Final Exam

1.

$\sum M_B = 0$: $[(1500)(12)(\frac{1}{2})](4) - 12 R_A = 0$ $\qquad R_A = 3000\ lb$

$\sum M_A = 0$: $12 R_B - [(\frac{1}{2})(1500)(12)](8) = 0$ $\qquad R_B = 6000\ lb$

(a) $V_r = 3000 - (\frac{1}{2})(\frac{1500X}{12})(X) = (-62.5X^2 + 3000)\ lb$ #

$M_r = 3000X - [(\frac{1}{2})(\frac{1500X}{12})X](\frac{X}{3}) = [-20.83X^3 + 3000X]\ lb \cdot ft$ #

(b) $\frac{dV_r}{dx} = -125X = 0$, solve $X = 0$

$V_{x=0} = 3000\ lb$, $V_{x=12} = -6000\ lb$

$V_{max} = V_{x=12} = (-6000)\ lb$ #

$\frac{dM_r}{dx} = -62.5X^2 + 3000 = 0$, solve $X = 6.928\ ft$

$M_{x=0} = 0\ lb.ft$, $M_{x=6.928} = 13828\ lb.ft$, $M_{x=12} = 0\ lb.ft$

$M_{max} = M_{x=6.928} = 13858\ lb.ft \cong 13.86\ kip.ft$ #

2.

$$EI\,v'''' = -w \sin \frac{\pi x}{L}$$

$$EI\,v''' = \frac{wL}{\pi} \cos \frac{\pi x}{L} + C_1$$

$$EI\,v'' = \frac{wL^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$EI\,v' = \frac{-wL^3}{\pi^3} \cos \frac{\pi x}{L} + \frac{C_1}{2} x^2 + C_2 x + C_3$$

$$EI\,v = \frac{-wL^4}{\pi^4} \sin \frac{\pi x}{L} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

B.C.

$x = 0,\quad M = EIv'' = 0 \quad \Rightarrow \quad C_2 = 0$

$x = L,\quad M = EIv'' = 0 \quad \Rightarrow \quad C_1 = 0$

$x = 0 \quad v = 0 \quad \Rightarrow \quad C_4 = 0$

$x = L \quad v = 0 \quad \Rightarrow \quad C_3 = 0$

(a) $$v = \frac{-wL^4}{\pi^4 EI} \sin \frac{\pi x}{L}$$

(b) $$\delta_M = v_{x=L/2} = \frac{-wL^4}{\pi^4 EI} = \frac{wL^4}{\pi^4 EI} \downarrow$$

(c) $$\theta_A = v'_{x=0} = \frac{-wL^3}{\pi^3 EI} = \frac{wL^3}{\pi^3 EI}$$

(d) $$R_A = V_{x=0} = EI\,v''' = \frac{wL}{\pi} = \frac{wL}{\pi} \uparrow$$

$$R_B = -V_{x=L} = EI\,v'''_{x=L} = -\left[-\frac{wL}{\pi}\right] = \frac{+wL}{\pi} = \frac{wL}{\pi} \uparrow$$

3.

$$J_{AB,s} = \pi d^4/32 = \pi(160)^4/32 = 64.33982\,(10^6)\ mm^4$$

$$J_{BC,s} = \pi(160^4 - 100^4)/32 = 54.52234\,(10^6)\ mm^4$$

$$J_{BC,b} = \pi(100)^4/32 = 9.81748\,(10^6)\ mm^4$$

$$T_s + T_b = 75\ kN\cdot m$$

$$\theta_{BC,s} = \theta_{BC,b} \qquad \theta = TL/GJ$$

$$\frac{T_s(1.5)}{(54.52234\times10^{-6})(80\times10^9)} = \frac{T_b(1.5)}{(9.81748\times10^{-6})(40\times10^9)}$$

$$T_s = 11.10720\ T_b$$

$$T_b = 6.19466\ kN\cdot m \qquad T_s = 68.80534\ kN\cdot m$$

In AB: $\tau_s = \dfrac{Tc}{J} = \dfrac{(85000)(0.08)}{(64.33982\times10^{-6})} = 105.6888\,(10^6)\ N/m^2$

$= 105.6888\ MPa$

In BC: $\tau_s = \dfrac{(68805.34)(0.08)}{(54.52234\times10^{-6})} = 101.0\,(10^6)\ N/m^2 = 101.0\ MPa$

$$\tau_b = \frac{(6194.66)(0.05)}{(9.81748\times10^{-6})} = 31.5\,(10^6)\ N/m^2 = 31.5\ MPa$$

(a) $\tau_{max,s} = 105.7\ MPa$ #

$\tau_{max,b} = 31.5\ MPa$ #

(b) $\theta_D = \theta_{B/A} + \theta_{C/B} + \theta_{D/C}$, $\theta_D = TL/GJ$

$$\theta_D = \frac{(85000)(2)}{(64.33982\times10^{-6})(80\times10^9)} + \frac{(-68805.34)(1.5)}{(54.52234\times10^{-6})(80\times10^9)} + 0$$

$= 0.00937\ rad$ #

4.

$A = \pi (450)^2 = 636172.5124 \ mm^4 = 0.6362 \ m^2$

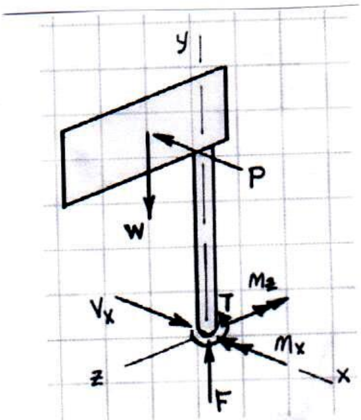
$Q = \dfrac{2(450)^3}{3} = 6.075 \times 10^7 \ mm^4 = 0.06075 \ m^3$ $\Rightarrow$ from CH 7-16 Page 458

$Q = \bar{y}_c A = \left(\frac{4r}{3\pi}\right)\left(\frac{\pi r^2}{2}\right)$

$= \frac{2}{3} r^3$

$I = \dfrac{\pi (450)^4}{4} = 3.2206 \times 10^{10} \ mm^4 = 0.0322 \ m^4$

$J = 2I = 0.0644 \ m^4$



$F = W = mg = (250)(9.81) = 2453 \ N \ (C)$

$V_x = P = pA = (1500)(8 \times 3) = 36000 \ N$

$M_x = W(3) = (2453)(3) = 7359 \ N.m$

$M_z = P(9) = (36000)(9) = 324000 \ N.m$

$T = P(3) = (36000)(3) = 108000 \ N.m$

$\sigma_F = \dfrac{F}{A} = \dfrac{2453}{0.6362} = 3.856 \times 10^3 \ N/m^2 = 3.856 \ kPa$

$\sigma_{M_x} = \dfrac{M_x c}{I} = \dfrac{(7359)(0.450)}{0.0322} = 102.824 \ kPa$

$\sigma_{M_z} = \dfrac{M_z c}{I} = \dfrac{(324000)(0.450)}{0.0322} = 4527.106 \ kPa$

$\tau_T = \dfrac{Tc}{J} = \dfrac{(108000)(0.450)}{0.0644} = 754.51 \ kPa$

$\tau_{V_x} = \dfrac{V_x Q}{It} = \dfrac{(36000)(0.06075)}{0.0322(0.900)} = 75.451 \ kPa$

At A:

$\sigma_x = 0$ kPa ,

$\sigma_y = \frac{F}{A} + \frac{M_x c}{I} = 3.856 + 102.824 = 106.68$ kPa (C)

$\tau_{xy} = \frac{Tc}{J} + \frac{V_x Q}{It} = 754.51 + 75.451 = 830$ kPa

$$\sigma_{P_1,P_2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 - 106.68}{2} \pm \sqrt{\left(\frac{0+106.68}{2}\right) + (830)^2}$$

$\sigma_{P_1} = 885.063$ kPa

$\sigma_{P_2} = -778.36$ kPa

$\sigma_{P_3} = 0$ kPa

$\tau_{max} = \tau_p = \frac{\sigma_{P_1} - \sigma_{P_2}}{2} = 831.71$ kPa #

At B:

$\sigma_x = 0$ kPa

$\sigma_y = \sigma_F + \sigma_{M_z} = 4523$ kPa

$\tau_{xy} = \tau_T = 754.5$ kPa

$\sigma_{P_1} = 4645.6$ kPa

$\sigma_{P_3} = 0$ kPa

$\sigma_{P_2} = -122.6$ kPa

$\tau_{max} = 2384.1$ kPa #