8-27*

From overall equilibrium: $R_A = P \uparrow$ $\quad M_A = Pa \circlearrowleft$

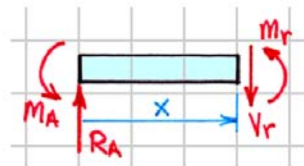
(a) $EIv'' = M_r = Px - Pa$

Boundary Conditions:

$$EIv' = \frac{Px^2}{2} - Pax + C_1 \qquad \text{At } x = 0, \quad v' = 0: \quad C_1 = 0$$

$$EIv = \frac{Px^3}{6} - \frac{Pax^2}{2} + C_1 x + C_2 \qquad \text{At } x = 0, \quad v = 0: \quad C_2 = 0$$

$$v = \frac{P}{6EI}\left(x^3 - 3ax^2\right)$$ .... **Ans.**



(b) $\delta_B = v_{x=L} = \dfrac{P}{6EI}\left(L^3 - 3aL^2\right)$

For $a = a_{\min} = \dfrac{L}{4}$: $\quad \delta_B = \dfrac{+PL^3}{24EI} = \dfrac{PL^3}{24EI} \uparrow$

For $a = a_{\max} = \dfrac{3L}{4}$: $\quad \delta_B = \dfrac{-5PL^3}{24EI} = \dfrac{5PL^3}{24EI} \downarrow$

$a = \dfrac{3L}{4}$: .... **Ans.**

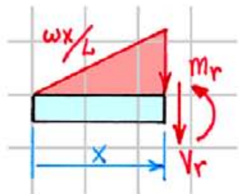
(c) $v = 0$ when $\quad L^3 - 3aL^2 = 0$

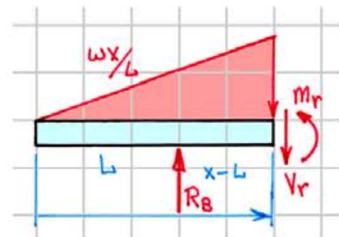
$a = L/3$ .... **Ans.**

8-30

From overall equilibrium:

$$R_B = R_C = (3wL/4)\uparrow$$





$0 \le x \le L$:

$$EIv_1'' = M_r = \frac{-wx^3}{18L}$$

$$EIv_1' = \frac{-wx^4}{72L} + C_1$$

$$EIv_1 = \frac{-wx^5}{360L} + C_1 x + C_2$$

$L \le x \le 3L$:

$$EIv_2'' = M_r = \frac{-wx^3}{18L} + \frac{3wL(x-L)}{4}$$

$$EIv_2' = \frac{-wx^4}{72L} + \frac{3wL(x-L)^2}{8} + C_3$$

$$EIv_2 = \frac{-wx^5}{360L} + \frac{3wL(x-L)^3}{24} + C_3 x + C_4$$

Boundary Conditions:

At $x = L$, $v_1 = 0$: $\quad C_1 L + C_2 = \dfrac{wL^5}{360L}$ (a)

At $x = L$, $v_2 = 0$: $\quad C_3 L + C_4 = \dfrac{wL^5}{360L}$ (b)

At $x = 3L$, $v_2 = 0$: $\quad C_3 3L + C_4 = \dfrac{-117wL^4}{360}$ (c)

Matching Condition:

At $x = L$, $v_1' = v_2'$: $\quad \dfrac{-wL^3}{72} + C_1 = \dfrac{-wL^3}{72} + C_3$ (d)

Solving Eqs. (a), (b), (c), and (d) gives $\quad C_1 = C_3 = \dfrac{-59wL^3}{360} \qquad C_2 = C_4 = \dfrac{wL^4}{6}$

(a) $0 \le x \le L$: $\quad v_1 = \dfrac{w}{360EIL}\left(-x^5 - 59L^4 x + 60L^5\right)$

$$\delta_A = v_{1,x=0} = \frac{+60wL^5}{360EIL} = \frac{wL^4}{6EI}\uparrow \text{ .... Ans.}$$

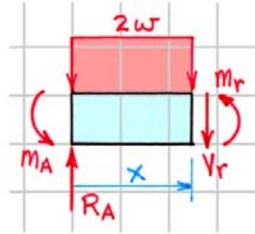
(b) $L \le x \le 3L$: $\quad v_2 = \dfrac{w}{360EIL}\left[-x^5 + 45L^2(x-L)^3 - 59L^4 x + 60L^5\right]$

$$\delta_M = v_{2,x=2L} = \frac{w}{360EIL}\left[-32L^5 + 45L^5 - 118L^5 + 60L^5\right]$$

$$\delta_M = \frac{-wL^4}{8EIL} = \frac{wL^4}{8EI}\downarrow \text{ .... Ans.}$$

8-35

From overall equilibrium:



$$R_A = 3wL \uparrow \qquad M_A = \left(5wL^2/2\right) \circlearrowleft$$

$0 \le x \le L$:

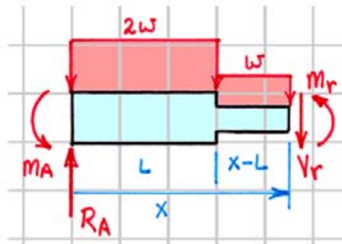
$$E(4I)v_1'' = M_r = 3wLx - \frac{5wL^2}{2} - wx^2$$

Boundary Conditions:

$$4EIv_1' = \frac{3wLx^2}{2} - \frac{5wL^2x}{2} - \frac{wx^3}{3} + C_1 \qquad \text{At } x=0,\quad v_1'=0:\quad C_1 = 0$$

$$4EIv_1 = \frac{wLx^3}{2} - \frac{5wL^2x^2}{4} - \frac{wx^4}{12} + C_1x + C_2 \qquad \text{At } x=0,\quad v_1=0:\quad C_2 = 0$$

$L \le x \le 2L$:



$$EIv_2'' = M_r = 3wLx - \frac{5wL^2}{2} - wx^2 + \frac{w(x-L)^2}{2}$$

$$EIv_2' = \frac{3wLx^2}{2} - \frac{5wL^2x}{2} - \frac{wx^3}{3} + \frac{w(x-L)^3}{6} + C_3$$

$$EIv_2 = \frac{wLx^3}{2} - \frac{5wL^2x^2}{4} - \frac{wx^4}{12} + \frac{w(x-L)^4}{24} + C_3x + C_4$$

Matching Conditions:

At $x = L$, $v_1' = v_2'$: $\quad \dfrac{1}{4}\left[\dfrac{3wL^3}{2} - \dfrac{5wL^3}{2} - \dfrac{wL^3}{3}\right] = \dfrac{3wL^3}{2} - \dfrac{5wL^3}{2} - \dfrac{wL^3}{3} + C_3$

At $x = L$, $v_1 = v_2$: $\quad \dfrac{1}{4}\left[\dfrac{wL^4}{2} - \dfrac{5wL^4}{4} - \dfrac{wL^4}{12}\right] = \dfrac{wL^4}{2} - \dfrac{5wL^4}{4} - \dfrac{wL^4}{12} + C_3L + C_4$

Therefore: $\qquad C_3 = wL^3 \qquad C_4 = -3wL^4/8$

(a) $\qquad v_1 = \dfrac{w}{48EI}\left(-x^4 + 6Lx^3 - 15L^2x^2\right)$

$$\delta_B = v_{1,x=L} = \frac{w}{48EI}\left(-L^4 + 6L^4 - 15L^4\right) = \frac{-10wL^4}{48EI} = \frac{5wL^4}{24EI}\downarrow$$ .... **Ans.**

(b) $\qquad v_2 = \dfrac{w}{24EI}\left[-2x^4 + 12Lx^3 - 30L^2x^2 + (x-L)^4 + 24L^3x - 9L^4\right]$

$$\delta_C = v_{2,x=2L} = \frac{w}{24EI}\left[-2\left(16L^4\right) + 12\left(8L^4\right) - 30\left(4L^4\right) + \left(L^4\right) + 24\left(2L^4\right) - 9L^4\right]$$

$$\delta_C = \frac{-16wL^4}{24EI} = \frac{2wL^4}{3EI}\downarrow$$ .... **Ans.**

8-44

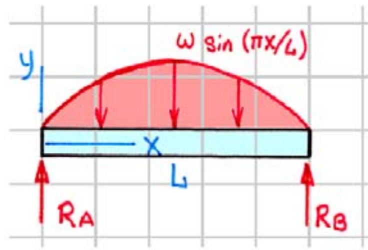
$$EIv'''' = -w \sin \frac{\pi x}{L}$$

$$EIv''' = \frac{wL}{\pi} \cos \frac{\pi x}{L} + C_1$$

$$EIv'' = \frac{wL^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$EIv' = \frac{-wL^3}{\pi^3} \cos \frac{\pi x}{L} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EIv = \frac{-wL^4}{\pi^4} \sin \frac{\pi x}{L} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$



Boundary Conditions:

At $x = 0$, $M = EIv'' = 0$: $C_2 = 0$

At $x = L$, $M = EIv'' = 0$: $C_1 = 0$

At $x = 0$, $v = 0$: $C_4 = 0$

At $x = L$, $v = 0$: $C_3 = 0$

(a) $v = \frac{-wL^4}{\pi^4 EI} \sin \frac{\pi x}{L}$ .... **Ans.**

(b) $\delta_M = v_{x=L/2} = \frac{-wL^4}{\pi^4 EI} = \frac{wL^4}{\pi^4 EI} \downarrow$ .... **Ans.**

(c) $\theta_A = v'_{x=0} = \frac{-wL^3}{\pi^3 EI} = \frac{wL^3}{\pi^3 EI}$ ↻ .... **Ans.**

(d) $R_A = V_{x=0} = EIv'''_{x=0} = \frac{wL}{\pi} = \frac{wL}{\pi} \uparrow$ .... **Ans.**

$R_B = -V_{x=L} = -EIv'''_{x=L} = -\left[\frac{-wL}{\pi}\right] = \frac{+wL}{\pi} = \frac{wL}{\pi} \uparrow$ .... **Ans.**