Chapter 1
Introduction to Modern Measuring Techniques of Thermal Fluid Mechanics

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Preface

實驗是檢驗理論的尺，也是發覺真理最直接的方法。

「工欲善其事，必先利其器」，本課 程之目的即在奠定對實驗量測有興趣之大學部及研究所同學們正確而廣泛的熱流實驗量測基礎知識，以避免經常可見的殺雞只會用牛刀，或是犯了以「變形的尺來檢驗理論」之誤謬。在本課程中，除了將講授目前一般精密熱流實驗量測所依據的相關學理與其應用，以及不同實驗方法間作比較分析外，並將探討目前奈/微米機電系統(NEMS/MEMS)技術所帶來對未來熱流精密實驗量測與應用之衝擊與影響，配合利用給予目標實際動手實驗、影帶觀賞與實驗室參觀等方式，以加強同學們對此課程之了解與訓練 。

Course Contents

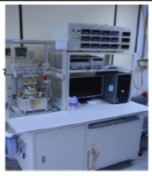
- Introduction to modern measuring techniques of thermal fluid mechanics
- Design of a new test facility for micro/macro-experiments
- Flow Visualization techniques
- Pressure Measurements
- Temperature measurements
- Velocity measurements
- Particle/droplet size measurements
- Flow rate Measurements
- Miscellaneous Material property measurements (viscosity, surface tension, density, refractive index, thermal conductivity …etc.)

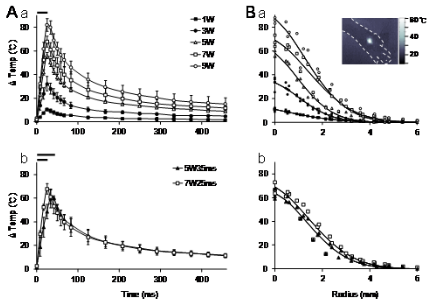
Reference & Grading Policy

- **Reference: Springer Handbook of Experimental Fluid Mechanics, Tropea, Cameron; Yarin, Alexander L.; Foss, John F. (Eds.) , 2007, XXVIII, 1557 p. 1240 illus. in color. With DVD., Hardcover, ISBN: 978-3-540-25141-5**
- (Authors Introduction) **Book in NTU-library available**
- **Lecture Notes on Web:** http://bernoulli.iam.ntu.edu.tw/tw/index.htm, 進入「學術課程」，再進入『現代熱流量測技術』
- **Grading Policy:** term project report and presentation: 100%

Recent Topics of Term-projects (I)

1. **Thermal conductivity measurement** for high k thin plate (CSI
(“以暫態平面熱源量測技術快速量測材料熱性質之探討”, 2012台灣熱管理協會年
2. **High speed** **dynamic temperature measurement** for skin sur
(Temporal and spatial temperature distribution of rat’s glabrous skin
pulse CO_2 laser, cooperation with Profs. 嚴震東& 趙福杉, published
Biomedical Optics, 17 (11), 117002, 1-8; (SCI [**IF=3.157**])

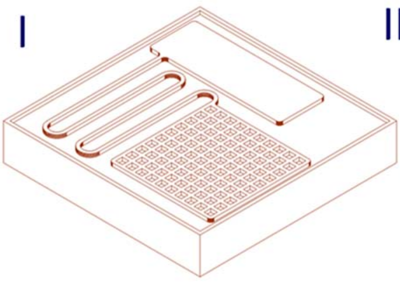
方法	穩態量測	暫態量測
設備示意圖		 $R_x(t)=R_0\left[1+\alpha\overline{\Delta T(t)}\right]$
時間	90分鐘	5秒-20分鐘
範圍	<10W/mk	0.005-500W/mk
成本	七十萬	★兩百萬★
體積	大	小



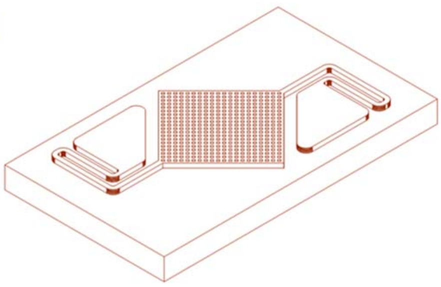
Recent Topics of Term-projects (II)

3. Design of **constant velocity micro-channel & measurement**
4. **Microsphere generation & measurement**
5. **Acoustic measurement** for drop impact and **image analysis**
6. **Instantaneous patterning of UV-curable adhesive & measurement**

I

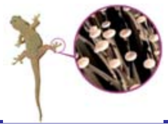


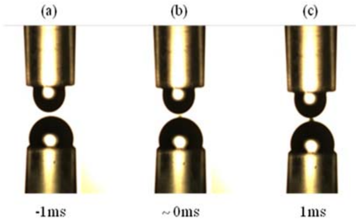
II



Recent Topics of Term-projects (III)

1. **Drop impact dynamics** Drop-drop coalescence **and measuring techniques** (undergraduates)
   (Illusive Effects of Generating Nipples between Two Coalescing Water Droplets, The twelfth Asian Symposium of Visualization (ASV#12), 2013)
2. Unsteadiness **flow-rate measurement** & **control** (T-junction project)
3. **Viscosity measurement** with minimum liquid volume
4. Viscoelastic **Adhesion measurement** (Gecko project)
5. **Instantaneous patterning of UV-curable adhesive & measurement**
6. Dynamics of Interfacial layers (Touch panel project) & measurement
7. Bubble generation & coalescence dynamics
8. **Microsphere generation & measurement**
9. Optical drying related thermal measurements





Recent Topics of Term-projects (IV)

- **Real-time pressure monitoring in a micropump**
- **Splashing of different drops onto liquid film**
- **Visualization of flow within coating die**
- **Preventing Ice Adhesion to Metal**

About “Preventing Ice Adhesion to Metal”

The successful technology will:

- Demonstrate anti-adhesion and easy release of ice from metal surfaces, in the presence of air flow (surface area 4 in^2)
- Metal surfaces including but not limited to: aluminum, stainless steel, titanium, and nickel
- Allow application on complex three-dimensional shapes
- Maintain anti-ice adhesion characteristics after exposure to particulates and water struck at high velocities
- Maintain properties at extreme low temperatures, -50 ˚F, with exposure to occasional temperatures of 200 ˚F
- Not be susceptible to cracking or aging or degradation by exposure to UV light
- Long maintain performance (minimum 10-20 year time period)
- Preferably be effective at minimum cost and weight

APPROACHES NOT OF INTEREST: Use of hot fluid to prevent ice buildup

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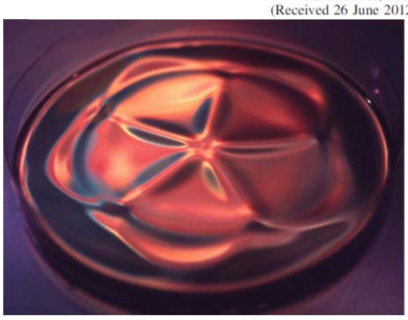
Experiment 1:

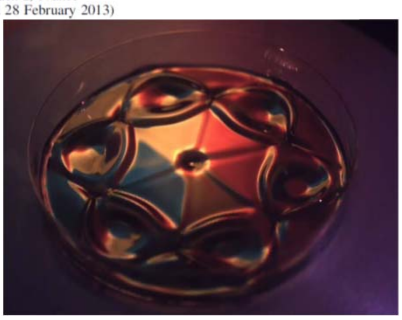
PRL 110, 094502 (2013) PHYSICAL REVIEW LETTERS week ending 1 MARCH 2013

Observation of Star-Shaped Surface Gravity Waves

Jean Rajchenbach,[1,*] Didier Clamond,[2] and Alphonse Leroux[1]
[1]*Laboratoire de Physique de la Matière Condensée (CNRS UMR 7336) Université de Nice—Sophia Antipolis, Parc Valrose, 06108 Nice Cedex 2, France*
[2]*Laboratoire Jean-Alexandre Dieudonné (CNRS UMR 7351) Université de Nice—Sophia Antipolis, Parc Valrose, 06108 Nice Cedex 2, France*
(Received 26 June 2012; published 28 February 2013)



a vibration amplitude of 1.95 mm (filling level 7 mm, $\Omega/2\pi$= 8 Hz)



Symmetry of 6th order, (filling level 8 mm, vibration amplitude 2.90 mm, $\Omega/2\pi$ =12 Hz

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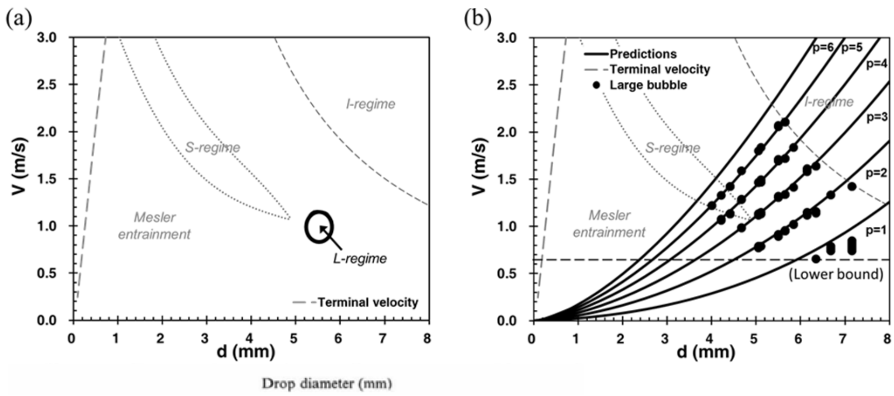
Experiment 2:
Drop impact induced bubbles in different fluids

A.-B. Wang, C.-C. Kuan, P.-H. Tsai

Physics of Fluids 25, 101702 (2013)



Experiment 3:

- ???
- If you need help, please let me know.

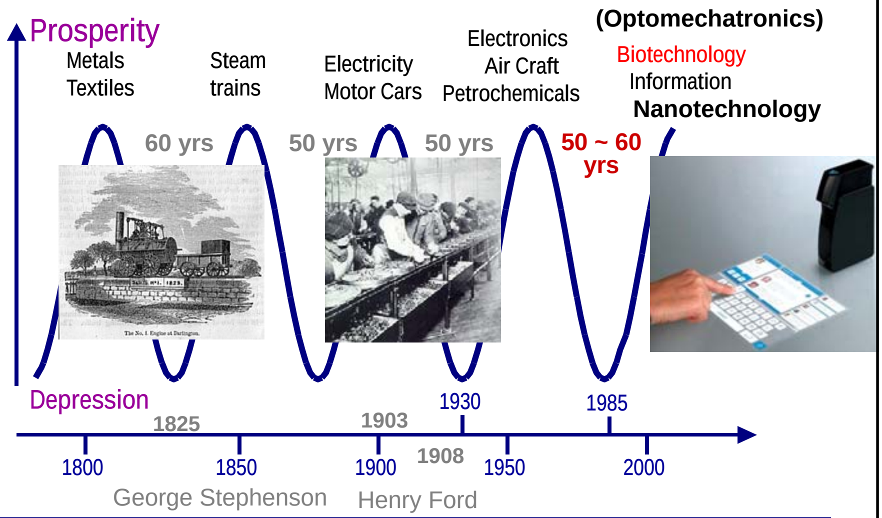
Schedule of term project

1. 2014/3/5 Determination of groups of term-projects
2. 2014/4/9 1st monthly report
3. 2014/5/14 2nd monthly report
4. 2014/6/18 final project presentation

Lecture Contents of Chapter 1

- Why thermal fluid mechanics ?
- Tools for solving problems
- The needs and design of modern measuring techniques of Thermal Fluid Mechanics
- Dimensional analysis & Π-theorem
- Modelling and Similarity
- Accuracy Analysis

Trend of the world



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IA-products

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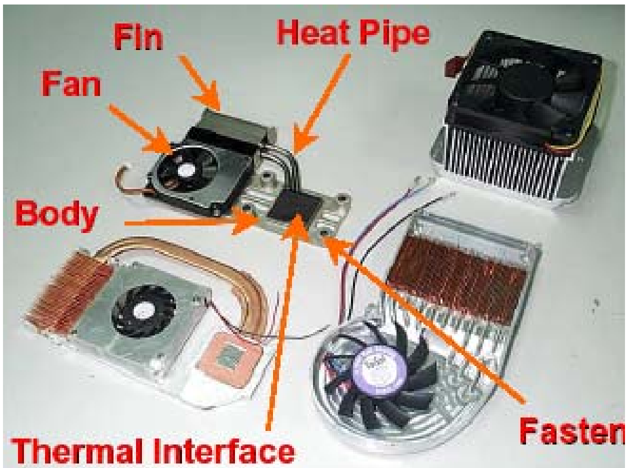
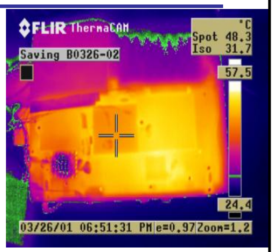
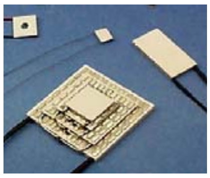
Thermal solutions





Notebook Bottom Skin

TE-Cooler



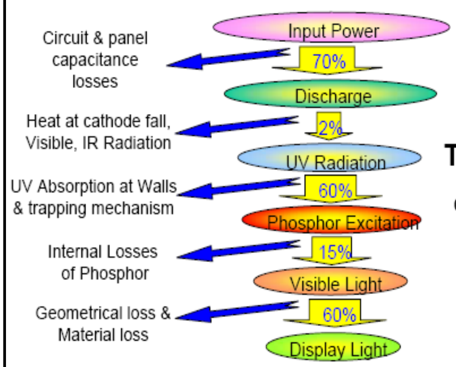
影像顯示科技(*FPD Technology*)

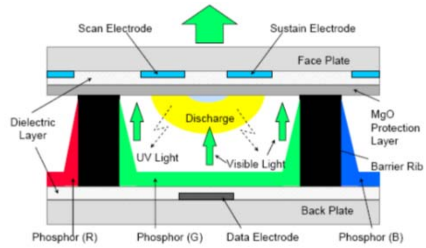
- 視覺為五覺之首，是接受資訊與知識之大門

影像顯示科技
- 內涵 (數位內容)
- 外表
  - 軟顯示器
  - 硬顯示器

PDP

PDP: Plasma Display Panel
(電漿/等離子體)





Total Efficiency of PDP=0.15%

2.0 lm/W

103” 松下 @ 2006 CeBIT

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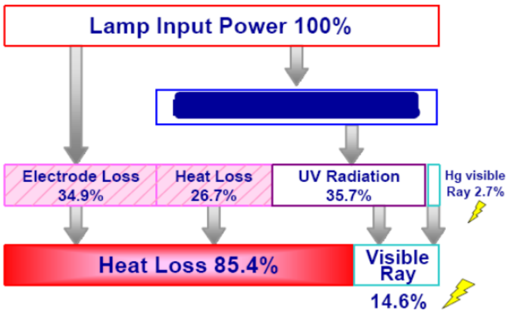
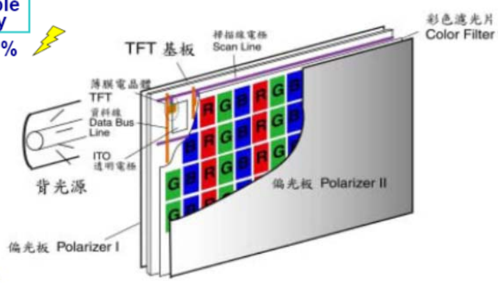
TFT-LCD





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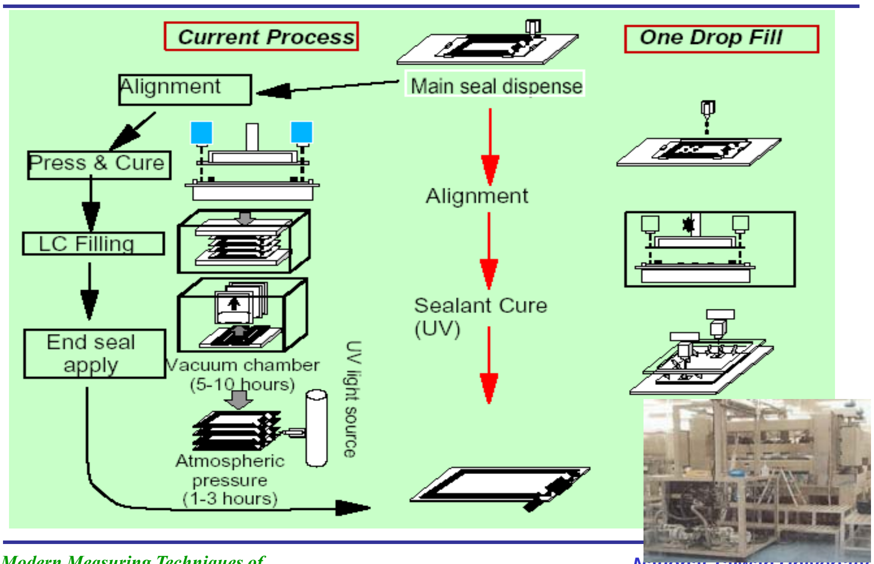
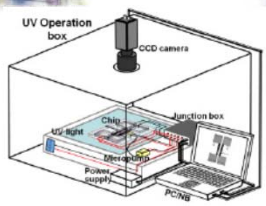
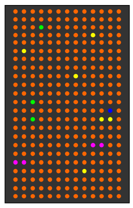
Old Process & ODF for FPD Manufacture

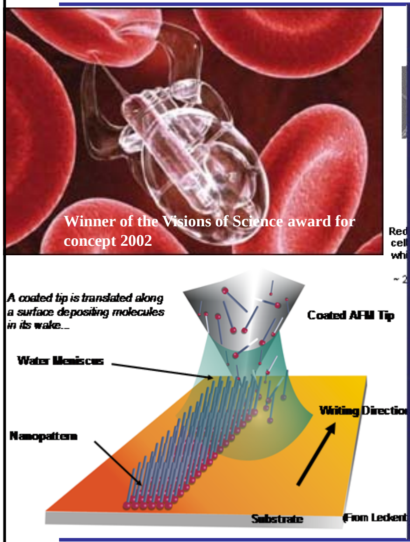


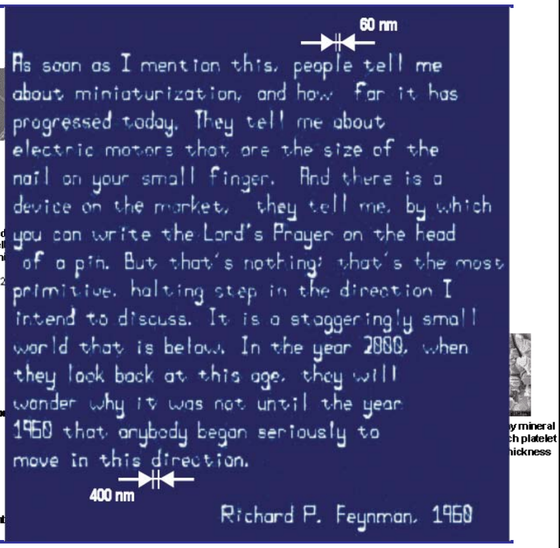
Bio-technology



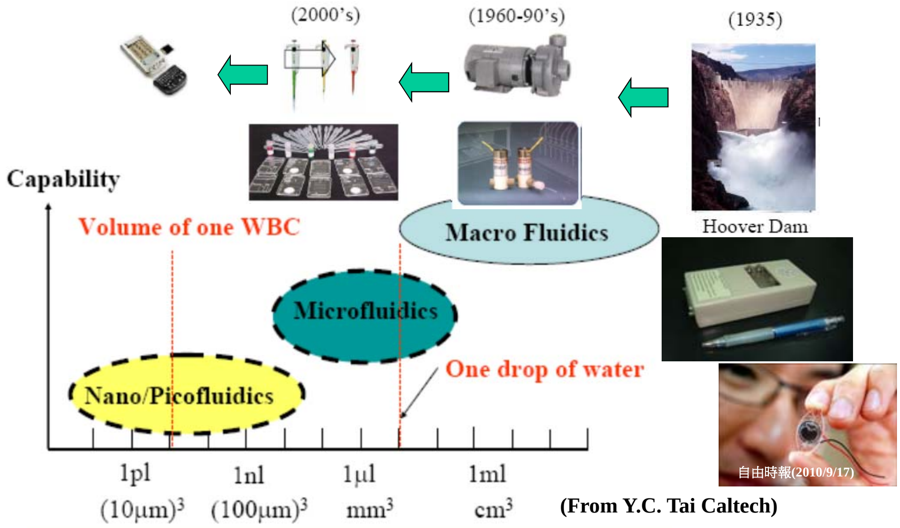


Nano-technology



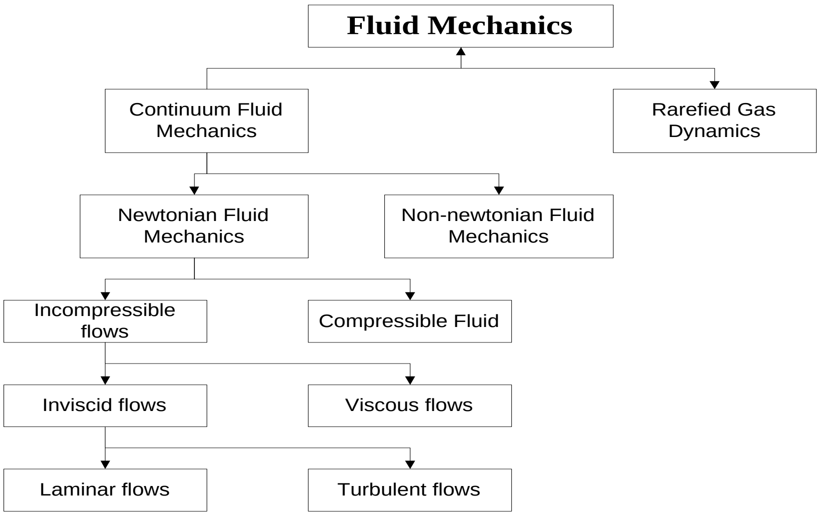


Evolution of Fluidics

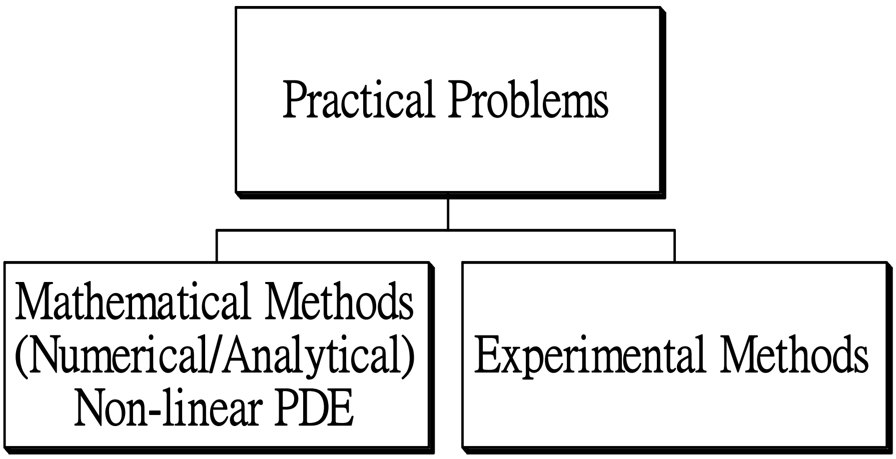


(From Y.C. Tai Caltech)

Scope of Fluid Mechanics



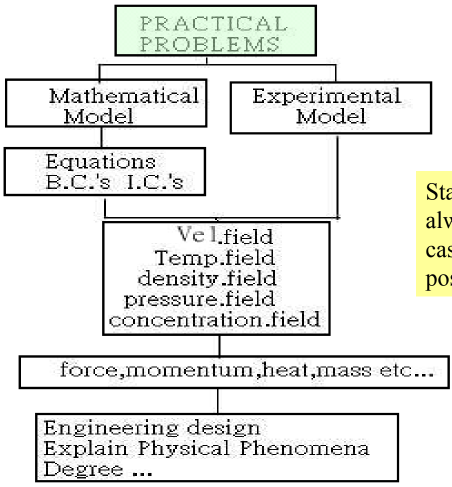
Tools for Solving Problems



Approaches to solve Fluid Mechanics

Numerical
Analytical

Nonlinear PDE
simplification is commonly needed



Status: Experiments are always needed in any cases, except it is not possible to be pursued.

Experiments & Simulations

Speedo LZR Racer swimsuit

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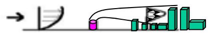
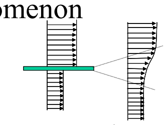
Why experiments ?

The need for experiments in fluid mechanics arises from a variety of aims :

- **Basic research** 
  - Extend physical understanding of a particular flow phenomenon
  - Test new theoretical results
  - Verify numerical models (e.g. in CFD)



- **Model studies** (inexpensive compared to prototype tests in most cases)
  - Investigate an unknown flow situation or test new apparatus designs
  - conduct a systematic parameter study and/or optimization
  - establish scaling laws

- **Flow Measurement**
  - E.G. Volume flow rate, drag, lift ...etc
  - Measurement of a quantity for control feedback purposes.

Design of an Experiment (I)

In designing an experiment, a number of questions must be asked :

- ***Which quantities*** are important to measure (both independent and dependent)?
  - This is answered partly through the ***aim of the experiment*** and partly through ***experience.***
  - ***Dimensional analysis*** is ***indispensable*** for complex systems.
- In ***what range*** will the measured quantities vary ?
  - This information,and information about the required accuracy is necessary for the choice of measuring technique.
- Which quantities must be controlled ? ---operating conditions must be well defined in order to control the experiments to be ***repeatable, stationary*** and simulated to the given conditions.

Design of an Experiment (II)

- What is the required dynamic response of the measuring instrument?
  - Is any correction or compensation necessary ?
- What is the measurement time ?
  - For time mean averaged Quantities,a given statistical variance should be specified.
- Has this experiment already been performed ?
  Does a theoretical solution exist ?

Dimensional analysis

Dimensional analysis facilitates the interpretation and extends the range of application of experimental data by correlating them in terms of dimensionless groups. Its most serious limitation is that it gives ***no*** information about the nature of a phenomenon.Rather it is necessary to know before hand which variables influence the phenomenon.

Typically ,the primary dimensions are chosen as

- M : mass
- L :length
- T :time
- θ: temperature

Why dimensional analysis (I)?

Dimensional analysis (***DA***) is enormous time- and money-saving !
Example: force on an immersed body

$$F= f(L,V,\rho,\mu)$$

generally speaking , we need 10 experimental points to define a curve, this means that we need

$$10*10*10*10=10^4 \text{ experiments.}$$

assuming NT \$10 /experiment we need 10^5 NT - dollars
and 0.1 day /experiment, we need 10^3 days =2.7years !!!
However , by using ***DA***

$$C_f = F/ \rho V^2L^2 = g(\rho VL/\mu) = g(Re)$$

Nothing is lost , but with
NT \$ 10 /experiment x 10 experiments =100 NT-dollars
and 0.1 day /experiment x 10 exp. = 1 day!!
It's a big difference !

Why dimensional analysis (II)?

2. ***DA*** helps thinking and planning (not only experiment but also theory)!
   It suggest variable which can be discarded and often give a great deal of insight into the form of the physical relationship .
3. ***DA*** provides ***scaling law*** (or similarity) which may convert data from a cheap, small model into design information for an expensive ,large prototype.

- A method for describing dimensionless parameter is generally credited to E. Buckingham in 1914, and is commonly called "***Buckingham Π-theorem***".
- From the governing Equations, boundary conditions, we may also get the governing dimensionless parameters

Dimensionless Groups in Fluid Mechanics (I)

Parameter	Definition	Qualitative ratio of effects	Importance
Reynolds number	$\mathrm{Re} = \frac{\rho U L}{\mu}$	$\frac{\text{Inertia}}{\text{Viscosity}}$	Always
Mach number	$\mathrm{Ma} = \frac{U}{a}$	$\frac{\text{Flow speed}}{\text{Sound speed}}$	Compressible flow
Froude number	$\mathrm{Fr} = \frac{U^2}{gL}$	$\frac{\text{Inertia}}{\text{Gravity}}$	Free-surface flow
Weber number	$\mathrm{We} = \frac{\rho U^2 L}{\Upsilon}$	$\frac{\text{Inertia}}{\text{Surface tension}}$	Free-surface flow
Cavitation number (Euler number)	$\mathrm{Ca} = \frac{p - p_v}{\rho U^2}$	$\frac{\text{Pressure}}{\text{Inertia}}$	Cavitation
Prandtl number	$\mathrm{Pr} = \frac{\mu c_p}{k}$	$\frac{\text{Dissipation}}{\text{Conduction}}$	Heat convection
Eckert number	$\mathrm{Ec} = \frac{U^2}{c_p T_0}$	$\frac{\text{Kinetic energy}}{\text{Enthalpy}}$	Dissipation
Specific-heat ratio	$\gamma = \frac{c_p}{c_v}$	$\frac{\text{Enthalpy}}{\text{Internal energy}}$	Compressible flow

White (1986)

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Dimensionless Groups in Fluid Mechanics (II)

Parameter	Definition	Qualitative ratio of effects	Importance
Strouhal number	$\mathrm{St} = \frac{\omega L}{U}$	$\frac{\text{Oscillation}}{\text{Mean speed}}$	Oscillating flow
Roughness ratio	$\frac{\epsilon}{L}$	$\frac{\text{Wall roughness}}{\text{Body length}}$	Turbulent, rough walls
Grashof number	$\mathrm{Gr} = \frac{\beta \Delta T g L^3 \rho^2}{\mu^2}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural convection
Temperature ratio	$\frac{T_w}{T_0}$	$\frac{\text{Wall temperature}}{\text{Stream temperature}}$	Heat transfer
Pressure coefficient	$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2}$	$\frac{\text{Static pressure}}{\text{Dynamic pressure}}$	Aerodynamics, hydrodynamics
Lift coefficient	$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics
Drag coefficient	$C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics

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Buckingham π-theorem

Process :

1. list down the all variables involved in the problem

(n = ?) $F(B_1, B_2, B_3, \ldots\ldots B_n) = C$ (Most critical and difficult process!)

2. choose the primary j variables, which cover the whole needed dimensions, but do not form a π-product

(j = ?) $B_1, B_2, B_3, \ldots, B_j$ (In fluid mechanics M, L, T, Θ!)

3.Add each additional variable to your j variables to form a power product, then find the exponents which make the n-j product dimensionless ,

$$\Pi_1 = B_1^{a1,1} B_2^{a1,2} \ldots\ldots B_j^{a1,j} B_{j+1}$$
$$\Pi_2 = B_1^{a2,1} B_2^{a2,2} \ldots\ldots B_j^{a2,j} B_{j+2}$$
$$\Pi_{n-j} = B_1^{an-j,1} B_2^{an-j,2} B_j^{an-j,j} B_n$$

4. we get finally $F_1(\Pi_1, \Pi_2, \ldots\ldots \Pi_{n-j}) = C_1$

(the reduction of variables : j)

Example of Buckingham π-theorem (I)

Consider the problem of pressure drop in steady pipe flow

(1) $F = F(\Delta p, V, L, D, \rho, \mu, e) = C$ ⇦ n = 7

(2) choose primary j variables: V, D, ρ ⇦ j = 3

(Which one is better? It's a matter of taste, custom and user's choice.)

(3) Find the n-j Π: ⇦ n-j = 7-3 =4

$$\Pi_1 = V^{a1,1} D^{a1,2} \rho^{a1,3} \Delta p$$
$$\Pi_2 = V^{a2,1} D^{a2,2} \rho^{a2,3} L$$
$$\Pi_3 = V^{a3,1} D^{a3,2} \rho^{a3,3} \mu$$
$$\Pi_4 = V^{a4,1} D^{a4,2} \rho^{a4,3} e$$

$$\Pi_1 = V^{a1,1} D^{a1,2} \rho^{a1,3} \Delta p$$

Example of Buckingham π-theorem (II)

$$\Pi_1 = M^0L^0T^0 = V^{a1,1}D^{a1,2}\rho^{a1,3}\,\Delta p$$

$$= (LT^{-1})^{a1,1}(L)^{a1,2}(ML^{-3})^{a1,3}\,(ML^{-1}T^{-2})$$

for M: $0 = a1,3 + 1$

L: $0 = a1,1 + a1,2 - 3 \times a1,3 - 1$

T: $0 = -a1,1 - 2$

so that $a1,1 = -2,\ a1,2 = 0,\ a1,3 = -1$

$$\Pi_1 = M^0L^0T^0 = V^{-2}D^0\rho^{-1}\,\Delta p = \Delta p/\rho V^2$$

Similarly, $\Pi_2 = L/D$, $\Pi_3 = \mu/\rho VD$ (= Re), $\Pi_4 = e/D$

⇨ $F_1(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = F_1(\Delta p/\rho V^2, L/D, \mu/\rho VD, e/D) = C_1$

or $\boxed{\Delta p/\rho V^2 = F2(L/D, \mu/\rho VD, e/D)}$

However, DA says nothing about the functional form of F1 or F2!!

Modelling and Similarity

$$\Pi_1 = F(\Pi_2, \Pi_3, \ldots, \Pi_k)$$

model $\Pi_{2m} = \Pi_{2p}$ prototype

$$\Pi_{3m} = \Pi_{3p}$$

$$\ldots$$

$$\Pi_{km} = \Pi_{kp}$$

⇨ $\Pi_{1m} = \Pi_{1p}$ ⇨ Complete Similarity

But in engineering, instead of complete similarity, We consider:

Geometric similarity (L-scale)

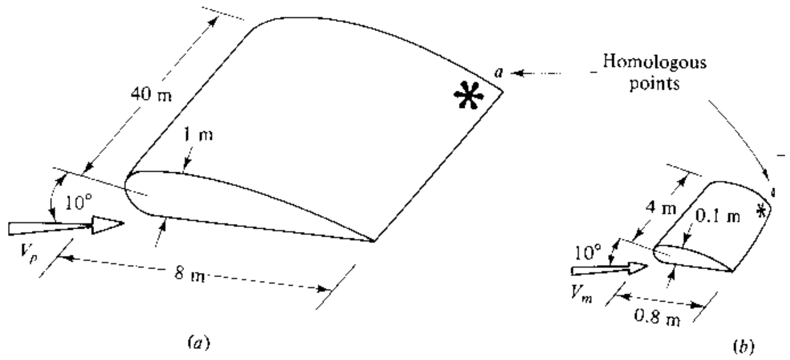
Kinematic similarity (L- & t-scale)

Dynamic similarity (L- & t- & m-scale)

(Thermal similarity)

Geometric Similarity

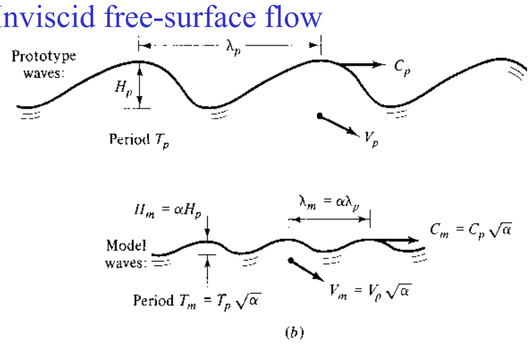
Geometric Similarity:
It requires all body dimensions in all three coordinates have the same linear scale ratio



Kinematic Similarity

Kinematic Similarity
It requires the model and prototype have the same length-scale ratio (geometric similarity) and also the same time scale ratio

Inviscid free-surface flow



$\lambda_m/\lambda_p = \alpha = H_m/H_p$

since $Fr_m = Fr_p$

$V_m^2/g\lambda_m = V_p^2/g\lambda_p$

⇨ $V_m/V_p = (\lambda_m/\lambda_p)^{1/2} = \alpha^{1/2} = \beta$

α & β are dependent!

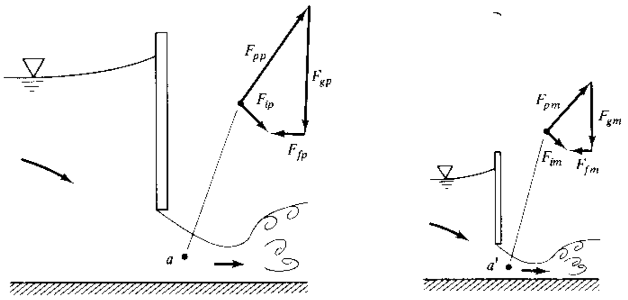
also $c_m/c_p = \alpha^{1/2}$

period: $T_m/T_p = \alpha^{1/2}$

Dynamic Similarity

•Dynamic Similarity

It requires the same ***length-scale ratio***, ***time scale ratio*** and also ***force-scale ratio***



$$F_{p(ressure)} + F_{g(ravity)} + F_{f(riction)} = F_{i(nertia)}$$

$Fr_p = Fr_m$ $\quad$ $Re_p = Re_m$

Discrepancies in model-prototype similarity (I)

- The perfect dynamic similarity is more of a dream than a reality because true equivalence of characteristic dimensionless parameters can be achieved only by dramatic changes in fluid properties , whereas in most model testing is simply done with air and water, the cheapest fluids available.
- Example: consider hydraulic model testing with free surface:

  Geometric similarity:

  $$\lambda_m / \lambda_p = \alpha = 1/10$$

  Froude number (Fr):

  $$V_m^2/g\lambda_m = V_p^2/g\lambda_p \Rightarrow V_m/V_p = (\lambda_m/\lambda_p)^{1/2} = \alpha^{1/2} = 0.32$$

  Reynolds number (Re):

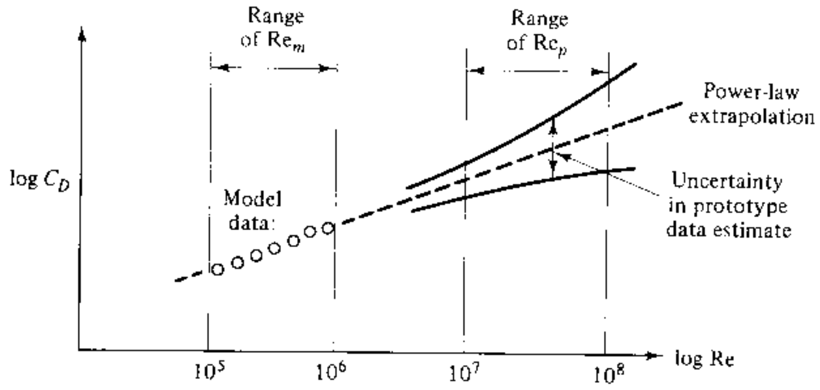
  $$V_m\lambda_m/\nu_m = V_p\lambda_p/\nu_p \Rightarrow \nu_m/\nu_p = (\nu_m/\lambda_p)^{3/2} = \alpha^{3/2} = 0.032$$

$\nu_p = 1\ mm^2/s$ for water ($\nu_{mercury} = 0.12\ mm^2/s$)

⇨Re-similarity is unavoidably violated!!

Discrepancies in model-prototype similarity (II)

In practice, water is used for both model and prototype!



Terminology for Error Analysis

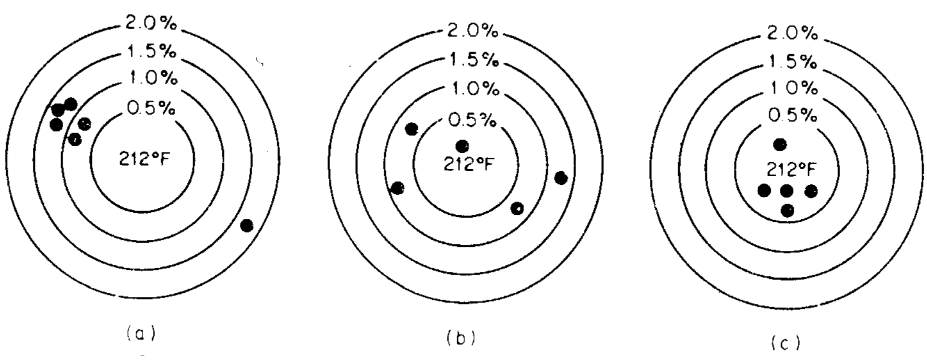
Terminology

- ***Error***: deviation of the reading from a known input
- ***Accuracy***: Error, usually expressed as a percentage of full-scale reading.for industrial or laboratory instruments.
- ***Uncertainty***: range within the error is likely to fall with specified confidence limits (or fiducial limit).
- ***Precision*** (Repeatability):
  - ***reproducibility*** of the reading for a given input.
  - an instrument can be precise, but ***not*** calibrated or misused
  - accuracy of an instrument cannot be better than its precision.
- ***Traceability***: The ability to trace the accuracy of a standard back to its ultimate source in the fundamental standards (e.g., NIST)
- ***Sensitivity*** : ratio of instrument output to input.

Precision(repeatability) & Accuracy

Precision, Bias error & Accuracy (I)

Calibration Data for Three Temperature-Measuring Devices

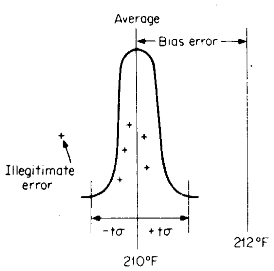


†Possible illegitimate error or an outlier.

(From Miller, 1983)

Modern Measuring Techniques of Thermo-fluids Mechanics *By An-Bang Wang* *National Taiwan University Institute of Applied Mechanics*

Precision, Bias error & Accuracy (II)



(From Miller, 1983)

With the outlier omitted, the average of the reading

$\bar{I} = \frac{\sum I_i}{n}$ **=(210.1+210.0+209.8+210.2+209.9)/5=210.0**

the standard deviation is:

$\sigma = [\frac{\sum (I_i - \bar{I})/\bar{I} \times 100\%}{n-1}]^{0.5}$

=[(0.0023+0+0.0090+0.0090+0.0023)/(5-1)]$^{0.5}$ =0.0753%

∴ the *precision* at the 95% confidence level is then:

$\sigma_p = t_{st}\,\sigma$, where t_{st} is two-tailed student's t-value, could be found from table (t_{st} = 2.776 for n=5).

∴ the precision is then σ_p=2.776×0.0753%=0.21%

The *direction bias error* is

$B = \frac{\bar{I} - I_t}{I_t} \times 100\%$ **=[(210-212)/(212)] × 100%=-0.94%**

∴ the *accuracy* is $A_{cc} = B \pm \sqrt{(1+\frac{1}{n})}\sigma_p$ **=-0.7%~-1.2%**

Error Analysis (I)

- Almost all fluid flow measurements are indirect. (e.g. Pitot-tube for pressure, hot-wire for velocity ...)
- In experimental work, errors of two different types can occur :
  - Systematic Errors
    - poorly adjusted instruments
    - improper calibration
    - false instrument specification
    - incorrect or biased statistical estimators
  - Statistical or random errors
    - improper reading from a scale
    - statistical variance of the measured quantity.
- Some errors can be corrected or controlled, others (uncertainty) cannot. All must be estimated !

Error Analysis (II)

- The influence of the errors on the end result can be determined using the error propagation rule from Gauss , i.e. ,
  A quantity $\Phi = f(P_1, P_2 \ldots)$ dependent on individual parameters P_i,and with individual uncertaintiesδ_{pi}, the uncertainty $\delta\Phi$ could be written as

  $$\delta\phi = \{(\frac{\partial\phi}{\partial p_1}\delta p_1)^2 + (\frac{\partial\phi}{\partial p_2}\delta p_2)^2 + \ldots\}^{1/2}$$

  In applying this relation, Gaussian error distributions are assumed.

Error analysis (III)

The Prandtl tube is used to measure flow velocity:

$$U = \sqrt{\frac{2\Delta P R T_1 g}{P_1}}$$

$\Delta P = P_1 - P_2$
Where

R:gas constant, P_1:stagnation pressure
T_1:gas temperature, g:acceleration due to gravity

The uncertainty in U becomes

$$\delta u = \{(\frac{1}{4}\frac{2RT_1 g}{(\Delta p) p_1})\delta\Delta p^2 + (\frac{1}{4}\frac{2\Delta p R T_1 g}{p_1^{\,3}})\delta p_1^{\,2} + (\frac{1}{4}\frac{2\Delta p R g}{p_1 T_1})\delta T_1^{\,2}\}^{1/2}$$

The normalized uncertainty is then

$$\frac{\delta u}{u} = \{(\frac{1}{2}\frac{\delta(\Delta p)}{\Delta p})^2 + (\frac{1}{2}\frac{\delta(p_1)}{p_1})^2 + (\frac{1}{2}\frac{\delta(T_1)}{T_1})^2\}^{1/2}$$

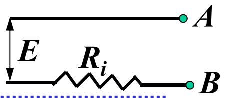
Clearly, a 0.1% accuracy in reading the temperature is not worthwhile, if for instant the ΔP can only be determined with 5% accuracy !

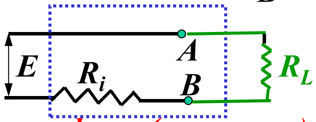
Some notes for electronics

- Impedance Matching

It is continually necessary to connect various electronic instruments in different combinations

Every instrument has an internal resistance R_i, in series with an externally connected load resistance R_L ,





The voltage

$$E_{AB} = E\frac{R_L}{R_L + R_i}$$

- If E_{AB} is to be measured, R_L should be chosen ***large (e.g. scopes)***.
- If power is to be transmitted

  $P = E_{AB}^2 / R_L$ or $P = E^2 R_L / (R_L + R_i)^2$

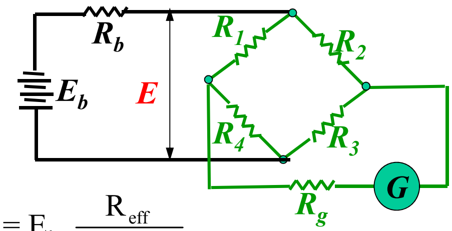
For a maximum transmission : $dP/dR_L = 0$ then $R_L = R_i$

i.e. resistive part of impedance should be match.The inductive and capacitive terms must also be considered when dynamic response is important.

Wheatstone bridge

The Wheatstone bridge is a basic building block of many measuring instruments. It allows precise measurement of minute change in resistance, capacitance or inductance. It has many applications in

- hot-wire anemometry
- strain gauges
- Inductive pressure transducers or condenser microphones



- effective bridge resistance:

$$R_{eff} = \frac{(R_1 + R_4)(R_2 + R_3)}{R_1 + R_2 + R_3 + R_4} \quad , \quad E = E_b \frac{R_{eff}}{R_{eff} + R_b}$$

- Voltage measured at galvanometer:

$$E_g = E\left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3}\right)$$

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