150\right)\left(10^{-3}\right)}{17181\left(10^{-6}\right)} \cong 26.2\ \text{MPa}$$

$$\theta = \frac{TL}{GJ} = \frac{300\left(10^3\right)\left(2\right)}{80\left(10^9\right)\left(17181\right)\left(10^{-6}\right)} \cong 0.00437\ \text{rad}$$ (***not*** degree!)

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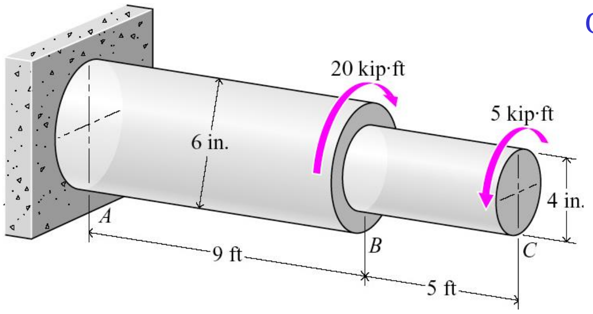
Example Problem 6-3 (I)



Given:

- G = 12,000 ksi

Find:

- τ_{max} = ?
- $\theta_{B/A}$ = ?
- $\theta_{C/B}$ = ?
- $\theta_{C/A}$ = ?

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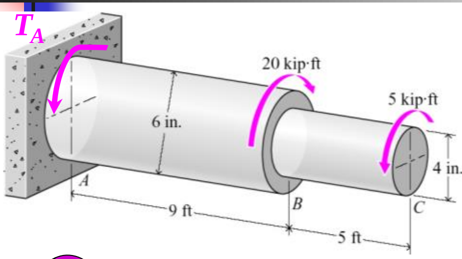
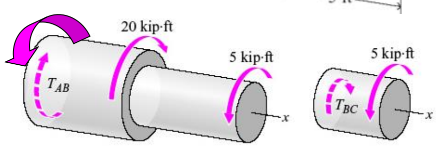
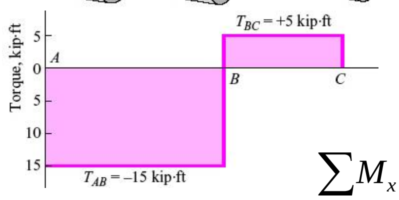
Example Problem 6-3 (II)



$$\sum M_x = 0: \ \ -T_{AB} - 20 + 5 = 0$$
$$T_{AB} = -15 \text{ kip}\cdot\text{ft}$$
$$\sum M_x = 0: \ \ -T_{BC} + 5 = 0$$
$$T_{BC} = 5 \text{ kip}\cdot\text{ft}$$



$$T_{AB} = -15 \text{ kip}\cdot\text{ft}$$
$$T_{BC} = 5 \text{ kip}\cdot\text{ft}$$



Or Alternatively,

$$\sum M_x = 0: \ \ T_A - 20 + 5 = 0$$
$$T_A = 15 \text{ kip}\cdot\text{ft}$$

$$\sum M_x = 0: \ \ T_A - T_{AB} = 0 \ ; T_{AB} = 15 \text{ kip}\cdot\text{ft}$$

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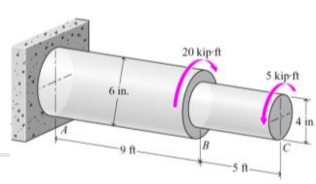
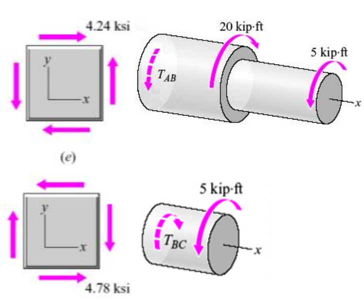
Example Problem 6-3 (III)



$$J_{BC} = \frac{\pi}{2} c^4 = \frac{\pi}{2}\left(2^4\right) = 25.13 \text{ in.}^4$$
$$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2}\left(3^4\right) = 127.23 \text{ in.}^4$$

$$T_{AB} = -15 \text{ kip}\cdot\text{ft}$$
$$T_{BC} = 5 \text{ kip}\cdot\text{ft}$$

$$\tau_{AB} = \frac{T_{AB}c_{AB}}{J_{AB}} = \frac{-15(12)(3)}{127.23} = -4.244 \text{ ksi}$$
$$\tau_{BC} = \frac{T_{BC}c_{BC}}{J_{BC}} = \frac{5(12)(2)}{25.13} = 4.775 \text{ ksi}$$



$$\tau_{max} = \tau_{BC} = 4.775\text{ksi} \cong 4.78\text{ksi}$$

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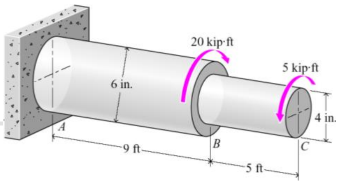
Example Problem 6-3 (IV)



$$\theta_{B/A} = \frac{T_{AB}L_{AB}}{G_{AB}J_{AB}} = \frac{-15(12)(9)(12)}{12,000(127.23)} \cong -0.01273 \text{ rad}$$

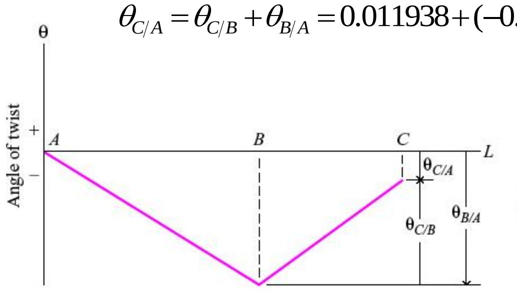
= - 0.73°

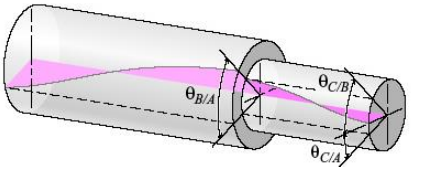
$$\theta_{C/B} = \frac{T_{BC}L_{BC}}{G_{BC}J_{BC}} = \frac{5(12)(5)(12)}{12,000(25.13)} \cong 0.01194 \text{ rad}$$

= 0.68°

$$\theta_{C/A} = \theta_{C/B} + \theta_{B/A} = 0.011938 + (-0.012733) = -0.000795 \text{ rad}$$

= -0.0456°





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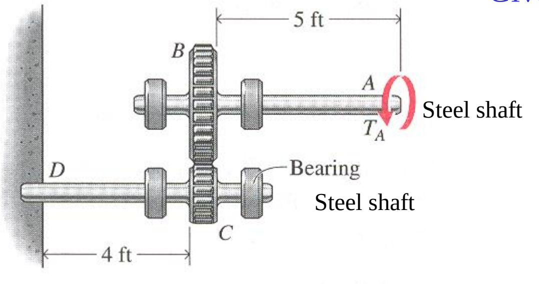
Example Problem 6-4 (I)



Given:

- d_s = 1.50 in.
- d_B = 10 in.
- d_C = 6 in.
- G = 12,000 ksi
- T_A = 750 ft·lb

Find:

- τ_{max} of shaft CD = ?
- θ_A = ?

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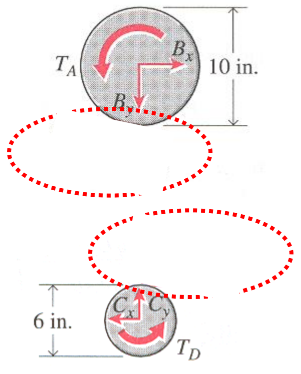
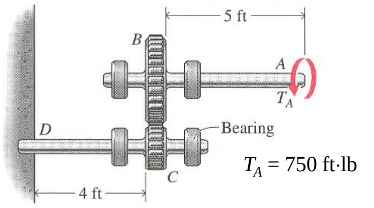
Example Problem 6-4 (II)

- $d_s = 1.50$ in.
- $d_B = 10$ in.
- $d_C = 6$ in.





$$T_A = r_B F$$
$$T_D = r_C F$$

$$T_D = \frac{r_C}{r_B} T_A = \left(\frac{3}{5}\right) 750 = 450 \text{ft} \cdot \text{lb}$$

$$\tau_{\max} = \frac{T_{CD} c_{CD}}{J_{CD}} = \frac{450(12)(0.75)}{(\pi/2)(0.75)^4} \cong 8150 \text{psi}$$

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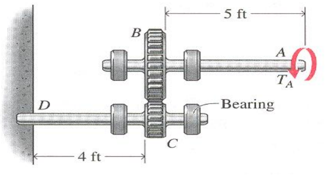
Example Problem 6-4 (III)

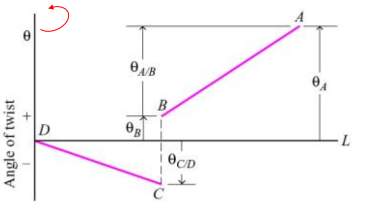


$$\theta_{C/D} = \frac{-T_{CD} L_{CD}}{G_{CD} J_{CD}} = \frac{-450(12)(4)(12)}{12(10^6)(\pi/2)(0.75^4)} = -0.04346 \text{ rad}$$

= –2.49°

$$s = r_B \theta_B = r_C \theta_{C/D}$$

Gears move the **same arc length**
i.e., the same number of teeth



$$\theta_B = \frac{r_C}{r_B} \theta_{C/D} = \frac{3}{5}(0.04346) = 0.02608 \text{rad}$$

= 1.49°

$$\theta_{A/B} = \frac{T_{AB} L_{AB}}{G_{AB} J_{AB}} = \frac{750(12)(5)(12)}{12(10^6)(\pi/2)(0.75^4)} = 0.09054 \text{rad}$$

= 5.19°

$$\theta_A = \theta_B + \theta_{A/B} = 0.02608 + 0.09054 = 0.11662 \text{rad} = 6.68°$$

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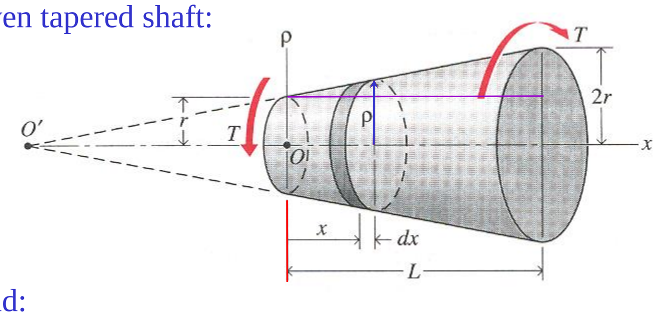
Example Problem 6-5 (I)

Given tapered shaft:



Find:

- θ(T, L, G, r)= ?

$$\rho = r + \frac{2r - r}{L} x = \frac{r}{L}(L + x) = r + \frac{r}{L} x$$

$\tau(\rho)$, $J(\rho)$

$$\theta = \frac{\gamma_c L}{c} = \frac{\gamma_\rho L}{\rho}$$

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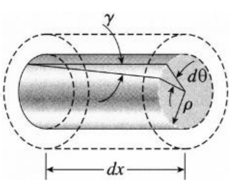
Example Problem 6-5 (II)



$$d\theta = \frac{\gamma}{\rho} dx \qquad \gamma = \frac{\tau}{G} \qquad \tau = \frac{T_r \rho}{J} = \frac{T\rho}{\pi\rho^4/2} = \frac{2T}{\pi\rho^3}$$

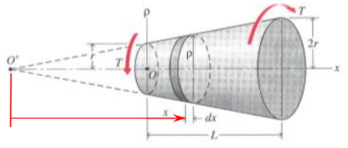
$$d\theta = \frac{\tau}{G\rho} dx = \frac{2T}{G\pi\rho^4} dx = \frac{2TL^4}{G\pi r^4 (L+x)^4} dx$$

$$\rho = \frac{r}{L}(L+x)$$

$$\theta = \frac{2TL^4}{G\pi r^4} \int_0^L \frac{dx}{(L+x)^4} = -\frac{2TL^4}{3G\pi r^4}\left(\frac{1}{8L^3} - \frac{1}{L^3}\right) = \frac{7TL}{12G\pi r^4}$$

$$\theta = \int_0^L \frac{T}{GJ(x)} dx \qquad J(x) = \frac{\pi}{2}\rho^4 = \frac{\pi}{2}\left[\frac{r}{L}(L+x)\right]^4$$

- Alternative : place the origin at O'



$$\rho = \frac{r}{L} x \qquad \theta = \frac{2TL^4}{G\pi r^4} \int_L^{2L} \frac{dx}{x^4} = \frac{7TL}{12G\pi r^4}$$

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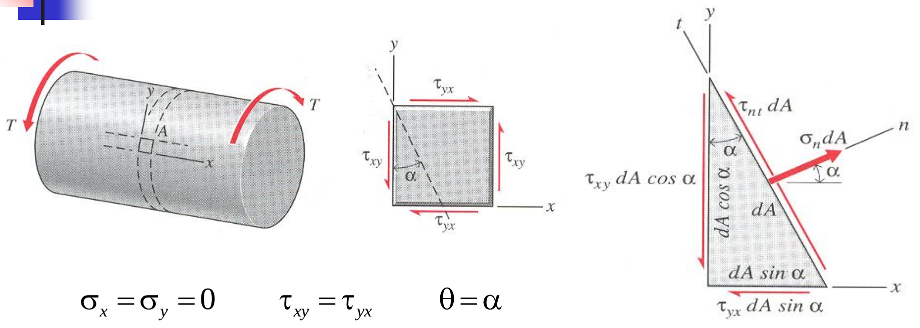
6-5 Stresses on Oblique Planes



$$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = \tau_{yx} \qquad \theta = \alpha$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta = \tau_{xy}\sin 2\alpha$$

$$\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = \tau_{xy}\cos 2\alpha$$

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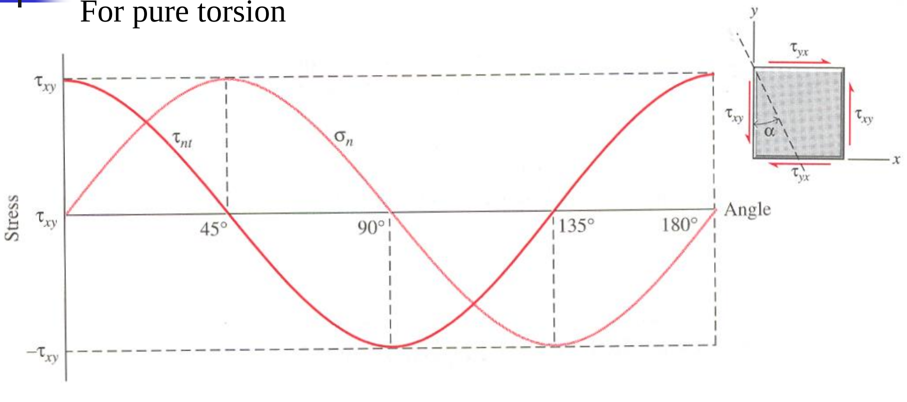
6-5 Stresses on Oblique Planes

For pure torsion



$$\max\sigma_n = \tau_{xy} \qquad \alpha = 45° \text{ (T)},\ 135° \text{ (C)}$$

$$\max\tau_{nt} = \tau_{xy} \qquad \alpha = 0°,\ 90°$$

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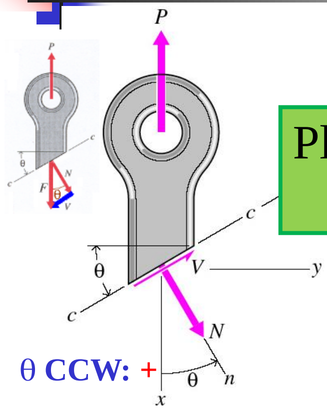
2-6 Stresses on an Inclined Plane in an Axially Loaded Member

Assumption: stress uniformly distributed



N P cosθ P … P (…2θ)
(2-7)

Please remind and compare by yourself

2θ
A_n $A/\cos\theta$ A $2A$
(2-8)

θ CCW: +

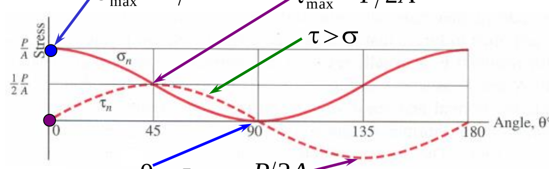
$N = P\cos\theta$

$V = -P\sin\theta$

Axially loaded only!

$\sigma_{max} = P/A$ $\tau_{max} = P/2A$

$\tau > \sigma$



$\sigma_{min} = 0$ $\tau_{min} = -P/2A$

σ and τ vanish at 90°

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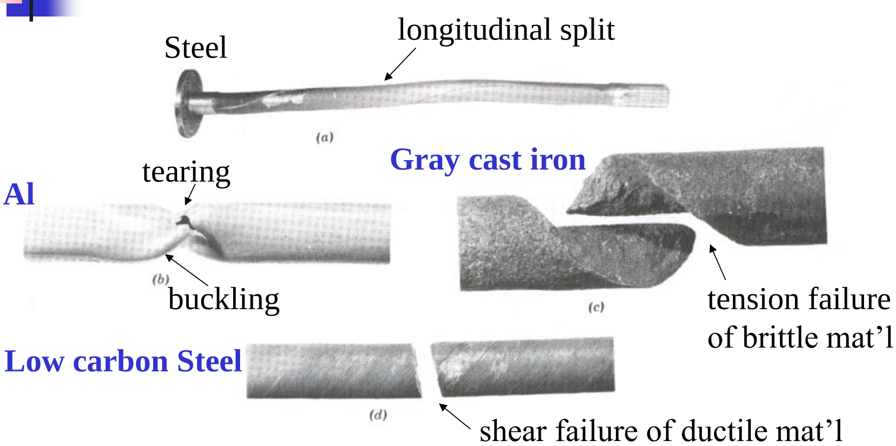
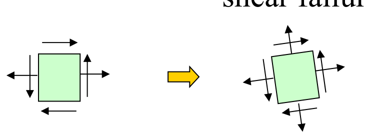
6-5 Stresses on Oblique Planes

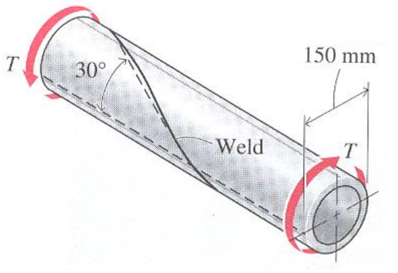




max τ

p.297

Example Problem 6-6 (I)



- t = 6 mm,
  σ_{max} (steel tube) = 80 Mpa
- T_{max} = ?
- T = 12 kN · m
  σ_{ult} (weld) = 345 Mpa
  τ_{ult} (weld) = 205 Mpa
  Determine factor of safety
  FS = ?

p.298

Example Problem 6-6 (II)

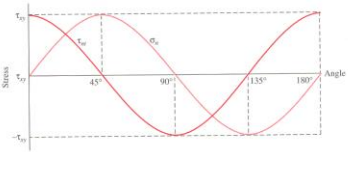
- σ_{max} (tube) = 80 Mpa, T_{max} = ?

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = \frac{\pi}{2}\left(75^4 - 69^4\right) = 14.096\left(10^6\right)mm^4 = 14.096\left(10^{-6}\right)m^4$$

$$\sigma_{max} = \tau_{max} = \frac{T_{max}c}{J} = 80\,\text{MPa} = 80\left(10^6\right)\text{N/m}^2$$

$$T_{max} = \frac{\sigma_{max}J}{c} = \frac{80\left(10^6\right)\left(14.096\right)\left(10^{-6}\right)}{75\left(10^{-3}\right)}$$

$$= 15.036\left(10^3\right)\text{N}\cdot\text{m} \cong 15.04\,\text{kN}\cdot\text{m}$$



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Example Problem 6-6 (III)

- T = 12 kN·m, σ_{ult} (weld) = 345 Mpa, τ_{ult} (weld) = 205 Mpa

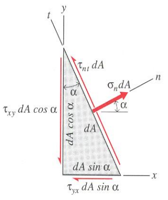
  FS = ?

$$\sigma_n = \tau_{xy}\sin 2\alpha = \frac{Tc}{J}\sin 2\alpha = \frac{12\left(10^3\right)\left(75\right)\left(10^{-3}\right)}{14.096\left(10^{-6}\right)}\sin 2\left(60°\right) = 55.29 \text{ MPa (T)}$$

$$\tau_{nt} = \tau_{xy}\cos 2\alpha = \frac{Tc}{J}\cos 2\alpha = \frac{12\left(10^3\right)\left(75\right)\left(10^{-3}\right)}{14.096\left(10^{-6}\right)}\cos 2\left(60°\right) = -31.92 \text{ MPa}$$

$$FS_\sigma = \frac{\sigma_{ult}}{\sigma_n} = \frac{345}{55.29} = 6.24 \quad \mathbf{= FS}$$

$$FS_\tau = \frac{\tau_{ult}}{\tau_n} = \frac{205}{31.92} = 6.42$$



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6-6 Power Transmission

- Work done by a constant torque T

  $W_k = T\phi$ $\quad$ ϕ : angular displacement (rad)

- Power: rate of doing work

$$\text{Power} = \frac{dW_k}{dt} = T\frac{d\phi}{dt} = T\omega \qquad \omega : \text{angular velocity}$$

T : N·m — **1 rpm = 2π rad/min**

ω : rad/min — **1 hp = 33000 ft · lb/min**

Power: N ·m /min — **1 watt = 1 N · m/s**

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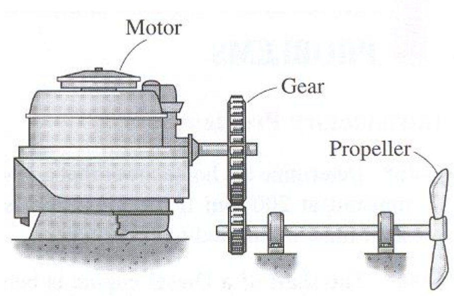
Example Problem 6-7 (I)



- ω_{motor} = 200 rpm
- Power = 800 hp
- Gear box ratio = 4 : 1
- G = 12,000 ksi
- τ_{ult} = 20 ksi
- θ_{max} (for 10’ propeller shaft) = 4°

Determine

- min diameters of two shafts = ?

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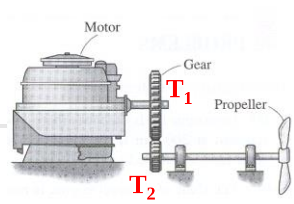
Example Problem 6-7 (II)



- ω_{motor} = 200 rpm
- Power = 800 hp
- Gear box ratio = 4 : 1
- G = 12,000 ksi

(a) **considering τ_{ult} = 20 ksi**

$$\text{Power} = T\omega \qquad 800(33{,}000) = T_1(200)(2\pi)$$

$$T_1(200)(2\pi) = T_2(800)(2\pi)$$

$$T_1 = 21{,}010\,\text{ft}\cdot\text{lb} \qquad T_2 = 5252\,\text{ft}\cdot\text{lb}$$

$$\frac{J_1}{c_1} = \frac{T_1}{\tau} = \frac{21{,}010(12)}{20(10^3)} = \frac{(\pi/2)c_1^4}{c_1} \qquad c_1 = 2.002\text{in.} \quad d_1 = 2c_1 \cong 4.00\text{in.}$$

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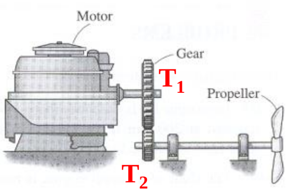
Example Problem 6-7 (III)

(b) **Considering θ_{max} (propeller shaft) = 4°**



- ω_{motor} = 200 rpm
- Power = 800 hp
- Gear box ratio = 4 : 1
- G = 12,000 ksi
- τ_{ult} = 20 ksi

$$\theta = \frac{TL}{GJ}$$

$$\frac{\pi}{180}(4) = \frac{5252(12)(10)(12)}{(12)(10^6)(\pi c_2^4/2)}$$

$$c_2^4 = 5.747 \qquad c_2 = 1.5483$$

$$\frac{J_2}{c_2} = \frac{T_2}{\tau} = \frac{5252(12)}{20(10^3)} = \frac{(\pi/2)c_2^4}{c_2} \qquad c_2 = 1.2612 in. \quad (d_2 = 2c_2 \cong 2.32 in.)$$

$$c_2 = 1.5483 > 1.2612$$

$$d_2 = 2c_2 = 2(1.5483) = 3.0966 in. \cong 3.10 in.$$

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6-7 Statically Indeterminate Members

- Statically indeterminate members
  - Equations of equilibrium
  - Distortion equations

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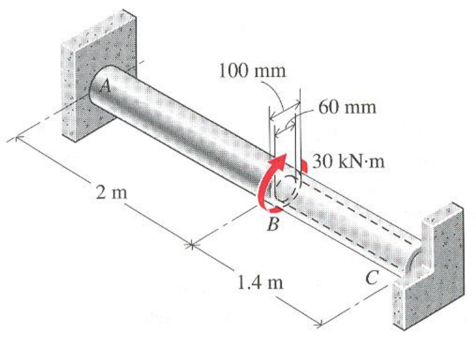
Example Problem 6-8 (I)



- Solid section *AB*
  - Annealed bronze
  - G_{AB} = 45 GPa
- Hollow section *BC*
  - Aluminum alloy
  - G_{BC} = 28 GPa

Determine

- τ_{max} = ? in both sections

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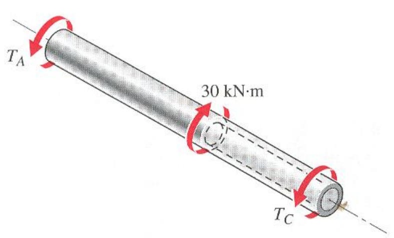
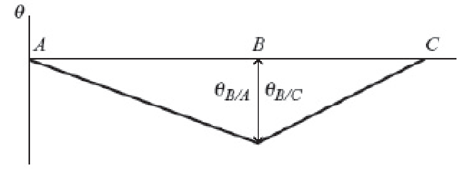
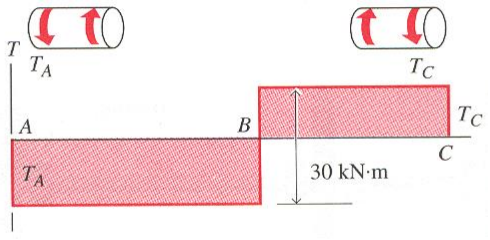
Example Problem 6-8 (II)







$$T_A + T_C = 30(10^3)$$

$$\theta_{B/A} = \theta_{B/C}$$

⇨ $$\frac{T_A L_{AB}}{G_{AB} J_{AB}} = \frac{T_C L_{BC}}{G_{BC} J_{BC}}$$

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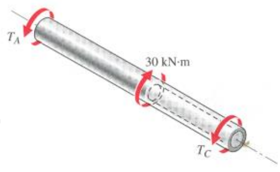
Example Problem 6-8 (III)



$T_A + T_C = 30\left(10^3\right)$

$$J_{AB} = (\pi/2)\left(50^4\right) = 9.817\left(10^6\right)\text{mm}^4 = 9.817\left(10^{-6}\right)\text{m}^4$$

$$J_{BC} = (\pi/2)\left(50^4 - 30^4\right) = 8.545\left(10^6\right)\text{mm}^4 = 8.545\left(10^{-6}\right)\text{m}^4$$

$$\frac{T_A(2)}{45\left(10^9\right)(9.817)\left(10^{-6}\right)} = \frac{T_C(1.4)}{28\left(10^9\right)(8.545)\left(10^{-6}\right)} \Rightarrow T_C = 0.7737 T_A$$

$\Rightarrow$ $T_A = 16.914\,\text{kN}\cdot\text{m}$ $\qquad$ $T_C = 13.086\,\text{kN}\cdot\text{m}$

$$\tau_{AB} = \frac{T_A c_{AB}}{J_{AB}} = \frac{16.914\left(10^3\right)(50)\left(10^{-3}\right)}{9.817\left(10^{-6}\right)} \cong 86.1\,\text{MPa}$$

$$\Rightarrow$$

$$\tau_{BC} = \frac{T_C c_{BC}}{J_{BC}} = \frac{13.086\left(10^3\right)(50)\left(10^{-3}\right)}{8.545\left(10^{-6}\right)} \cong 76.6\,\text{MPa}$$

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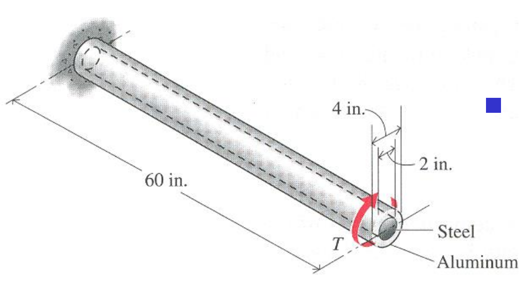
Example Problem 6-9 (I)



- Aluminum alloy tube
  - G_a = 4000 ksi
  - $\tau_{allowable}$ = 10 ksi
- Securely connected at the end of a Steel core
  - G_s = 11,600 ksi
  - $\tau_{allowable}$ = 14 ksi

Determine

- Max. T = ?
- θ = ? under max. T

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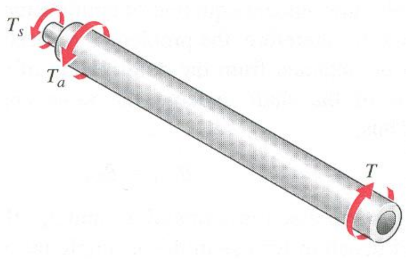
Example Problem 6-9 (II)

- Aluminum alloy tube
  - $G_a = 4000$ ksi
  - $\tau_{allowable} = 10$ ksi
- Securely connected at the end of a Stee core
  - $G_s = 11{,}600$ ksi
  - $\tau_{allowable} = 14$ ksi



$$\left(\theta = \frac{\gamma_\rho L}{\rho} = \frac{\tau_\rho L}{G\rho}\right)$$

$$\Rightarrow \frac{\tau_a L_a}{G_a c_a} = \frac{\tau_s L_s}{G_s c_s}$$

$$\Rightarrow \frac{\tau_a(60)}{4.0(10^6)(1)} = \frac{\tau_s(60)}{11.6(10^6)(2)}$$

$$\Rightarrow \tau_s = 1.45\tau_a$$

$$T_a + T_s = T$$

$$\theta_a = \theta_s$$

$$\tau_s(\text{allowable}) = 14\,\text{ksi}$$

$$= 1.4(10)\,\text{ksi} = 1.4\tau_a(\text{allowable})$$

$\Rightarrow$ τ_s controls

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Example Problem 6-9 (III)

$$\tau_s = 14\,\text{ksi}$$

$$\tau_a = 14/1.45 = 9.655\,\text{ksi} < 10\,\text{ksi}$$

$$T_s = \frac{\tau_s J_s}{c_s} = \frac{14{,}000(\pi/2)(1^4)}{1} = 21{,}990\ \text{in.}\cdot\text{lb}$$

$$T_a = \frac{\tau_a J_a}{c_a} = \frac{9{,}655(\pi/2)(2^4 - 1^4)}{2} = 113{,}750\ \text{in.}\cdot\text{lb}$$

- Max. $T =$?

$$T = T_a + T_s = 113{,}750 + 21{,}990 = 135{,}740\ \text{in.}\cdot\text{lb} \cong 135.74\ \text{in.}\cdot\text{kip}$$

$$\theta = \theta_a = \theta_s = \frac{\tau_s L_s}{G_s c_s} = \frac{14{,}000(60)}{11.6(10^6)(1)} = 0.0724\,\text{rad}$$

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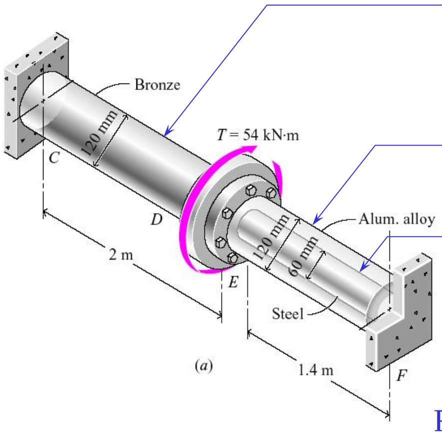
Example Problem 6-10 (I)



(a)

Given:

- Section *CD*
  - Solid, bronze, G_B = 45 GPa
- Section *EF*
  - Hollow aluminum alloy tube G_A = 28 GPa
  - Steel core, G_A = 80 GPa
- **Bolt clearance** = 0.03 rad before *EF* carries any load
- T = 54 kN·m

Find:

- τ_{max} = ? in each material

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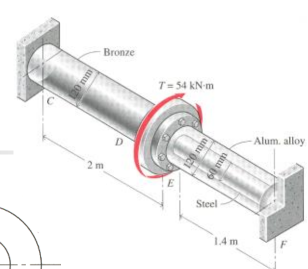
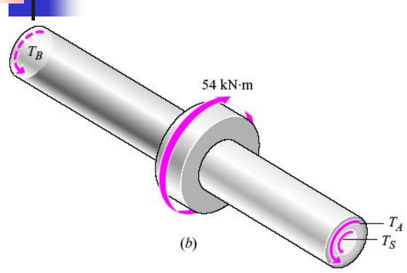
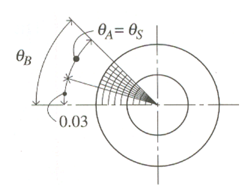
Example Problem 6-10 (II)

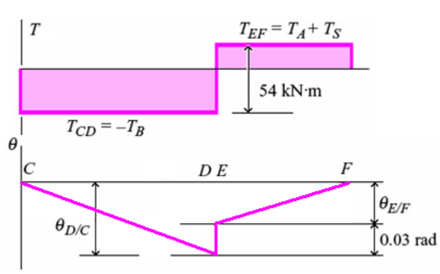




(b)



$$T_B + T_A + T_S = 54(10^3)$$



$$\theta_{\text{total}} = \theta_{F/E} + \theta_{E/D} + \theta_{D/C} = 0$$

$$\theta_{E/D} = 0.03 \text{ rad}$$

$$\left(\theta_{F/E}\right)_A = \left(\theta_{F/E}\right)_S$$

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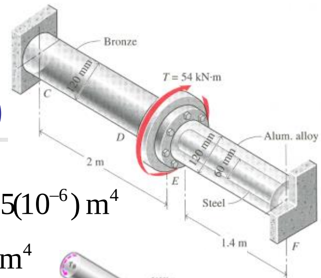
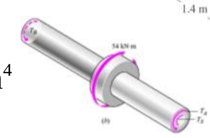
Example Problem 6-10 (III)



$$J_A = (\pi/2)(60^4 - 30^4) = 19.085(10^6)\ \text{mm}^4 = 19.085(10^{-6})\ \text{m}^4$$

$$J_B = (\pi/2)(60^4) = 20.36(10^6)\ \text{mm}^4 = 20.36(10^{-6})\ \text{m}^4$$

$$J_S = (\pi/2)(30^4) = 1.2723(10^6)\ \text{mm}^4 = 1.2723(10^{-6})\ \text{m}^4$$



$$T_B + T_A + T_S = 54(10^3) \quad \Rightarrow \quad \frac{\tau_B J_B}{c_B} + \frac{\tau_A J_A}{c_A} + \frac{\tau_S J_S}{c_S} = 54(10^3)$$

$$\frac{\tau_B(20.36)(10^{-6})}{60(10^{-3})} + \frac{\tau_A(19.085)(10^{-6})}{60(10^{-3})} + \frac{\tau_S(1.2723)(10^{-6})}{30(10^{-3})} = 54(10^3)$$

$$16\tau_B + 15\tau_A + 2\tau_S = 2.546(10^9)$$

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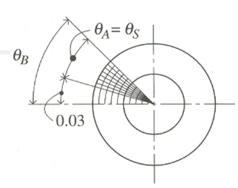
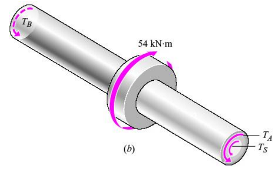
Example Problem 6-10 (IV)



$$\frac{-\tau_B L_B}{G_B c_B} + \frac{\tau_A L_A}{G_A c_A} + 0.03 = 0$$

$$\frac{-\tau_B(2)}{45(10^9)(60)(10^{-3})} + \frac{\tau_A(1.4)}{28(10^9)(60)(10^{-3})} + 0.03 = 0 \qquad 8\tau_B = 9\tau_A + 324(10^6)$$



$$\theta_A = \theta_s \qquad \frac{\tau_A L_A}{G_A c_A} = \frac{\tau_S L_S}{G_S c_S}$$

$$\frac{\tau_A(1.4)}{28(10^9)(60)(10^{-3})} = \frac{\tau_S(1.4)}{80(10^9)(60)(10^{-3})} \qquad 10\tau_A = 7\tau_S$$

$$\tau_A \cong 52.9\ \text{MPa} \qquad \tau_B \cong 100.0\ \text{MPa} \qquad \tau_S \cong 75.6\ \text{MPa}$$

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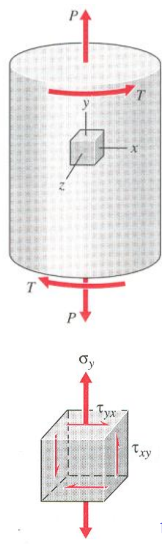
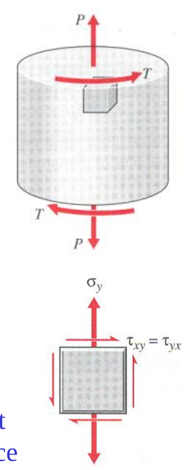
6-8 Combined Effects – Normal and Shearing Stress



small element at
the outside surface



$$\sigma_y = \frac{P}{A}$$

$$\tau_{xy} = \frac{Tc}{J}$$

plane stress

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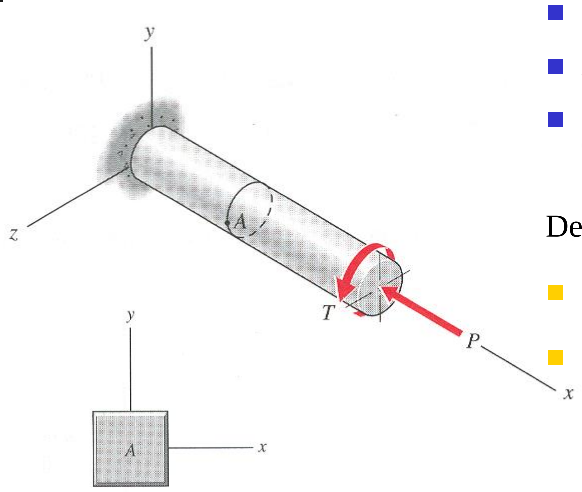
Example Problem 6-11 (I)



- d = 100 mm
- P =200 kN
- T = 30 kN·m

Determine

- stresses at point A
- σ_p , τ_{max} = ?

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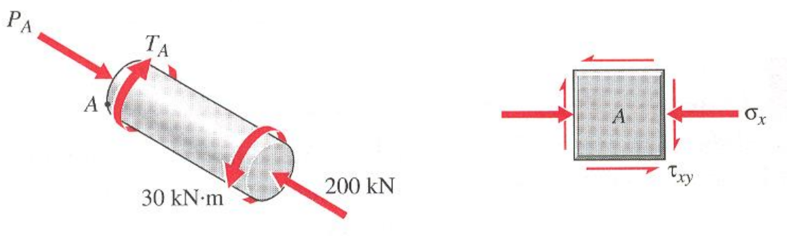
Example Problem 6-11 (II)



$$\sigma_x = \frac{P_A}{A} = \frac{200(10^3)}{(\pi/4)(0.100)^2}\ \text{N/m}^2 = 25.46\,\text{MPa} \cong 25.5\,\text{MPa(C)}$$

$$\tau_{xy} = \frac{T_A c}{J} = \frac{30(10^3)(0.050)}{(\pi/2)(0.050)^4}\ \text{N/m}^2 = 15279\,\text{MPa} \cong 1528\,\text{MPa}$$

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Example Problem 6-11(III)

$$\sigma_x = -25.46\,\text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -15279\,\text{MPa}$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-25.46 + 0}{2} \pm \sqrt{\left(\frac{(-25.46) - 0}{2}\right)^2 + (-15279)^2} = -12.73 \pm 15332$$

$$\sigma_{p1} = -12.73 + 15332 \cong 1406\,\text{MPa}$$

$$\sigma_{p2} = -12.73 - 15332 \cong -1661\,\text{MPa}$$

$$\sigma_{p3} = \sigma_z = 0$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{14059 - (-16605)}{2} \cong 1533\,\text{MPa}$$

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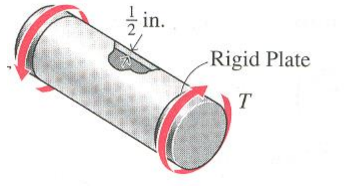
Example Problem 6-12 (I)

Given: a pressure vessel



- d_i = 24 in.
- t = 1/2 in.
- P = 250 psi
- T = 150 ft·kip

Determine

- σ_{max} , τ_{max} = ?

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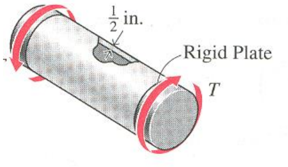
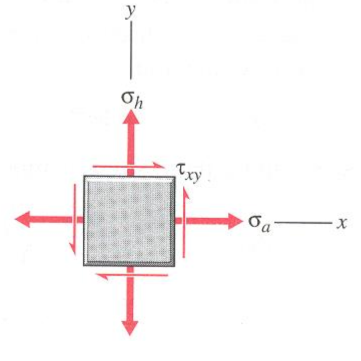
Example Problem 6-12 (II)



- d_i = 24 in.
- t = 1/2 in.



$$\sigma_a = \sigma_x = \frac{pr}{2t} = \frac{250(12)}{2(1/2)} = 3000\text{psi}$$

$$\sigma_h = \sigma_y = \frac{pr}{t} = \frac{250(12)}{1/2} = 6000\text{psi}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{150(10^3)(12)(12.5)}{(\pi/2)(12.5^4 - 12^4)} = 3894\text{psi}$$

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Example Problem 6-12 (III)

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{3000+6000}{2} \pm \sqrt{\left(\frac{3000-6000}{2}\right)^2 + (3894)^2} = 4500 \pm 4173$$

$$\sigma_{p1} = 4500 + 4173 \cong 8673\text{MPa}$$

$$\sigma_{p2} = 4500 - 4173 \cong 327\text{MPa}$$

$$\sigma_{p3} = \sigma_z = 0$$

$$\sigma_{max} = \sigma_{p1} \cong 8670\text{psi}(T)$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{8673-0}{2} \cong 4340\text{MPa}$$

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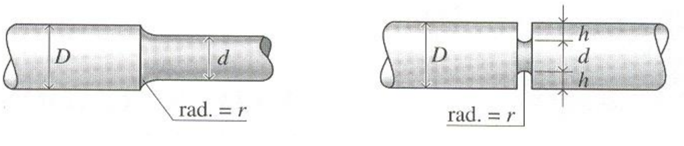
6-9 Stress Concentrations in Circular Shafts Under Torsional Loadings



- Uniform shaft

$$\tau_{max} = \frac{Tc}{J}$$

- Stress Concentration

$$K = K\left(\frac{D}{d}, \frac{r}{d}\right)$$

- K can be used to determine τ_{max} as long as τ_{max} does not exceed the proportional limit.

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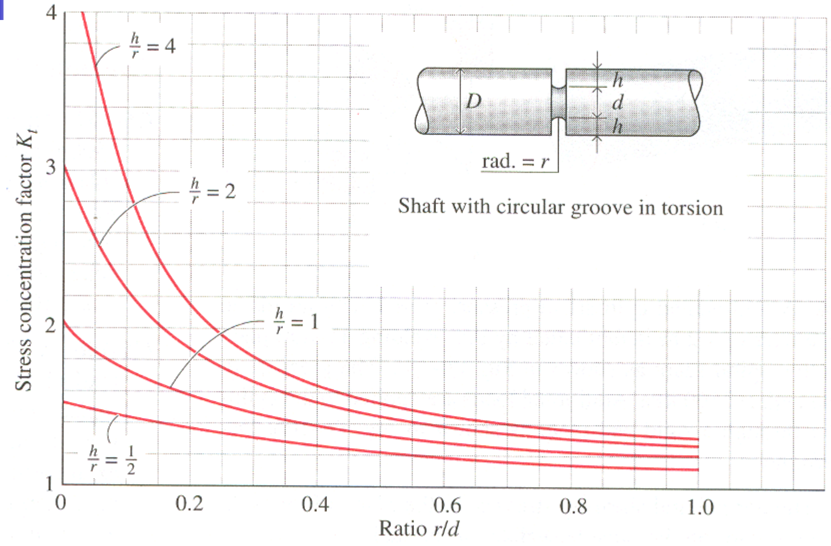
6-9 Stress Concentrations in Circular Shafts Under Torsional Loadings



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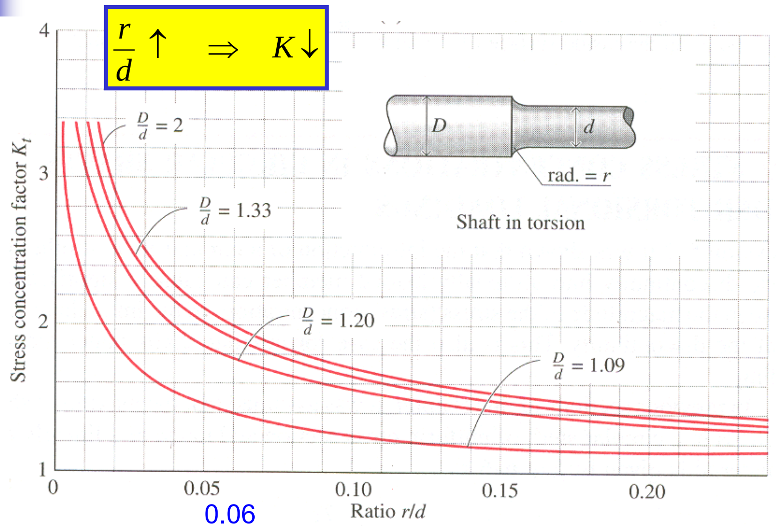
6-9 Stress Concentrations in Circular Shafts Under Torsional Loadings



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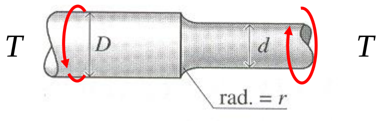
Example Problem 6-13



- D = 4 in.
- d = 2 in.
- T = 6280 in·lb
- $\tau_{allowable}$ = 8 ksi

Determine

- r_{min} = ?

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Example Problem 6-13

- In the 2in. diameter section

$$\tau_{max} = \frac{Tc}{J} = \frac{6280(1)}{(\pi/2)(1^4)} = 3998 \text{psi}$$

- In the fillet

$$\tau_{allowable} = K_t \frac{Tc}{J} = 8 \text{ksi}$$

$$K_t = 8/3.998 \cong 2$$

- D/d = 2, K_t = 2. From Fig. 6-25(b),

$$r = 0.06d = 0.06(2) = 0.1200 \text{in.}$$

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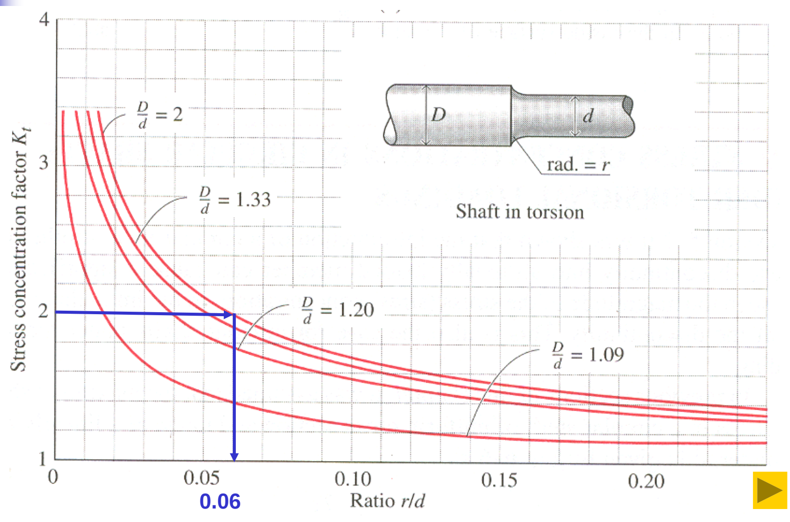
6-9 Stress Concentrations in Circular Shafts Under Torsional Loadings



8 Exercises

6-18,	**6- 22,**	**6-36,**	**6-47,**
6- 74,	**6- 87,**	**6-98,**	**6- 148**