
Chapter 6

Temperature measurements

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*Modern Measuring Techniques of
Thermo-fluids Mechanics*

By An-Bang Wang

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Lecture Contents

- Introduction
- Temperature Sensor Classification
- The International Practical Temperature Scale
- Evolution of Noncontact Temperature sensors
- Thermal Radiation Theory
- Infrared Radiometry
- Temperature Measurement by Radiant Techniques
- Applications of Infrared Sensing
- Practical Operation

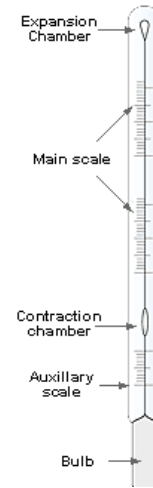
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Introduction

- One of the differences between temperature and other physical concepts, such as mass or length, is that it is subjective: different people will have different perceptions of what is hot and what is cold. To make objective measurements, we must use a thermometer in which some physical property of a substance changes with temperature in a reliable and reproducible way.(NPL)
- All material which is sensitive to temperature could be used as thermometer.



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The Early History of Temperature Sensing

Year	Events	
1592	The first instrument to measure temperature – The barothermoscope	Galilei, Italy
1611	Temperature scale was added (but uncalibrated).	Galen
1613	Using a thermoscope to record "degrees of heat" of winter snow, summer heat...	Sagredo
1624	"Thermometer" appeared in the literature.	Leurechon
1654	The first sealed thermometer filled with spirits of alcohol – The Florentian Thermometer that sensing temperature independent of pressure.	Ferdinand II, Italy
1661	The Florentian thermometer was exported to Rome, Paris, England.	
1663	Attempted to calibrate and standardize thermometers – using ice point.	Hooke
1694	Suggested ice point and boiling point as two fixed points and scale 12.	Renaldini, Italy
1701	Defined two fixed points: ice point and armpit temperature (labeled as also 12).	Newton, British
1708	Modified Romer (a Danish astronomer) scale to a Fahrenheit scale, substitute mercury for spirits, used a mixture of sea salt, ice, and water to produce the zero point. Ice point = 32, boiling point = 212.	Fahrenheit, Netherland
1742	Invented a scale with 0 at the steam point and 100 at the ice point.	Celsius, Sweden
1743	Inverted Celsius scale and named "centigrade" to indicate a scale divided into 100 parts.	Christin, France
1821	Discover the existence of thermoelectric current	Seeback, Estonia
1848	Define a thermodynamic (absolute) temperature scale with ideal gas (H ₂).	Kelvin, British

Contact measurement

(from Lee (2001) with some modifications)

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What is Temperature?

Qualitative Definitions:

1. The degree of hotness or coldness of a body.
2. All bodies have the same temperature if they are in thermal equilibrium.
3. The level of thermal energy. (cf. Electrical energy; Mechanical potential energy.)

Units of temperature (one of System International units, SI) :

	Unit	Symbol	Quantity measured
Dimensional	1. Meter	m	Length
	2. Kilogram	kg	Mass
	3. Second	s	Time
	4. Ampere	A	Electric current
	5. Kelvin	K	Temperature
	6. Mole	mol	Amount of substance
	7. Candela	cd	Luminous intensity
Dimensionless	8. Radian	rad	Plane angle
	9. Steradian	sr	Solid angle

(from Lee (2001))

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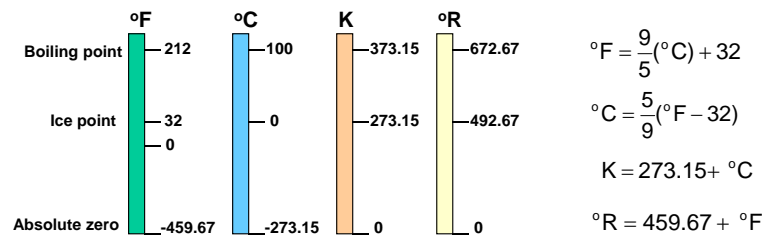
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How to Quantify the temperature?

Four common temperature scales:

Fahrenheit(1708), Celsius(1740), Kelvin(1848), Rankine(1730)



Fundamental temperature scales:

The relationship of measured variable to temperature depends only on fundamental physical constants, not on arbitrary calibrating constants.

Examples:

1. Thermodynamic, $PV = PV(T, N_A, k)$
2. Thermal radiance, $W = W(T, k, c, h)$

(from Lee (2001))

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Temperature spectrum



- 200,000,000 °C : The Joint European Torus (JET) nuclear fusion project
- 15,000,000 °C : Temperature of the centre of the Sun
- 6000 °C : Temperature of the surface of the Sun
- 1,200 ~ 1,500 °C : Molten glass / steel
- 1064 °C : Melting point of gold
- 100 °C : Boiling point (at 1 atm-pressure)
- 0 °C : Freezing point of pure water
- 89.2 °C : All time coldest point on earth
- 196 °C : Cryogenic storage in liquid nitrogen
- 270 °C : Cosmic background radiation



(<http://www.npl.co.uk/npl/publications/temperature/spectrum.html>)

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Temperature Sensor Classification

Sensor	Principle							
	Thermo-electric	Electrical Resistance	Carrier Mobility	Thermal Radiation	Electrical Capacitance	Thermal Expansion	Resonant Frequency	Others
Thermocouple								
Thermopile								
RTD(PTC, NTC)								
Thermistor								
P-N junction								
Optical pyrometer								
Pyro-electric								
Quantum								
Spectroradiometer								
Cooling IR imager								
Uncooled IR imager								
Gas								
Liquid								
Bi-metal								
Quartz								
Liquid Crystal								1
Others								2,3,4,5

[Remark]: 1.Reflectance, 2.Index of Refraction, 3.Ultrasonic, 4.Microwave, 5.Acoustic

(from Lee (2001))

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Temperature Sensor Classification

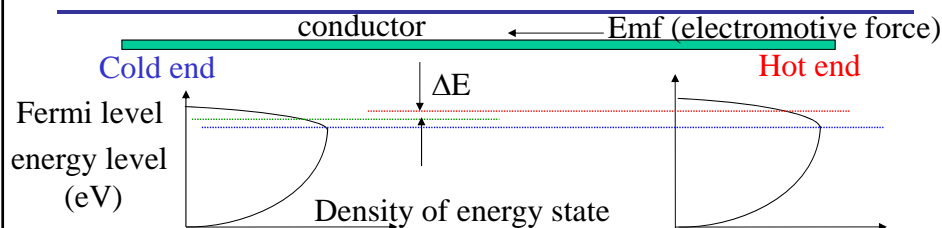
Transducer or probe	Temperature sensitive parameter	Contact method	Remarks
Resistor	Electrical resistance or voltage at constant current	Direct contact	Usually calibrated against a thermocouple. However Pt RTDs are one of the most accurate temperature sensors available.
Thermocouple	Open circuit voltage	Direct contact	Useful as a "point" sensor.
Diode or transistor	Voltage, usually with constant forward bias current	Direct contact	Usually employed to measure an active device or IC temperature.
Infrared or radiation	Detector voltage	Line-of-site or optical contact	Can yield either a point temperature or a thermal map or image. Not strictly quantitative unless sample emittance is known at the image points.
Fluorescent detector	Detector voltage	Direct contact (proximity)	Approximate point detector, contact resistance a problem.
Liquid crystal	Color	Direct contact	Yields a temperature map, semiquantitative unless a detailed calibration is performed to quantify color vs. temperature relation.

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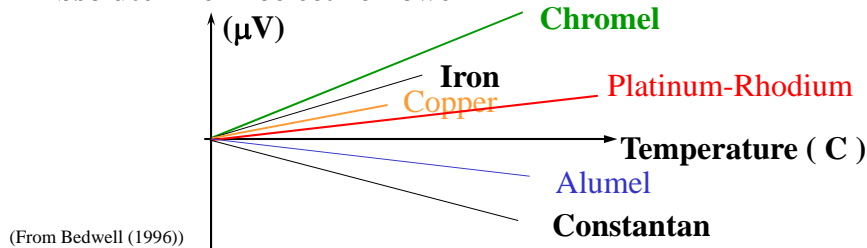
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Thermocouple Sensor (I)



Absolute Thermoelectric Power



(From Bedwell (1996))

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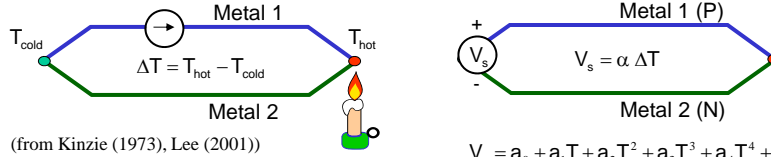
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Thermocouple Sensor (II)

□ Thermoelectric Effect (Thomas Seebeck, Germany, 1821):

Two wires composed of dissimilar metals are connected at both ends, to make a complete electrical circuit, an electrical current will flow in the circuit if one of the ends is heated.



(from Kinzie (1973), Lee (2001))

$$V_s = a_0 + a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4 + \dots$$

Letter	P wire	N wire	Range	Accuracy	Output($\mu V/^\circ C$)	Stability	Cost
T	Cu	Ni ₄₅ Cu ₅₅	-160~400 $^\circ C$	0.5 $^\circ C$	46	?	low
J	Fe	Ni ₄₅ Cu ₅₅	0~760 $^\circ C$	0.1 $^\circ C$	46	?	low
E	Ni ₉₀ Cr ₁₀	Ni ₄₅ Cu ₅₅	-100~1000 $^\circ C$	0.5 $^\circ C$	68	Low-mid	low
K	Ni ₉₀ Cr ₁₀	Ni ₉₅ Mn ₂ Al ₂ Si ₁	0~1370 $^\circ C$	0.7 $^\circ C$	42	low	low
N	Nicrosil	Nisil	0~1300 $^\circ C$		38	mid-high	low
S	Pt ₉₀ Rh ₁₀	Pt	0~1750 $^\circ C$	1.0 $^\circ C$			
R	Pt ₈₇ Rh ₁₃	Pt	0~1400 $^\circ C$	0.5 $^\circ C$	10	high	high
B	Pt ₇₀ Rh ₃₀	Pt ₉₄ Rh ₆	100~1750 $^\circ C$		5	high	high

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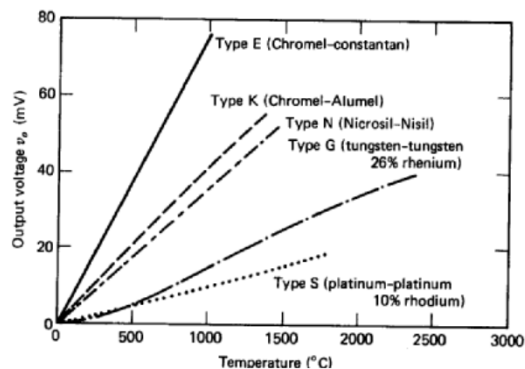
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Thermocouple Sensor (III)

□ Thermocouples rank only behind the clinical thermometer for popularity for temperature measurement.

□ Advantages:

- smaller and cheaper than resistance devices (allow for high number of individual measurements).
- simpler to condition than oscillating sensors, e.g., quartz crystals.
- wider operating range than thermistors or liquid crystals.

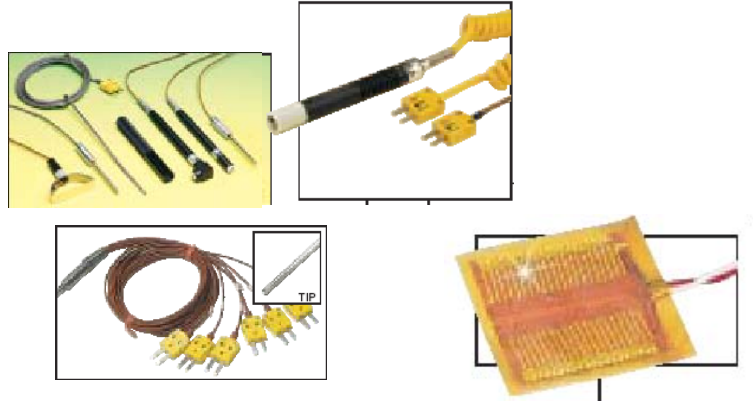


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Thermocouple Sensor (IV)



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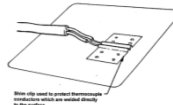
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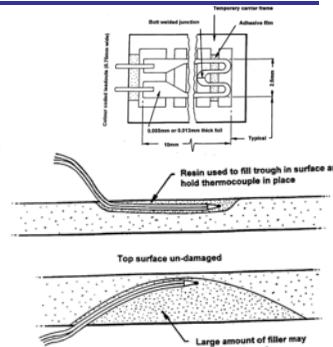
Thermocouple installation

Installation methods:

- Bonding the substrate mounted TC by adhesive or braze.
- direct welding
- Being recessed (cable and junction) under the surface.



Welds are used to protect thermocouple junctions which are welded directly to the surface.



Some restrictions:

- the adhesives and substrate materials are limited to around 300°C.
- The TC earths to the surface might cause problems
- The accuracy of TC bead position is the prime concern and the filled resin should have similar conductivity.

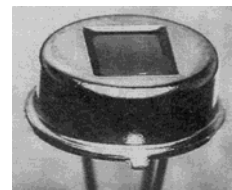
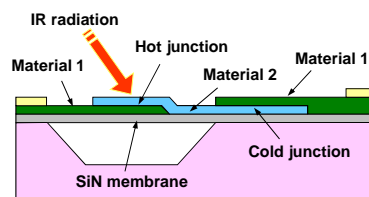
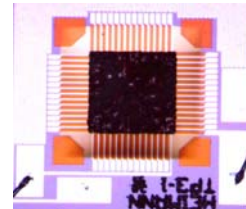
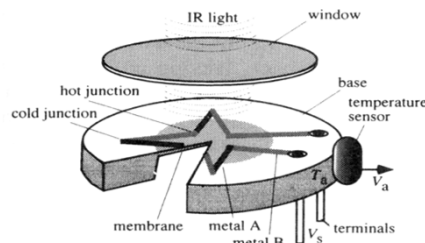
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Thermopile Sensor

□ **Thermopile (Melloni, 1833):** A thermopile is serially connected thermocouples.



(from Lee (2001))

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RTD

□ **Resistance Temperature Detector (RTD)**

For general metal or alloy (Pt, Ni, Balco (Ni-Cu alloy)),

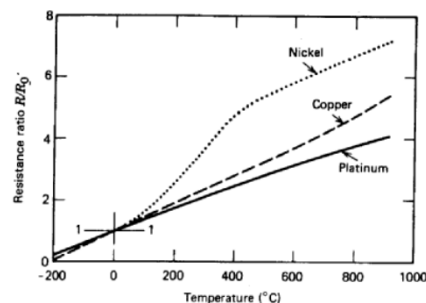
PTCR : Positive Temperature Coefficient of Resistance

$$R(T) = R_0[1 + \alpha_0(T - T_0) + \alpha_1(T - T_0)^2 + \dots]$$

$$\alpha = \left. \frac{1}{R} \frac{dR}{dT} \right|_{T=T_0} \quad \text{good linearity for a limited temperature range}$$

R_0 : resistance at 0°C

α : 0.0035/K for Pt

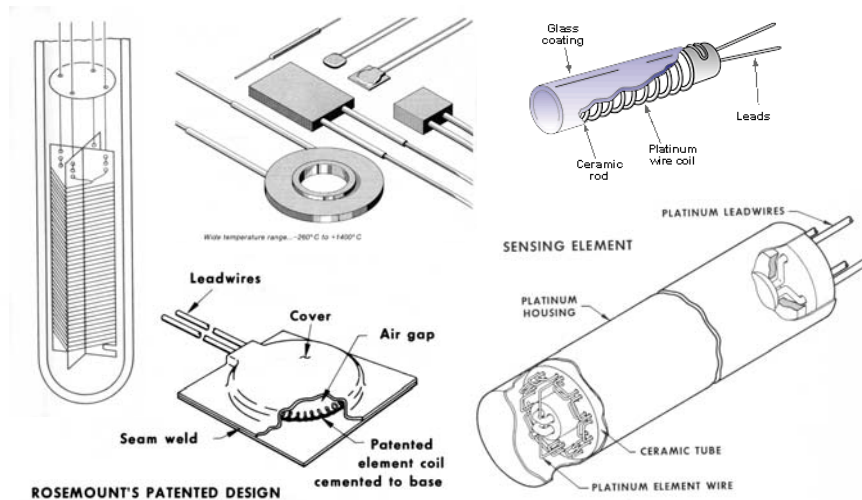


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RTD Sensors



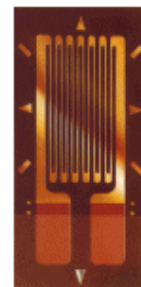
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RTD surface sensors

- Foil sensor (Polyimide or epoxy) : Easy to Install
- Compact, Flexible and Thin
- Fast Response Time
- High Output
- Capable of Measuring Temperatures from -200° to $+370^{\circ}$ C
- Excellent Accuracy and Repeatability
- Metal Foils: Nickel, Balco, Platinum



(Vishay Measurements Group)

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Thermistor

□ Thermal Sensitive Resistor (Thermistor)

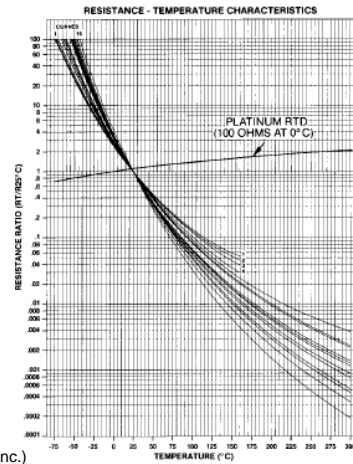
For **Semiconducting materials**,
 NTCR/PTCR : **Negative/Positive**
 Temperature Coefficient of Resistance

$$R(T) = R_0 e^{B\left(\frac{1}{T} - \frac{1}{T_0}\right)} \quad (T \text{ in } K)$$

$$\text{Sensitivity: } \alpha = \left. \frac{1}{R} \frac{dR}{dT} \right|_{T=T_0} \approx -\frac{\beta}{T^2}$$

In general, higher sensitivity,
 lower cost, smaller size, faster
 response, in comparison with
 RTD

Measuring range: -200 ~ 1000°C



(Thermometrics Inc.)

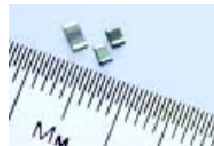
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Thermistor

NTC



PTC

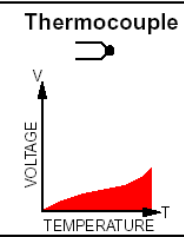
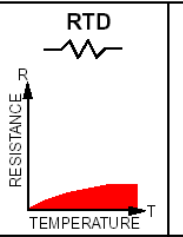
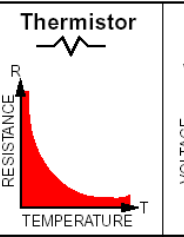
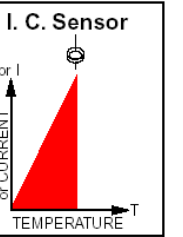


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Analysis of common temperature sensors

	Thermocouple 	RTD 	Thermistor 	I. C. Sensor 
Advantages	<input type="checkbox"/> Self-powered <input type="checkbox"/> Simple <input type="checkbox"/> Rugged <input type="checkbox"/> Inexpensive <input type="checkbox"/> Wide variety <input type="checkbox"/> Wide temperature range	<input type="checkbox"/> Most stable <input type="checkbox"/> Most accurate <input type="checkbox"/> More linear than thermocouple	<input type="checkbox"/> High output <input type="checkbox"/> Fast <input type="checkbox"/> Two-wire ohms measurement	<input type="checkbox"/> Most linear <input type="checkbox"/> Highest output <input type="checkbox"/> Inexpensive
Disadvantages	<input type="checkbox"/> Non-linear <input type="checkbox"/> Low voltage <input type="checkbox"/> Reference required <input type="checkbox"/> Least stable <input type="checkbox"/> Least sensitive	<input type="checkbox"/> Expensive <input type="checkbox"/> Current source required <input type="checkbox"/> Small ΔR <input type="checkbox"/> Low absolute resistance <input type="checkbox"/> Self-heating	<input type="checkbox"/> Non-linear <input type="checkbox"/> Limited temperature range <input type="checkbox"/> Fragile <input type="checkbox"/> Current source required <input type="checkbox"/> Self-heating	<input type="checkbox"/> $T < 200^{\circ}\text{C}$ <input type="checkbox"/> Power supply required <input type="checkbox"/> Slow <input type="checkbox"/> Self-heating <input type="checkbox"/> Limited configurations

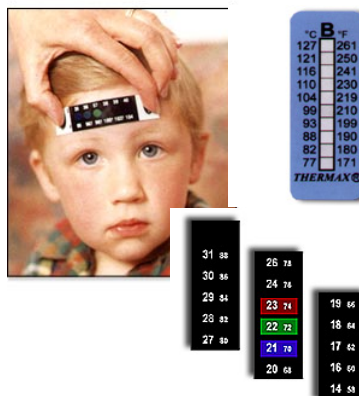
(Omega)

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Liquid crystal Thermometer

Forehead Thermometer
Fish box thermometer

- Safe
- convenient
- accurate
- Time constant: some seconds
- Mirror or observer is needed.



From: Taiwah Charts Corp.

Remote temperature sensing

- Too far to measure
- Too hot/dangerous
- Too small to access
- Moving body

Advantages of IR-sensing

- Fast response
- High accuracy available (1~3%)
- Wide measuring range (~3000°C)



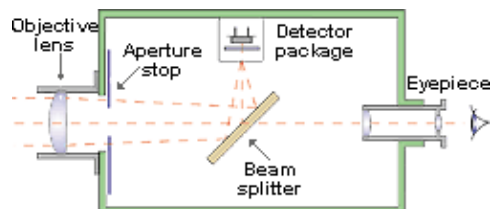
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Radiation (Pyrometer)

Radiation thermometers, or pyrometers, make use of the fact that all objects emit thermal radiation, as seen when looking at the bars of an electric fire or a light bulb. The amount of radiation emitted can be measured and related to temperature using the Planck law of radiation. Temperatures can be measured remotely using this technique, with the sensor situated some distance away from the object. Hence it is useful for objects that are very hot, moving or in hazardous environments.

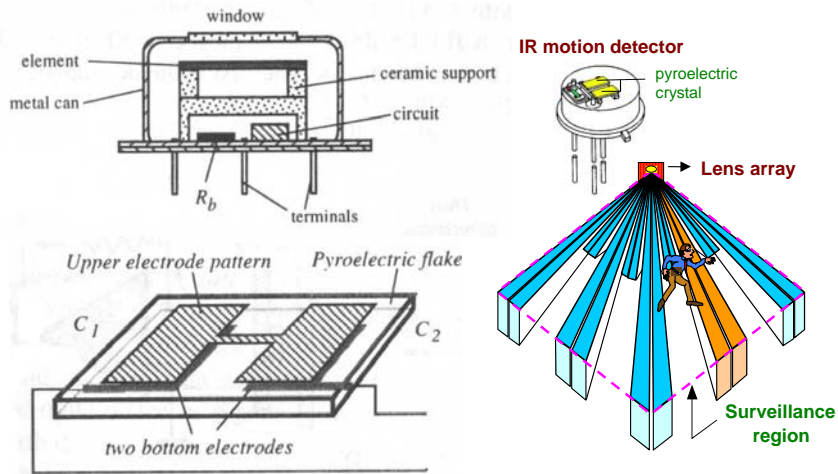


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Pyro-electric Sensor



(from Lee (2001))

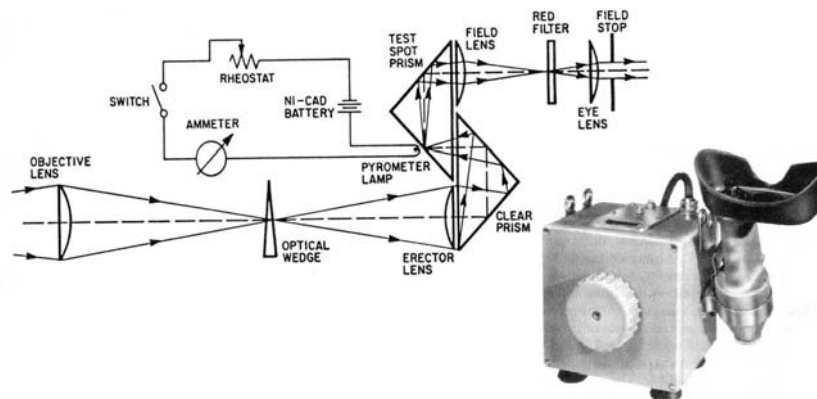
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Optical Pyrometer

□ The disappearing filament optical pyrometer



(from Lee (2001))

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Spectroradiometer/Radiometer



Blackbody radiation spectrum fitting

Spectroradiometer



Radiometer

(from Lee (2001))

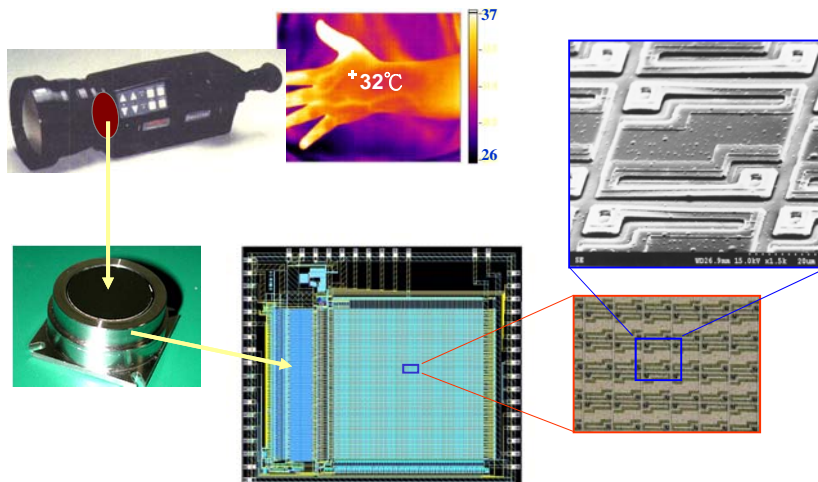
Items	Specifications
Spectral range	0.2 to 25 microns
FOV	0.3 mrad - 100 mrad
Focusing range	2.5m to infinity
Scan rates	0.015 to 30 scans/sec
Chopping Frequency	100 to 1800 Hz.

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Infrared Imager



(from Lee (2001))

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Temperature Scales

□ Requirements for establishing a temperature scale:

1. Set the fixed points and given the temperature value.
2. Choosing an appropriate instrument to interpolate scale.
3. Determine the relationship between the measurement variable and the temperature.

□ The International Practical Temperature Scale:

The International Committee of Weights and Measures(1927,.. 1960,.. 1968,.. 1990)
Fixed points: 6(ITS-27), 13(IPTS-68), 17(ITS-90)

Defining fixed points of the ITS-27 °C

Number	°C	Substance	State	Range	°C	Interpolating Instrument
1	-182.97	O ₂	Boiling Point	I	-182.97 ~ 0	Platinum resistance
2	0.000	H ₂ O	Freezing Point	II	0 ~ 660	Platinum resistance
3	100.000	H ₂ O	Boiling Point	III	660 ~ 1063.0	S-type thermocouple
4	444.60	S	Boiling Point	IV	>1063.0	Optical pyrometer
5	960.5	Ag	Freezing Point			
6	1063.0	Au	Freezing Point			

(from Lee (2001))

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ITS-90(1)

□ The International Temperature Scale of 1990 (ITS-90)

[Ref]:<http://www.omega.com/techref/inttemp.html>

1. Units of Temperature

The unit of the fundamental physical quantity known as thermodynamic temperature, symbol T, is the kelvin symbol K, defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

Because of the way earlier temperature scales were defined, it remains common practice to express a temperature in terms of its difference from 273.15 K, the ice point. A thermodynamic temperature, T, expressed in this way is known as a Celsius temperature, symbol t, defined by:

$$t / ^\circ\text{C} = T / \text{K} - 273.15$$

The unit of Celsius temperature is the degree Celsius, symbol °C, which is by definition equal in magnitude to the kelvin. A difference of temperature may be expressed in kelvins or degrees Celsius.

The International Temperature Scale of 1990 (ITS-90) defines both International Kelvin Temperatures, symbol T₉₀, and International Celsius Temperatures, symbol t₉₀. The relation between T₉₀ and t₉₀ is the same as that between T and t, i.e.:

$$t_{90} / ^\circ\text{C} = T_{90} / \text{K} - 273.15$$

The unit of the physical quantity T₉₀ is the kelvin, symbol K, and the unit of the physical quantity t₉₀ is the degree Celsius, symbol °C, as is the case for the thermodynamic temperature T and the Celsius temperature t.

(from Lee (2001))

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ITS-90(2)

2. Principles of the International Temperature Scale of 1990 (ITS-90)

The ITS-90 extends upwards from 0.65 K to the highest temperature practicably measurable in terms of the Planck radiation law using monochromatic radiation. The ITS-90 comprises a number of ranges and sub-ranges throughout each of which temperatures T_{90} are defined. Several of these ranges or sub-ranges overlap, and where such overlapping occurs, differing definitions of T_{90} exist: these differing definitions have equal status.

3. Definition of the International Temperature Scale of 1990

Between 0.65 K and 5.0 K T_{90} is defined in terms of the vapour-pressure temperature relations 3He and 4He.

Between 3.0 K and the triple point of neon (24.5561 K) T_{90} is defined by means of a helium gas thermometer calibrated at three experimentally realizable temperatures having assigned numerical values (defining fixed points) and using specified interpolation procedures.

Between the triple point of equilibrium hydrogen (13.8033 K) and the freezing point of silver (961.78°C) T_{90} is defined by means of platinum resistance thermometers calibrated at specified sets of defining fixed points and using specified interpolation procedures.

Above the freezing point of silver (961.78°C) T_{90} is defined in terms of a defining fixed point and the Planck radiation law.

(from Lee (2001))

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Defining Fixed Points of the ITS-90

Number	Temperature		Substance	State
	T_{90}/K	$t_{90}/^{\circ}C$		
1	3 to 5	-270.15 to -268.15	He	V
2	13.8033	-259.3467	e-H ₂	T
3	~17	~-256.15	e-H ₂ (or He)	V (or G)
4	~20.3	~-252.85	e-H ₂ (or He)	V (or G)
5	24.5561	-248.5939	Ne	T
6	54.3584	-218.7916	O ₂	T
7	83.8058	-189.3442	Ar	T
8	234.3156	-38.8344	Hg	T
9	273.16	0.01	H ₂ O	T
10	302.9146	29.7646	Ga	M
11	429.7485	156.5985	In	F
12	505.078	231.928	Sn	F
13	692.677	419.527	Zn	F
14	933.473	660.323	Al	F
15	1234.93	961.78	Ag	F
16	1337.33	1064.18	Au	F
17	1357.77	1084.62	Cu	F

V: vapor pressure point; T: triple point; G: Gas thermometer point;
M: melting point; F: Freezing point

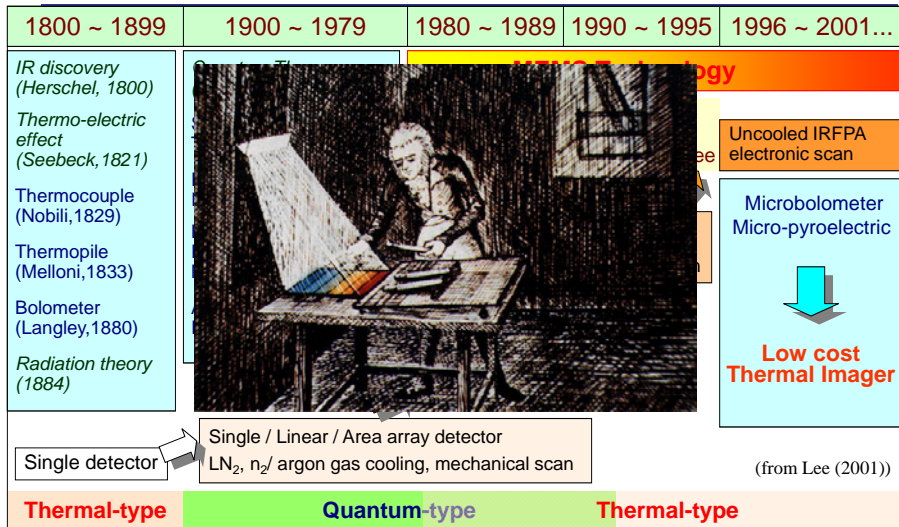
(from Lee (2001))

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Evolution of IR Sensor Technologies

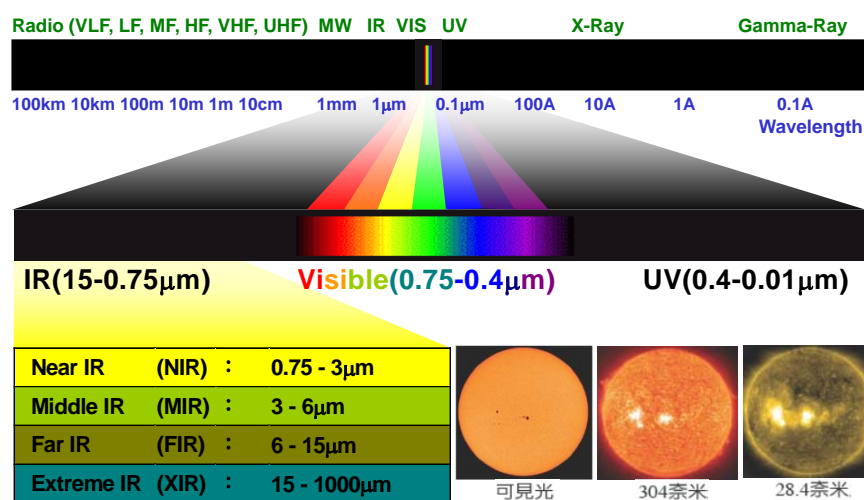


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The Electromagnetic Spectrum



(from Lee (2001) & K.L. Lin(2006))

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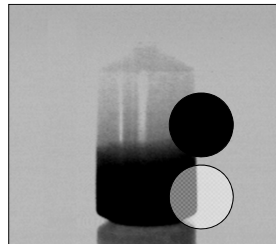
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Visible cf. Thermal Radiation (1)

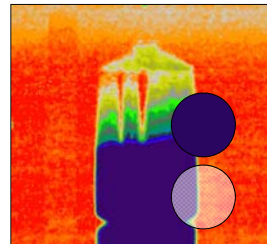
□ Target signature (Spatial Distribution)



Visible(0.4-0.75 μm)



FIR(8-12 μm)



FIR(8-12 μm)

- Active sensing : Visible, NIR
- Passive Sensing : Visible, NIR, MIR, FIR

(from Lee (2001))

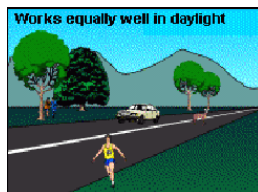
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Visible cf. Thermal Radiation(2)

□ Background signature (Spatial Distribution)



Ref. "Thermal imaging solutions", TI NIGHTSIGHT interactive explorer.

(from Lee (2001))

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Gas Thermometer

- Developed in the first half of 19th century.
- Least liable to uncertain variation and generally accepted as the standard for comparing all types of thermometers.
- Boyle-Mariotte law:

$$(PV)_t = K \quad \text{for ideal gases}$$

- Charles-Gay-Lussac Law:

$$\frac{1}{V_0} \left(\frac{V - V_0}{T - T_0} \right)_p = \alpha_{0p} \quad \text{or} \quad \frac{1}{P_0} \left(\frac{P - P_0}{T - T_0} \right)_V = \alpha_{0V}$$

- Clapeyron's equation of state:

$$PV = R_p \left(T - T_0 + \frac{1}{\alpha_{0p}} \right) \quad \text{or} \quad PV = R_v \left(T - T_0 + \frac{1}{\alpha_{0V}} \right)$$

- Regnault's

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