Chapter 5 Velocity Measurement

(by An-Bang Wang)

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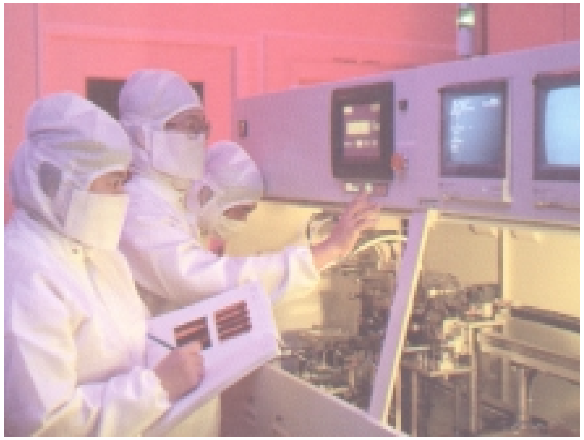
- Pressure-based probes
- Hot-wire anemometry
- Laser Doppler anemometry
- Particle image velocimetry

Velocity measurements in Industry



Traditional velocity Measurements

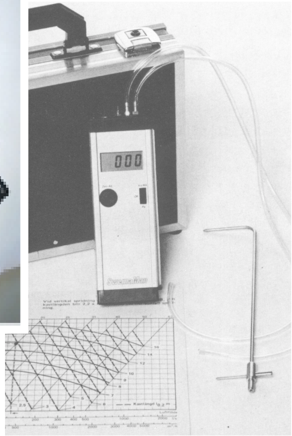
- **(flow rate ⇨) Average velocity**
  Q = U A

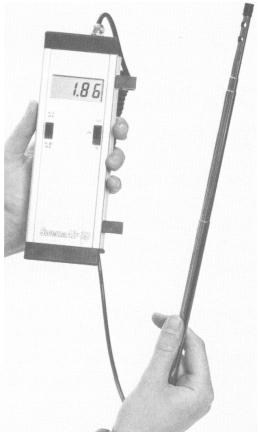
Bernoulli's Equation

$$\frac{p_0}{\rho_0} = \frac{p}{\rho} + \frac{V^2}{2}$$

- **Local velocity measurement:**

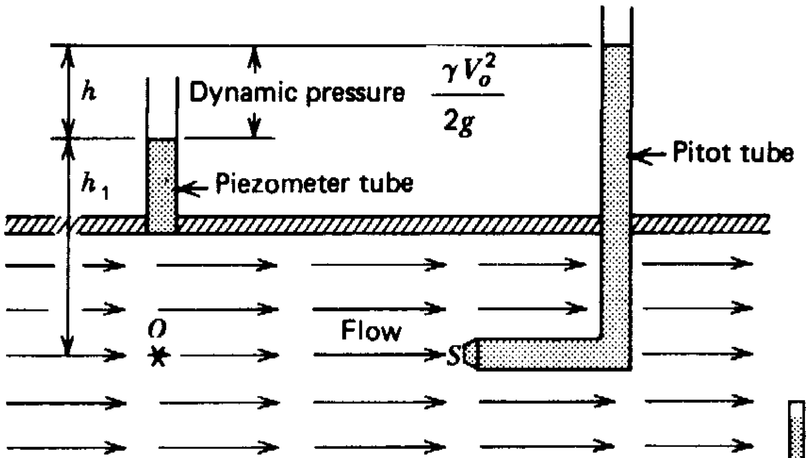
  - mechanical rotation*($)
  - Pitot-static tube ($$)
  - Hot-wire/-film*($$$)

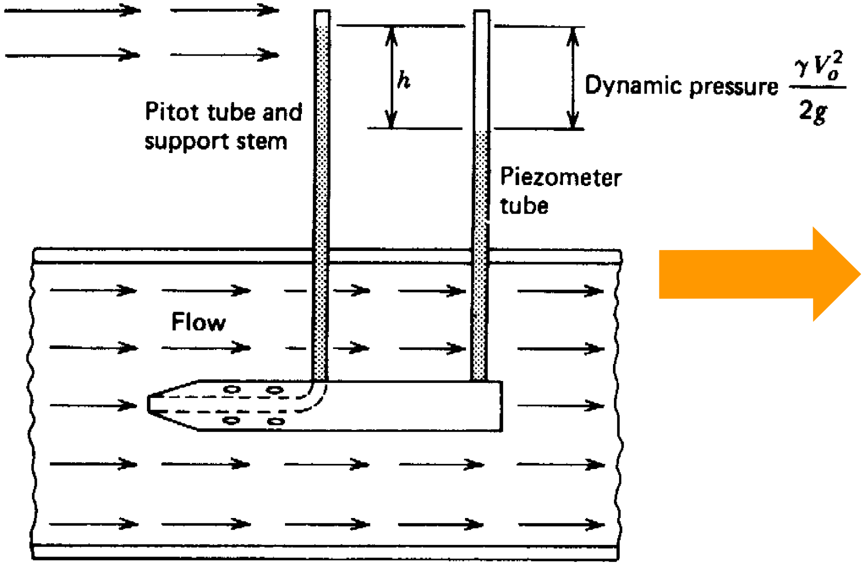


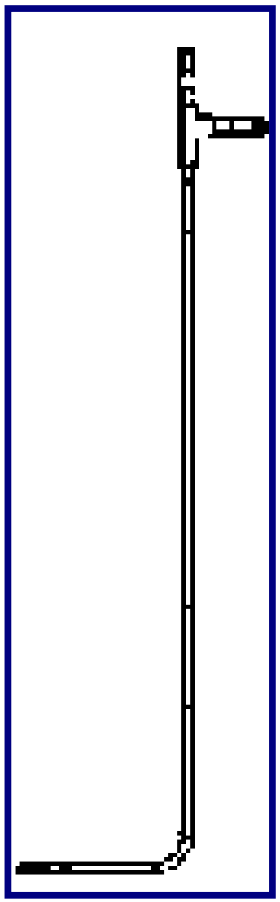


(* regular calibration needed)

Pitot Tube & Prandtl Tube



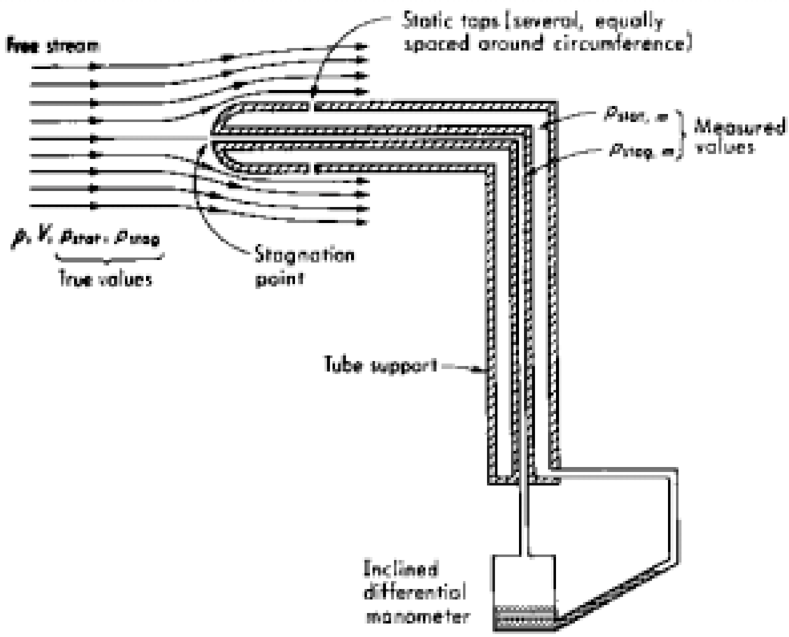


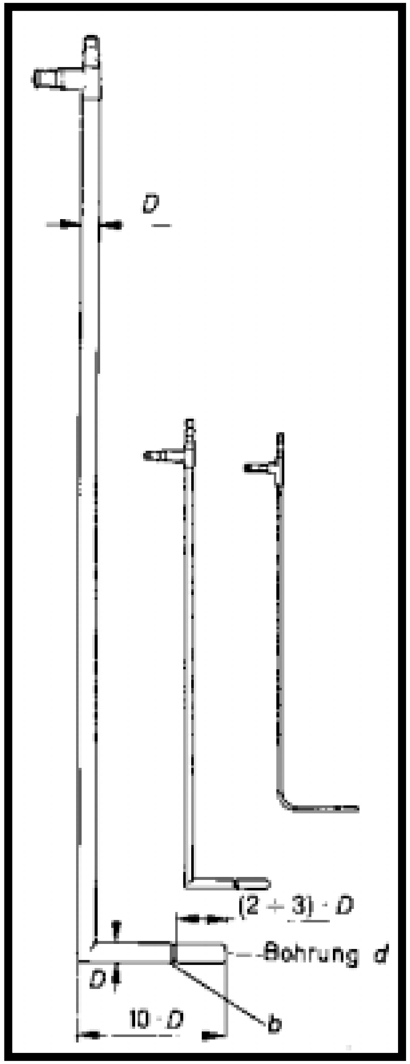


Dynamic pressure measurement

Measurement of dynamic pressure

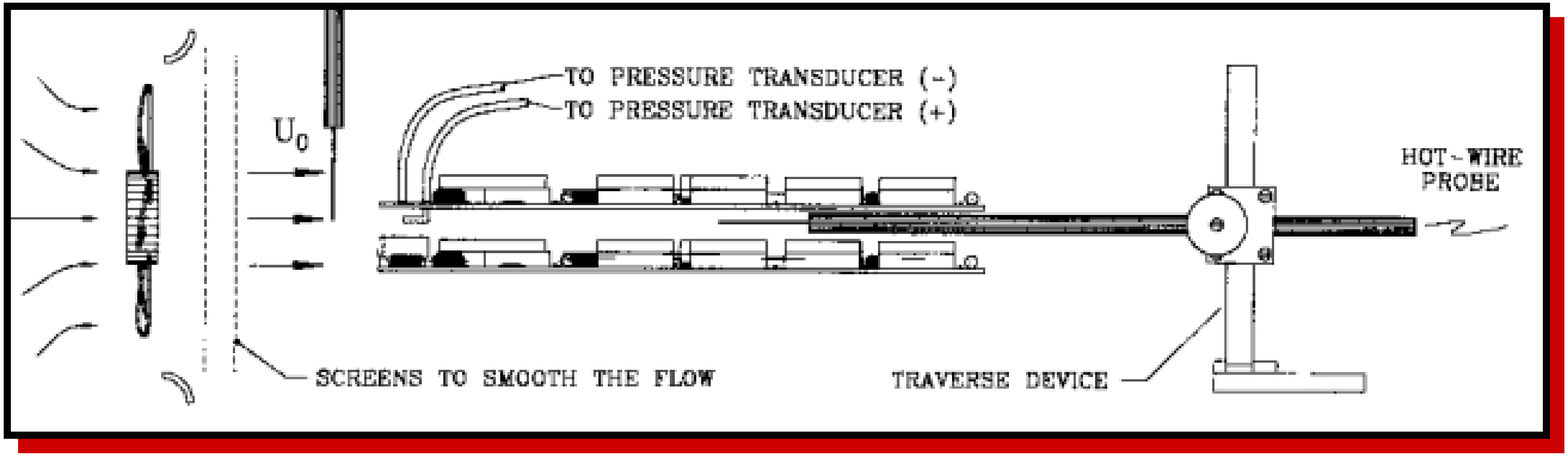
- Pitot-static tube (or Prandtl tube) is used to measure dynamic pressure and hence flow velocity.
- It should not be used at too low Reynolds numbers or too close to a wall.
- Advantages: robust, cheap
- **Limitation**: low dynamic response





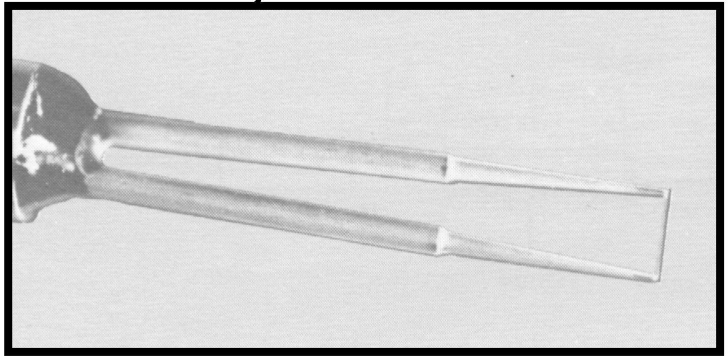
Introduction to Hot Wire/Film Anemometry

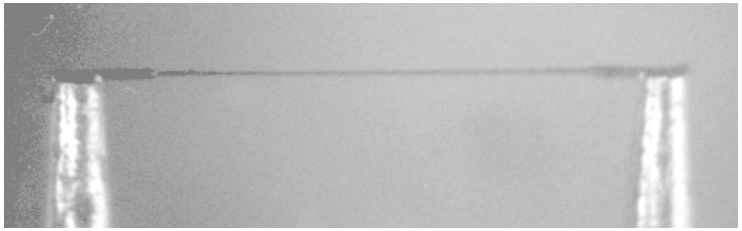
Velocity measurement by using hot wire/film anemometer



Thermal Anemometer

Hot wire and hot film are most commonly used sensor of thermal anemometers.





⦿**Advantages:**

- convenient usage

- fast response

⦿**Disadvantages of thermal anemometer**:

- intrusive
- calibration-required
- fragile
- blind to direction
- thermo-sensitive
- regular cleaning needed

Principle of thermal anemometers (I)

- Thermal anemometer is an indirect measuring technique (not the velocity but the heat loss from a thin, heated wire is measured and related to the flow velocity.)
- The heat loss of a hot wire (or hot film) is dependent on a number of factors:
  - relative velocity between sensor and fluid medium. (magnitude and direction)
  - temperature difference between sensor and medium.
  - material properties of sensor and medium.(e.g. thermal conductivity, film coefficient,…etc.)
  - dimensions of the sensor.
- If the last three factors are kept constant, a calibration can be given the relation between the heat loss and the flow velocity

Principle of thermal anemometers (II)

- The basic circuitry for hot-wire and hot-film anemometry is identical.
- The heated wire, whose resistance is dependent on the temperature.
- Temperature changes due to velocity fluctuations (⇨resistance changes) are detected by means of a bridge circuit.
- For the sensor by Joule heating:

heat loss

$$Q = IE = I^2 R = \frac{E^2}{R}$$

Constant current ⇨ ***R*** changes ⇨ wire temperature changes

constant temperature ⇨ ***R*** remains constant ⇨ ***E*** changes

CCA & CTA

Basically two methods of operation are possible.

- CCA : (Constant current Anemometer)

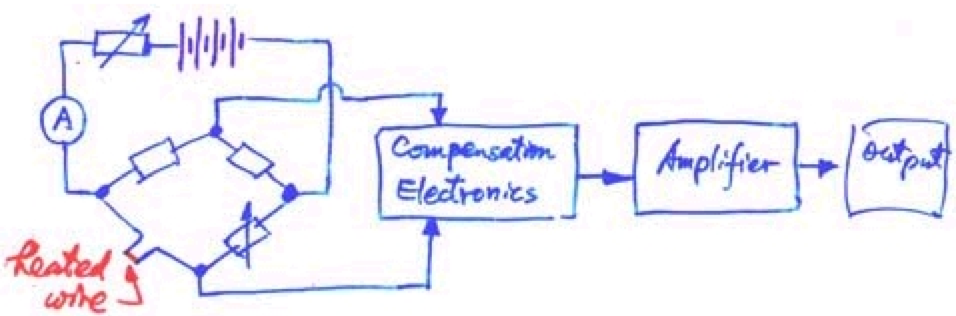
  The heating current ***I*** is held constant.

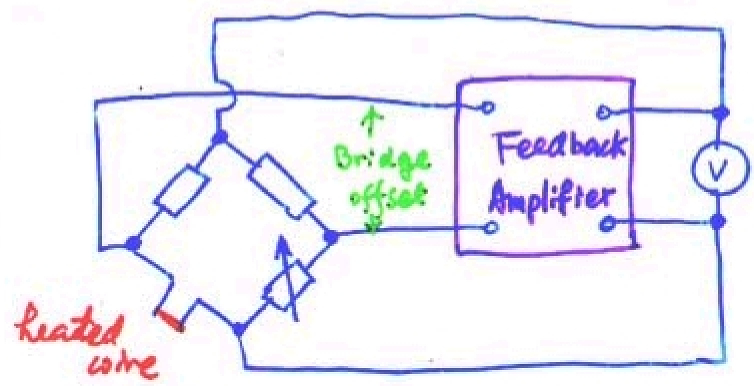
  ⇨ ***R*** is then a measure for heat loss ***Q***.

- CTA: (Constant temperature anemometer)

  The resistance R, and hence the temperature of the sensor is held constant.

  ⇨ The bridge voltage ***E*** is then a measure for Q.





CCA

Advantages :

- good signal-to-noise(SNR) ratio.
- useful for flow temperature (fluctuation) measurements.

  - heating the sensor only marginally over ambient makes it insensitive to velocity fluctuations, but still sensitive to ambient temperature changes.
  - linear temperature-voltage response.

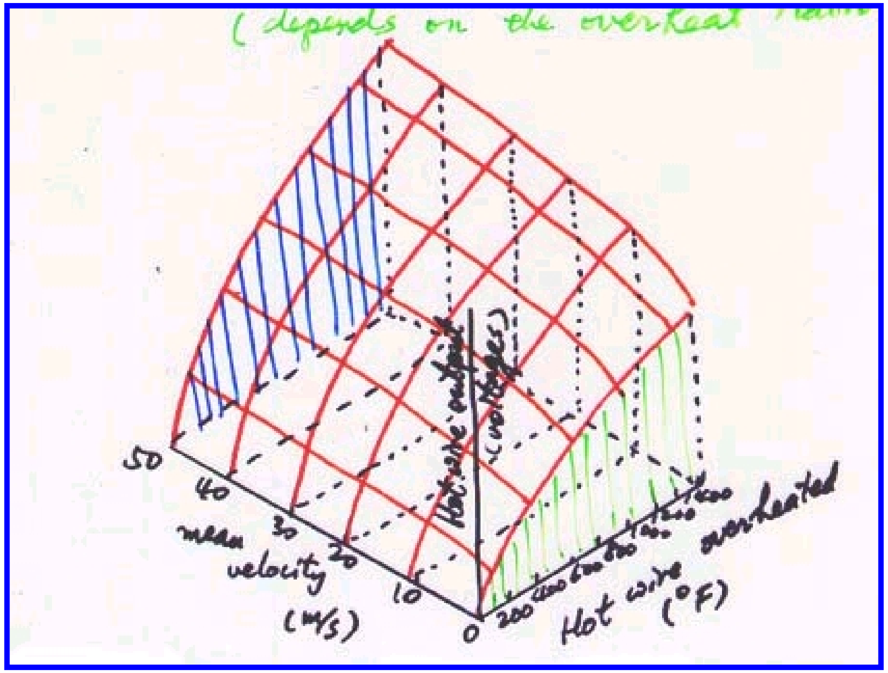
Disadvantages :

- low frequency response due to thermal inertia of sensor; thinner wires help but are more susceptible to breakage.
- large velocity fluctuations can cause burn out.
- relatively low sensitivity to velocity changes in normal range (>1.5m/s)
- compensation circuits (for thermal inertia of sensor) must be adjusted for <u>each</u> sensor and velocity range.

CTA

In CTA-mode the sensor is not required to change temperature, thus there is no problem of thermal inertia as in CCA.

- Advantages :
  - large bandwidth
    the limiting frequency is governed by the electronics (~150kHz)
  - very sensitive to velocity fluctuations
- Disadvantages: requires a stable feedback electronics.



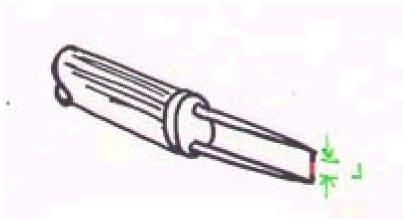
Hot-wire probe

- Hot-wire (***HW***) probe
  ϕ : 1~10μm, L : ~200 ϕ
- Wire materials are chosen mainly according to their temperature sensitivity

$$R = R_0 [1 + \alpha_1 (T-T_0) + \alpha_2 (T-T_0)^2 + \ldots]$$

for platinum: α_1 =3.5 x 10^{-3} /°C, α_2 = -5.5 x 10^{-7} /(°C)^2

for tungsten: α_1 =5.2 x 10^{-3} /°C, α_2 = 7.0 x 10^{-7} /(°C)^2



- In addition, the material must be mechanically robust.
- Comparison of different materials according to various criteria (1.- highest ranking):

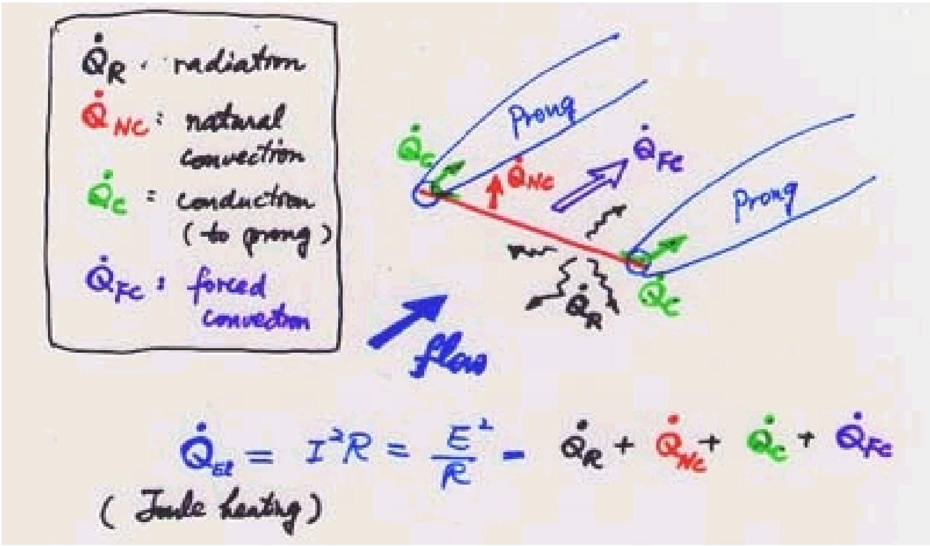
Material	α_1	Mech. strength	Time constant
Tungsten	2	1	1
Platinum	3	4	3
Nickel-Platinum	1	3	4
Iridium(80%Pt)	4	2	2

Heat transfer of a Hot-wire probe (I)

- Radiation and natural convection losses are negligible for most operating conditions.
- Conduction to the prongs can be up to 20% of Q_{FC} , and is given by Fourier as:

$$Q_C = -2k\left(\frac{dT}{dx}\right)_{sensorend} \frac{\pi d^2}{4}$$

- Forced convection (of a cylinder in parallel flow):



Nu = Nu(Re, Pr, Gr, Ma, l/d, ΔT,…)

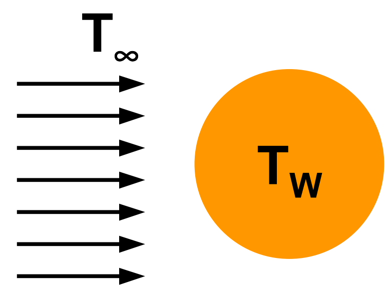
$$Q_{FC} = Nu\ \pi\ l\ k(Tw - Ta); \quad Nu = \frac{hd}{k}$$

Heat transfer of a Hot-wire probe (II)

- Many influentral parameters can be neglected under certain conditions.

$$Nu = Nu(Pr, Re)$$

(by constant reference temperature, fixed operation condition, excluding low velocities.)



- Most commonly used reference temperature is the film temperature

$$T_m = 0.5(T_w + T_a)$$

- The first theoretical solution, based on potential flow theory, for the heat transfer from circular cylinder was given by King(1914):

$$Nu = \frac{1}{\pi} + \sqrt{\frac{2}{\pi}\,\mathrm{Re}\,\mathrm{Pr}}$$

For $\mathrm{Re} \times \mathrm{Pr} > 0.08$

Heat transfer of a hot-wire probe (III)

- Today, many people are still using the relation in a modified form for hot-wire measurement. The constants A, B, n and s are determined by calibration.

$$Nu = \left[A(Pr, \Delta T) + B(Pr, \Delta T)\,\mathrm{Re}_f^{\;n} \right] \left(\frac{T_m}{T_a} \right)^s$$

Author	conditions	A	B	n	s
King(1914)	Pe=RePr>0.08	$1/\pi$	$(2Pr/\pi)^{0.5}$	0.5	0
Kramers(1946)	0.01,Re<1000	$0.42Pr^{0.2}$	$0.5Pr^{0.33}$	0.5	0
Collis & Williams (1959)	0.02<Re<44	0.24	0.56	0.45	0.17
	44<Re<140	0	0.48	0.51	0.17
Koch & Gartshore(1972)	Re<4.2	0.72	0.80	0.45	-0.67

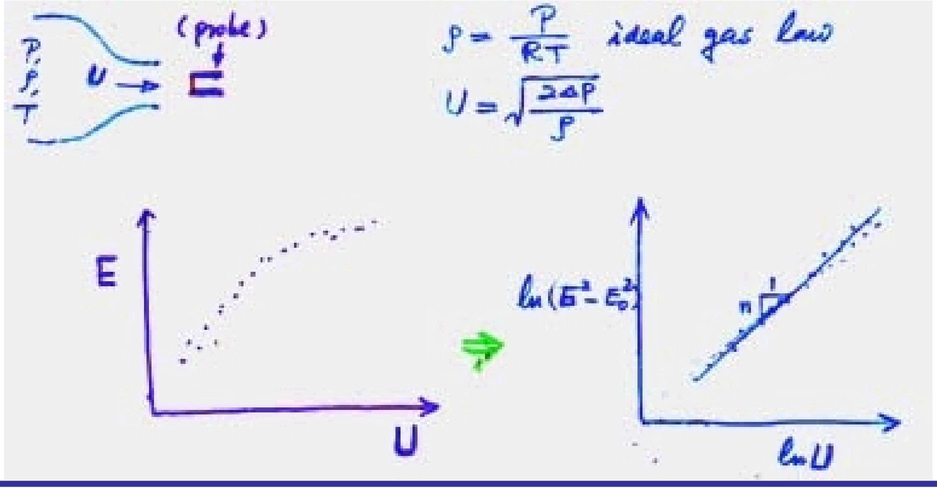
- A simple form has recently been proposed by Wang et al. (2001)

$$Nu = A' + B'\ \mathrm{Re}_f^{\;0.5} \left(\frac{T_w}{T_a} \right)^{0.125}$$

For laminar steady air flow: A'=0.502, B'=0.434
For laminar unsteady air flow:
A'=-0.153, B'=0.527, $T_w/T_a < 1.8$

Calibration of Thermal Anemometer

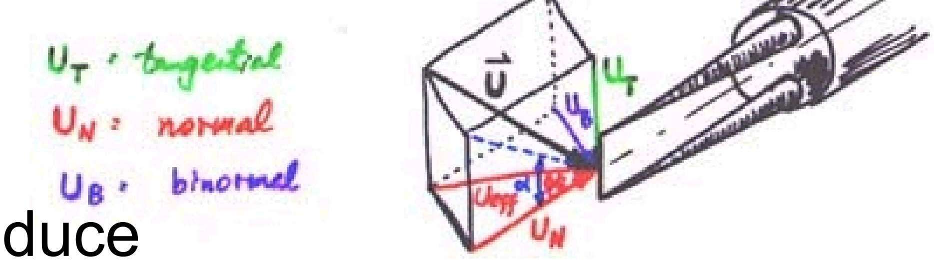
- Although there are plenty empirical values for the calibration constants in the literature, however, no (hot-wire) probe can be made absolutely the same as the other, therefore, it is recommended to calibrate all the hot-wire probe when it is newly used.
- Calibration of directional sensitivity of a probe is only necessary for multi-wire-probe or for inclined probe.
- A calibration nozzle or wind tunnel is best suited for multi-wire-probe or for inclined probe.



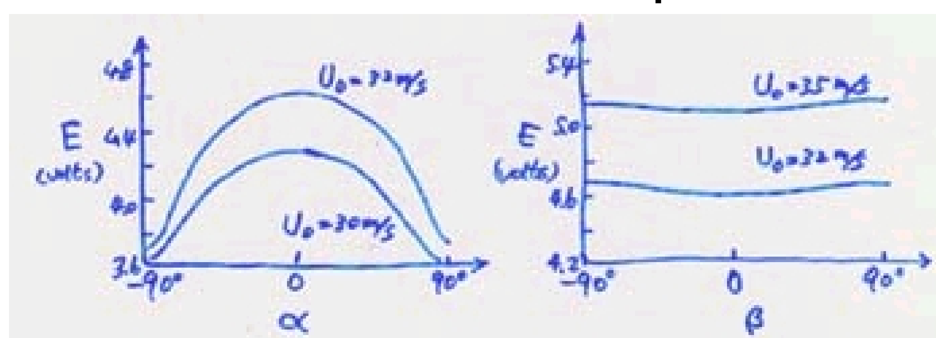
Directional sensitivity of HW

- The heat loss due to force convection is dependent on direction as well as the magnitude of velocity vector.

  U_T: tangential
  U_N: normal
  U_B: binormal



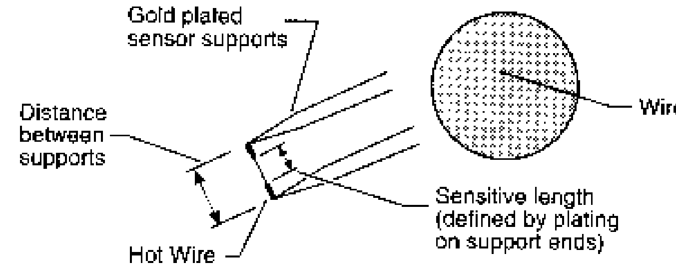
- Common practice is to introduce the effective cooling velocity $U_{eff} = f(\alpha, \beta)|U|$
  where α and β are the yaw and pitch angles respectively.
- It is customarily assuming $E^2 = A + BU_{eff}^n = A + B\, f(\alpha,\beta)^n|U|^n$
  i.e., the yaw and pitch influence can be separated from the speed influence.

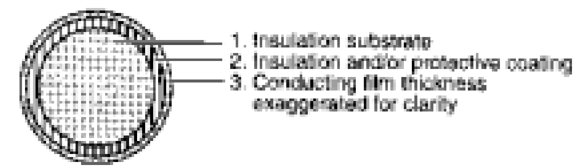


Hot wire and hot film

Comparison of hot-wire to hot-film probes:

- **Advantages of hot-wire probes:**
  - low thermal inertia
  - high bandwidth
  - high sensitivity





- **Advantages of hot-film probes:**
  - long time stability
  - uniform production possible (constant calibration coefficients for the same type of sensors)
  - very robust mechanically, not sensitive to contamination
  - can be easily insulated with quartz film (useful for conducting liquids)

Frequency response

- The dynamic response of hot-wire sensors as a first order process:

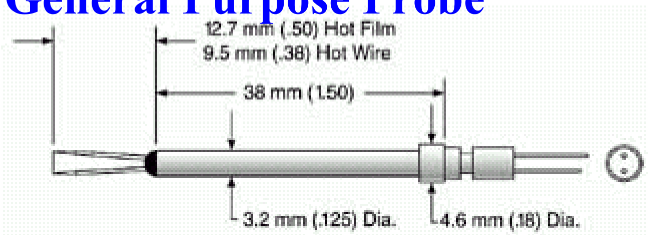
$$m_s C_s \frac{dT_s}{dt} = I^2 R[1+\alpha(T_s - T_e)] - \pi k_m l(T_s - T_e)$$

- However, as an anemometer, several other factors have an influence, such as:
  - the probe type (aspect ratio)
  - feedback amplification
  - dynamic response of electronic components
  - bridge adjustment (impedance matching…etc.)

therefore the dynamic response is usually determined experimentally, either directly or indirectly.

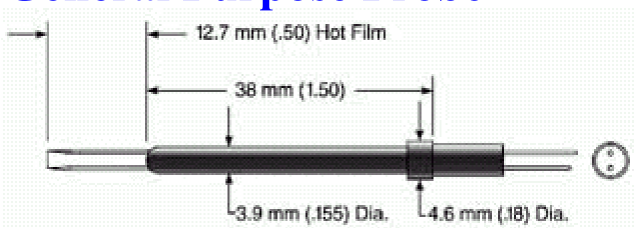
Commercial probes (I)

General Purpose Probe



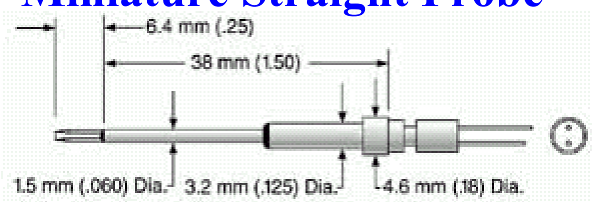
4μm ϕ

General Purpose Probe

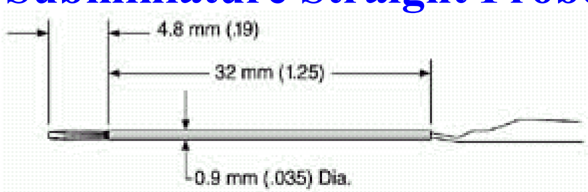


20μm ϕ

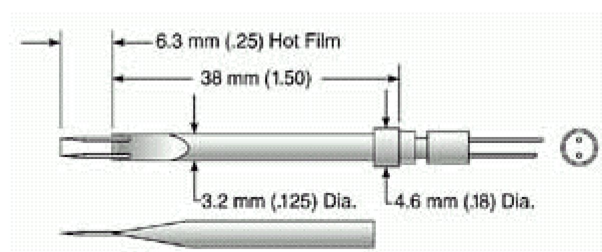
Miniature Straight Probe



Subminiature Straight Probe

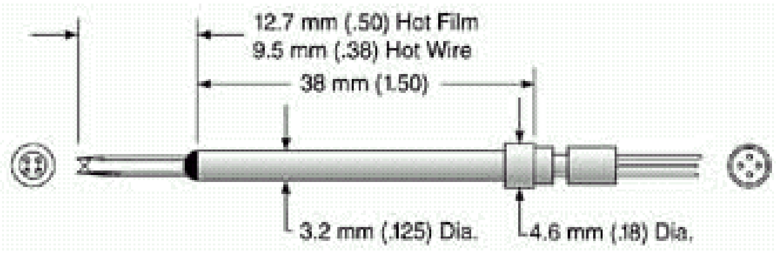


Streamlined Probe

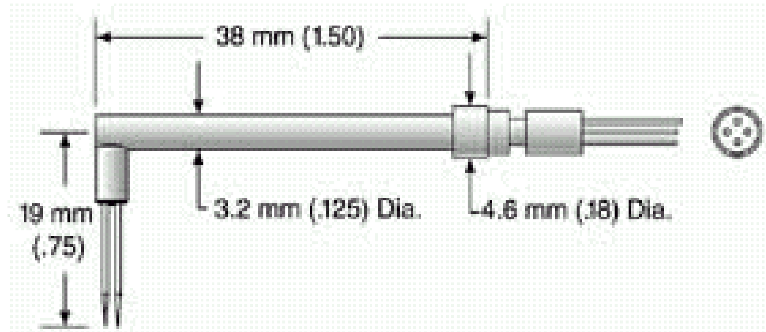


Commercial probes (II)

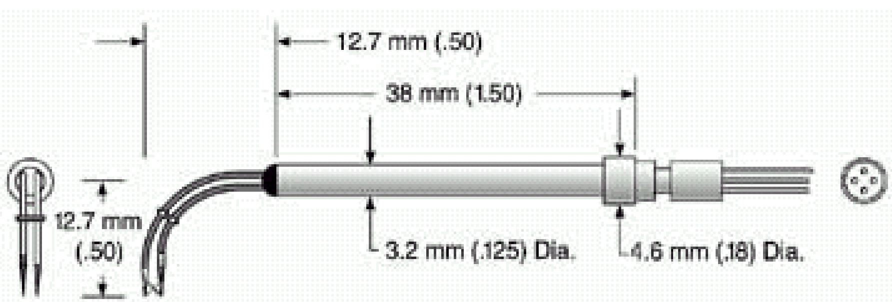
Standard "X" Probe



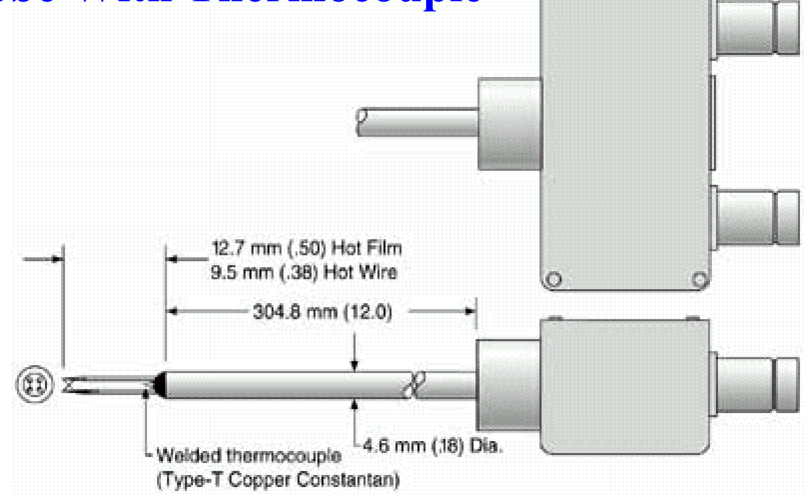
Standard 90° "X" Probe



Boundary Layer "X" Probe

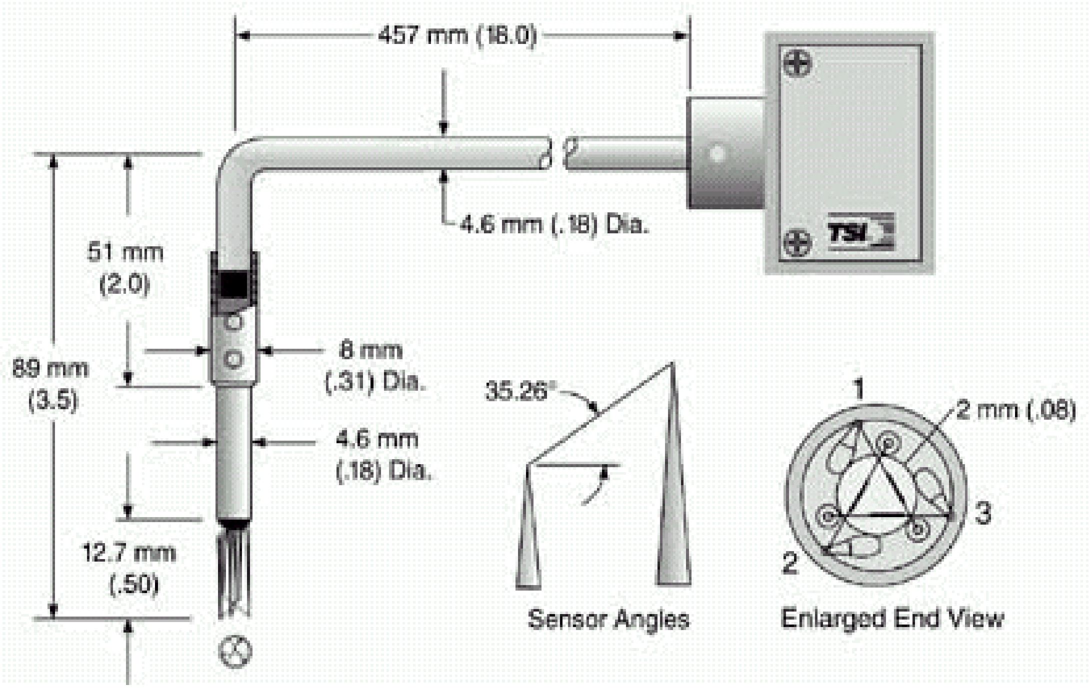


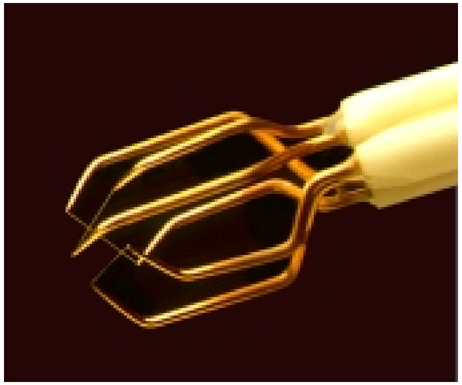
Probe With Thermocouple

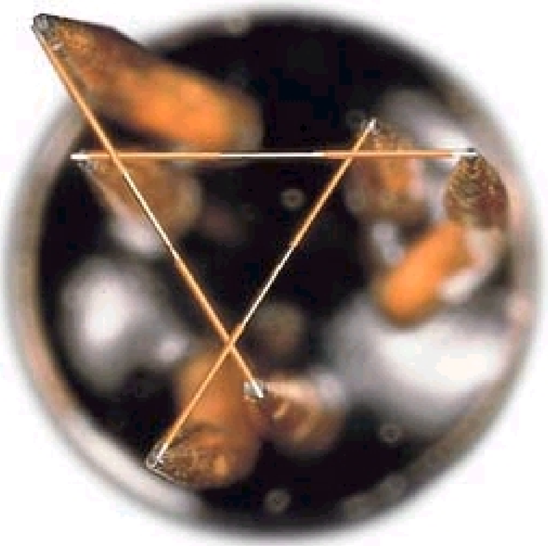


Commercial probes (III)

Standard probe for three component measurements. Sensors oriented 90° to probe body. Sensor orientation designed to minimize sampling volume and minimize flow disturbance. For velocity and turbulence measurements to 150°C.



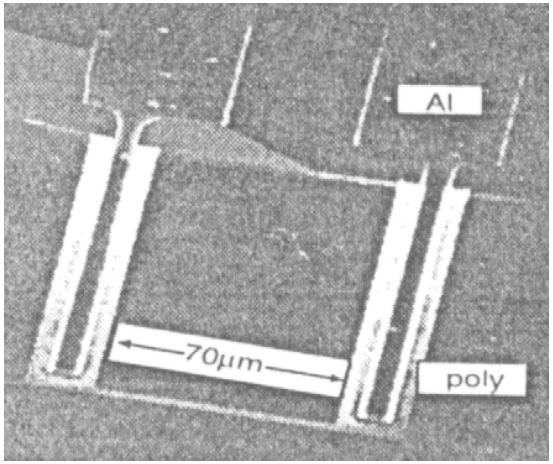




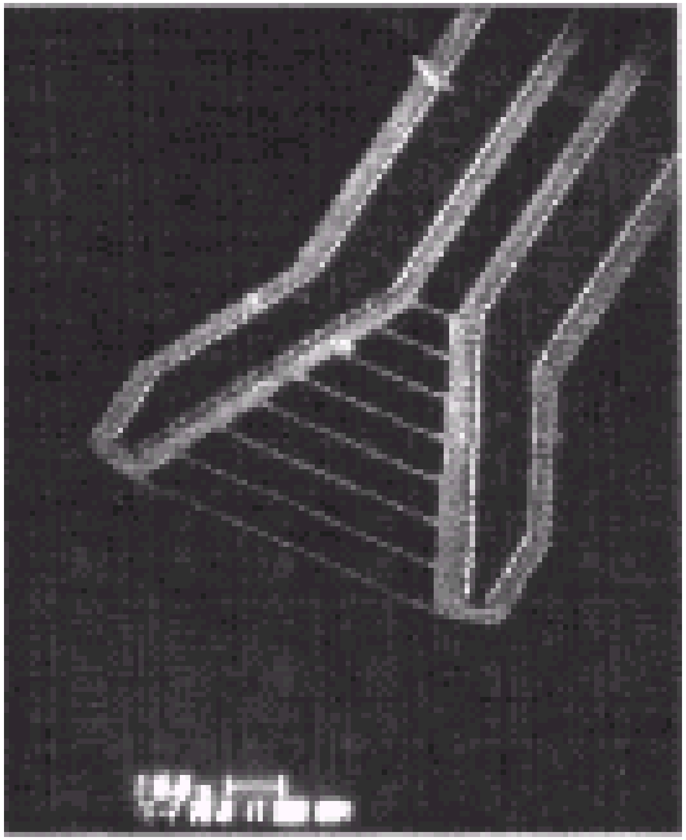
Hot-wire by IC-technology

(from Jiang et al. [1994])

- **Advantages:**
  high spacial resolution,
  high response (~MHz),
  disposable, cheap



(1um x 70 um single hot wire)

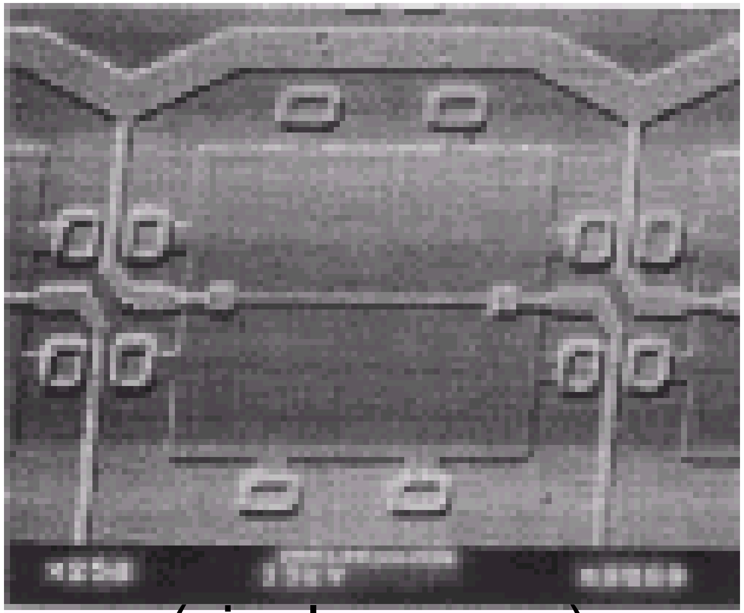


(hot- wire array)

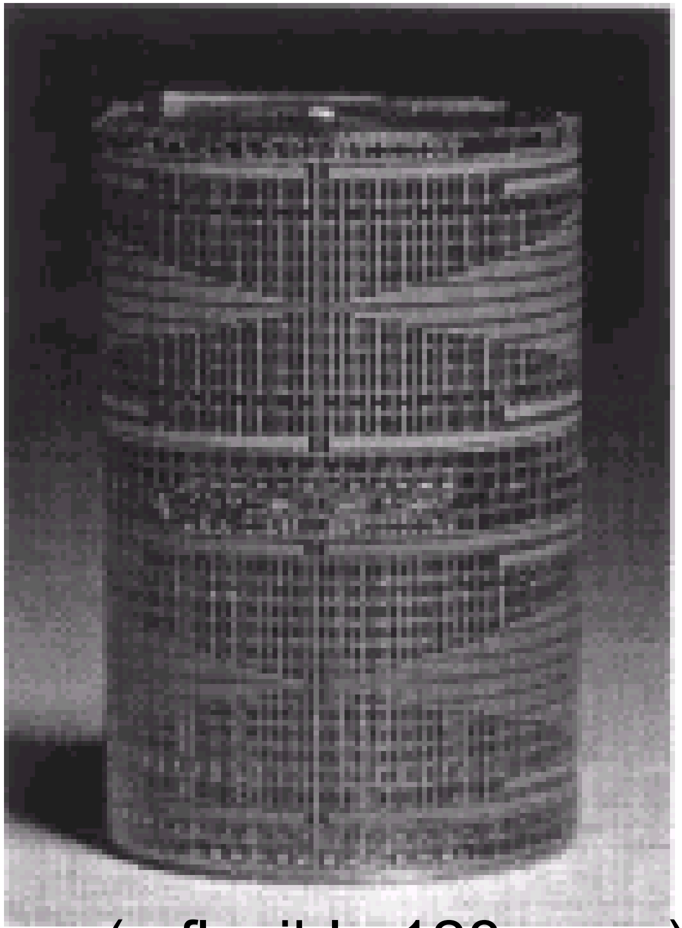
Shear stress sensor by IC-technology.

(from Jiang et al.[1996,1997])

- **Advantages:**
  high spacial resolution,
  high response (~MHz),
  disposable, flexible



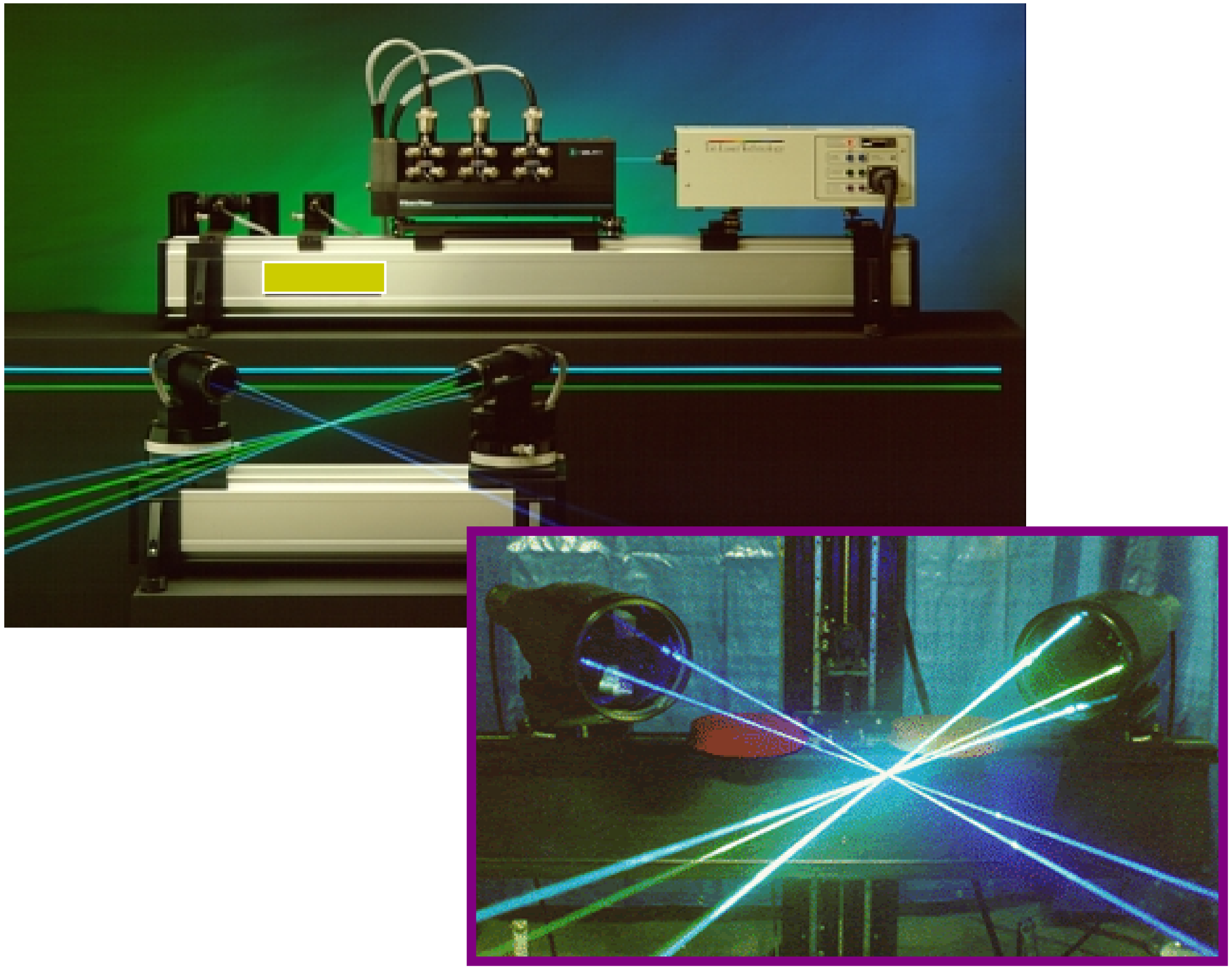
(single sensor)



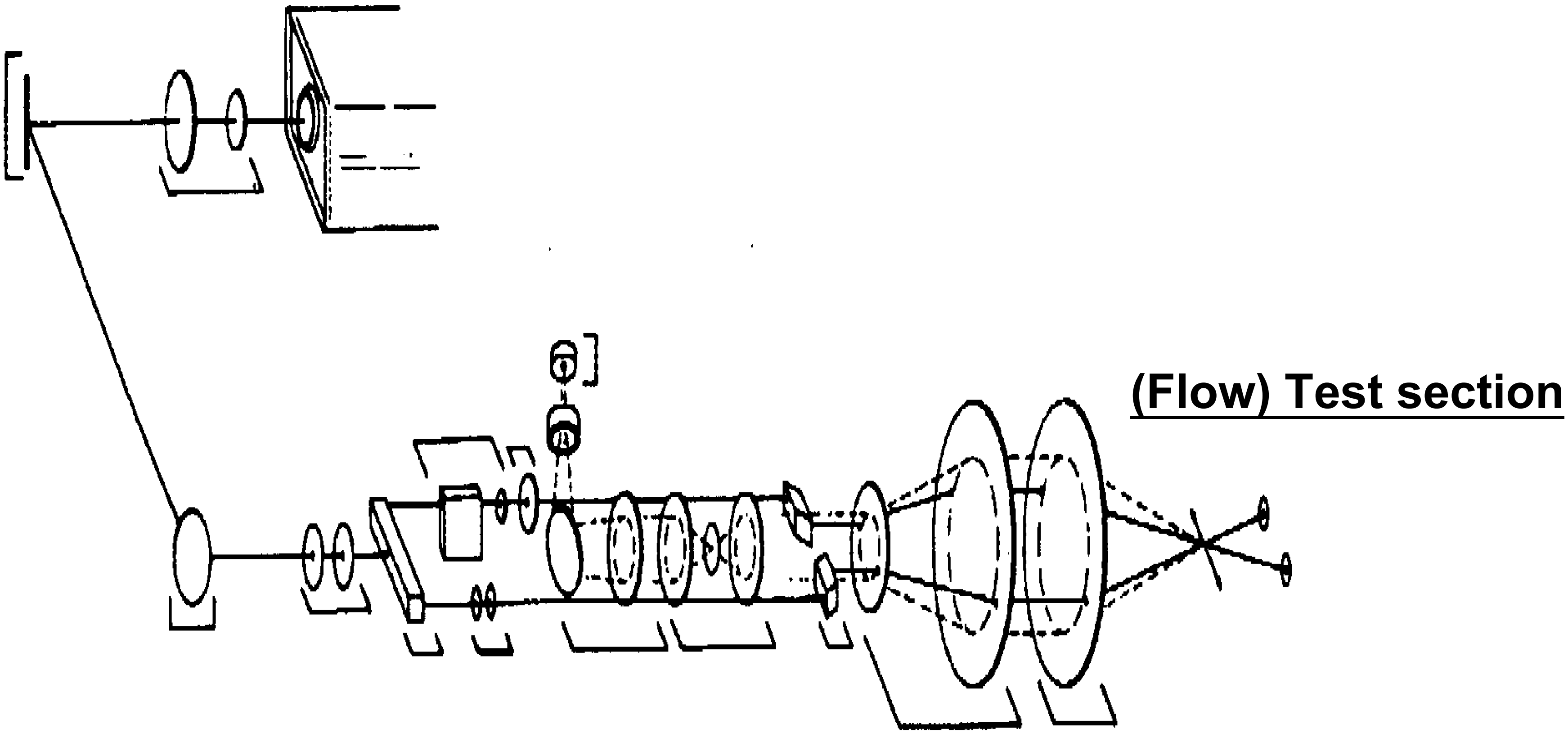
(a flexible 128 array)

Introduction to Laser Doppler Anemometry (LDA)

LDA systems



Principle and configuration of LDA



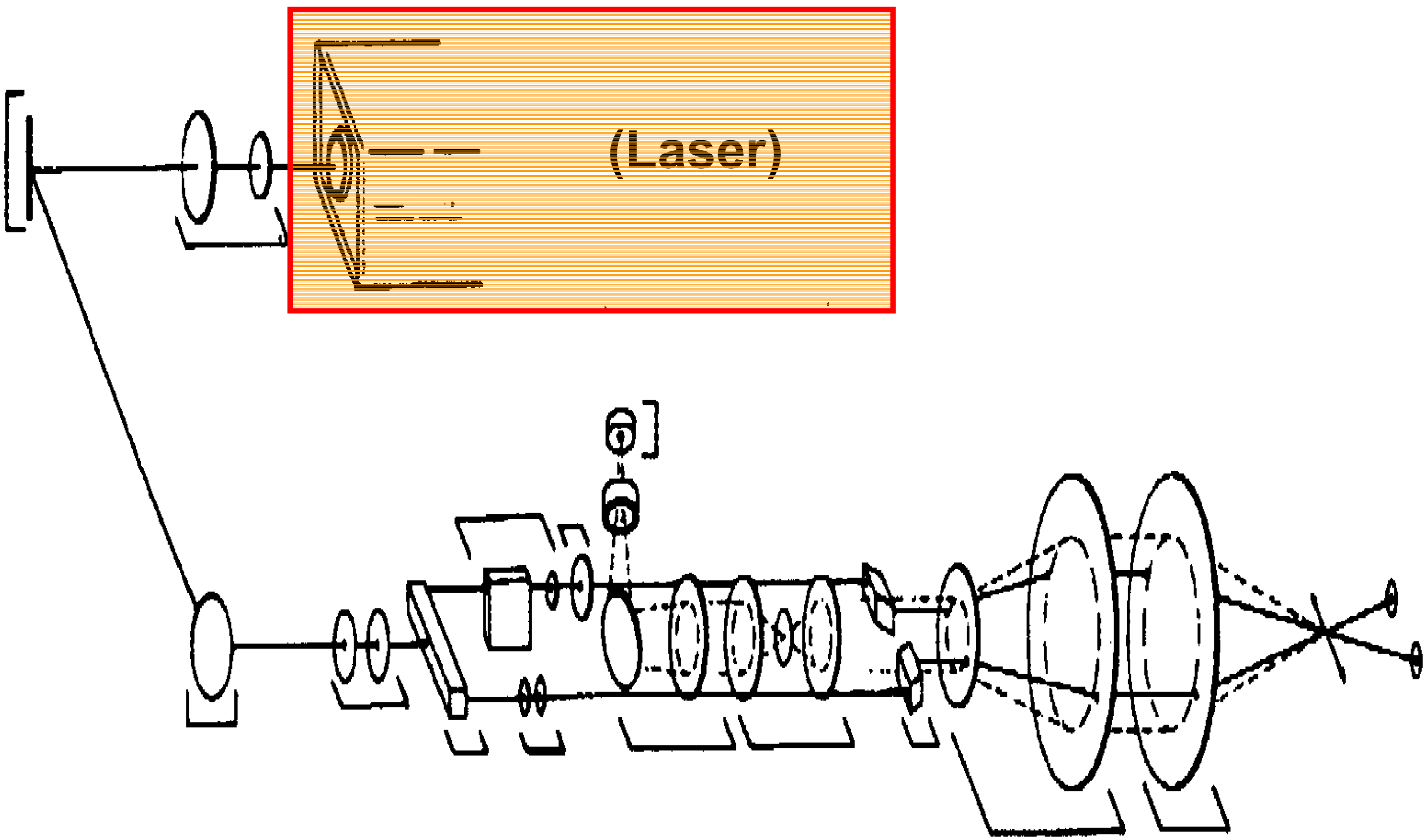
Characteristics of LDA (I)

- ***Advantages***:
  - ***Non-intrusive***
  - ***no calibration*** required (not strongly dependent on the temperature, density, composition of the flows)
  - sensitive to ***velocity*** magnitude and ***direction***
  - ***linear transfer function*** for velocity measurements
  - measures a ***single*** desired velocity component directly
  - ***high accuracy*** obtainable
  - ***very high frequency response***
  - very ***small*** measuring volume
  - ***high dynamic range*** (from μm/s to 1000 *m/s*)

Characteristics of LDA (II)

- **Disadvantages**:
  - relatively ***expensive*** for set-up and maintenance
  - ***seeding particles*** in the flow required
  - ***optical access*** to measuring point required
  - flow medium must be ***transparent***
  - ***experienced*** man-power recommended
  - ***spherical*** particles based
  - relatively huge and heavy for traditional LDA system

Principle and configuration of LDA



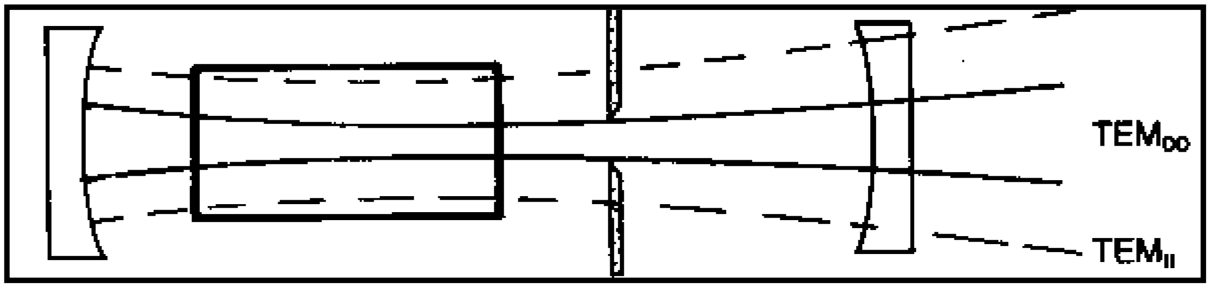
Introduction to Laser

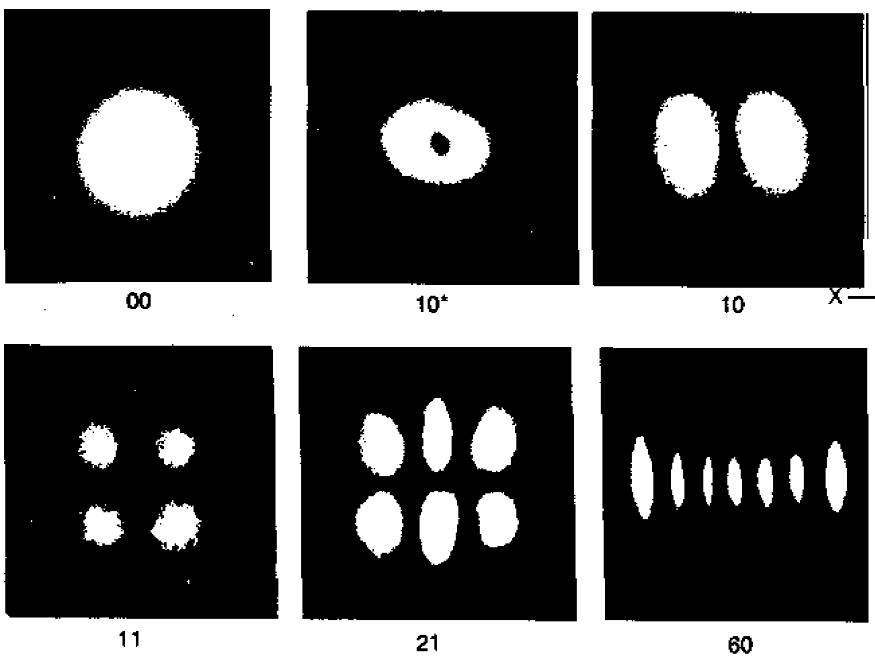
- ***Laser*** (雷射、激光、鐳射)：
  Light Amplification by Simulated Emission of Radiation
- **Characteristics of Laser:**
  - high light intensity
  - narrow monochromaticity
  - high coherence (temporal & spatial)
  - low divergence angle (0.1°: 360°)
  - short pulse time(*ns* ~ *ps*)

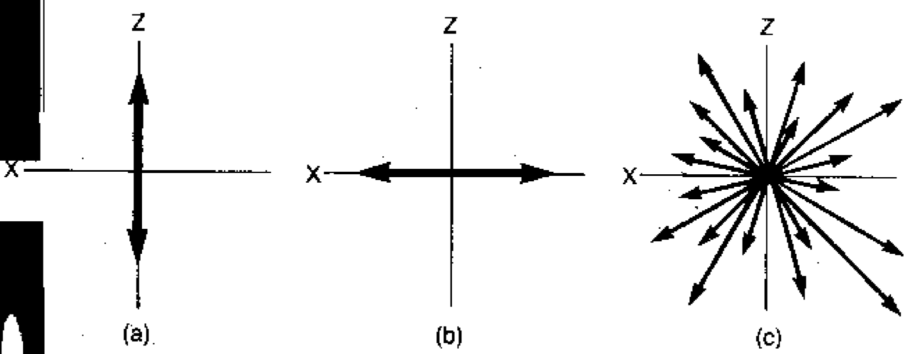
Development of LASER

1900 Planck's quantum theory
1905 Einstein's photon theory
1917 Einstein's stimulated radiation theory
1954 Townes produced the 1st Maser
1960/5 Maiman produced the 1st rubby Laser
1960/11 1st gas Laser (He-Ne)
1962 1st semi-conductor Laser(GaAs)
1964 CO_2-Laser, Ar^+-Laser, YAG-Laser, Dye-Laser
1970 Excimer Laser

Laser modes & polarization

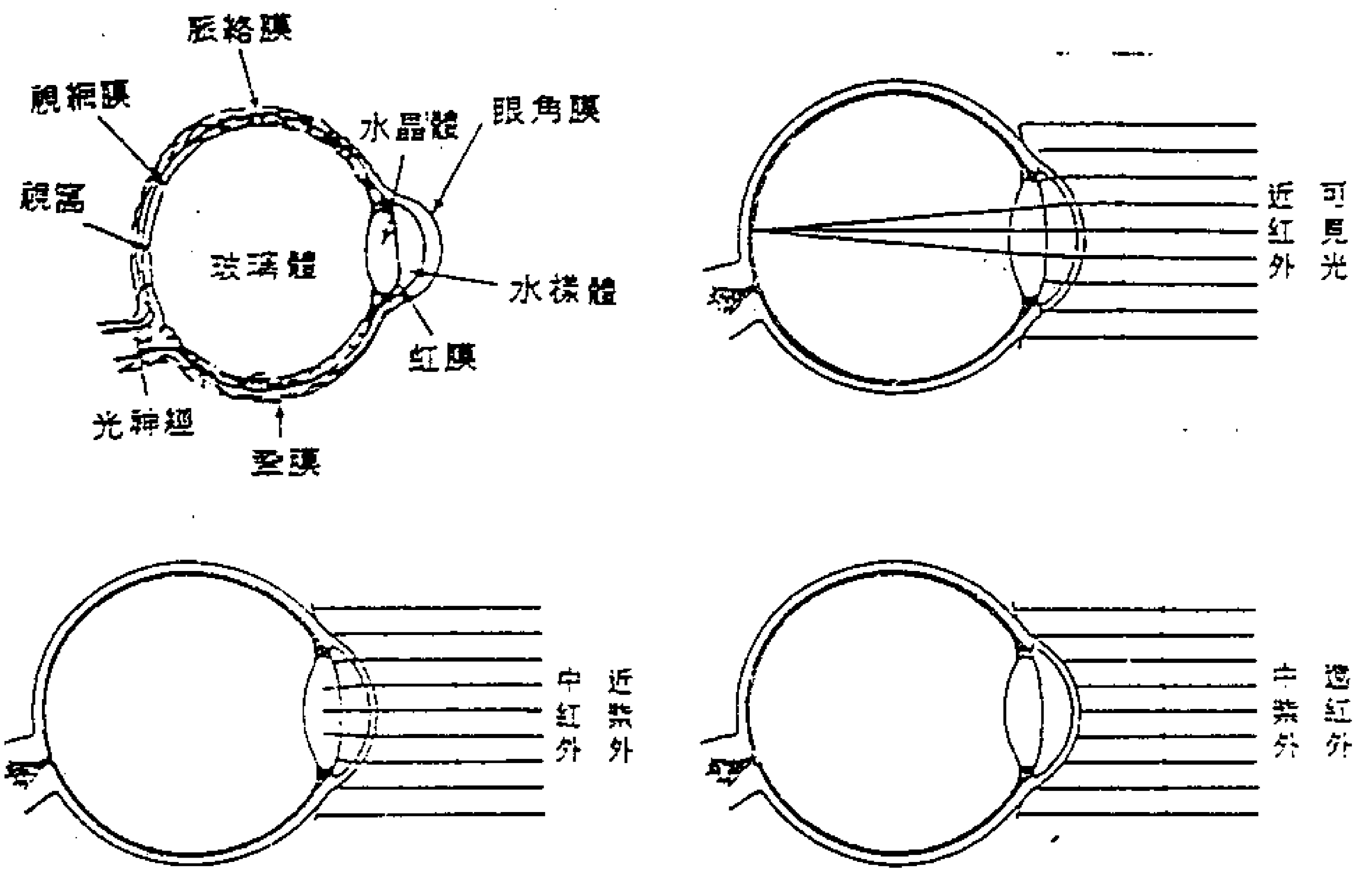






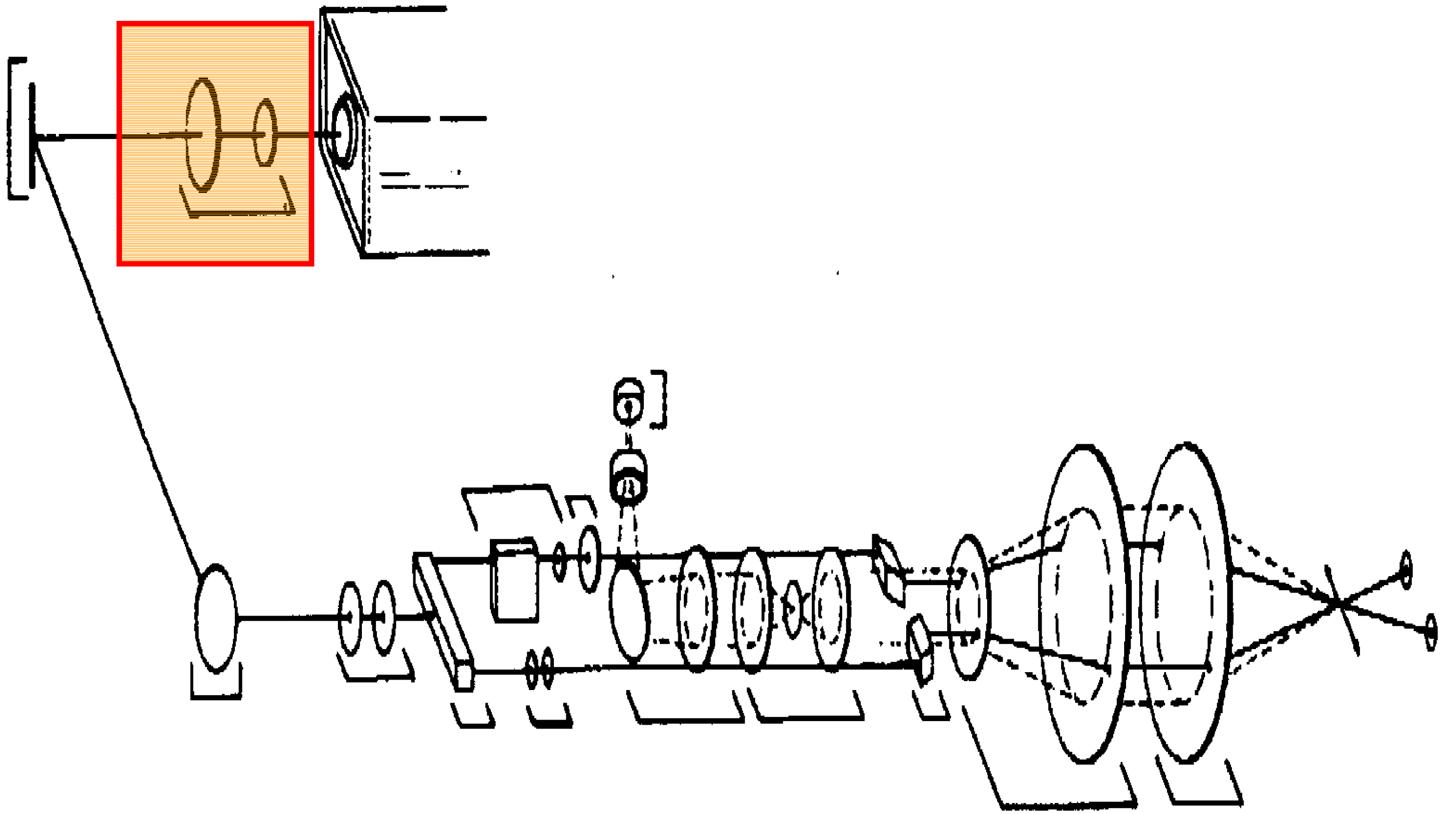
Laser safety

- Class I:
  no gangeous,
  <0.4mW
- Class II:
  dangerous for direct observation,
  < 1mW
- Class III:
  1~500mW
- Class IV:
  >500 mW



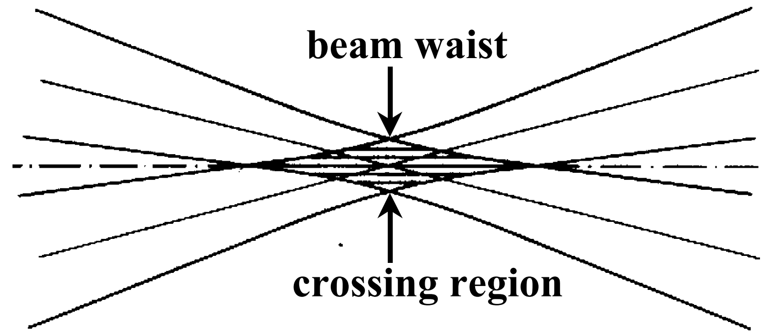
Principle and configuration of LDA

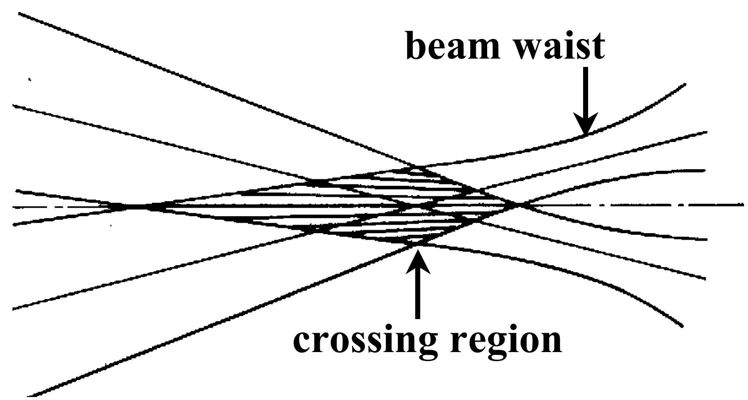
Beam collimator



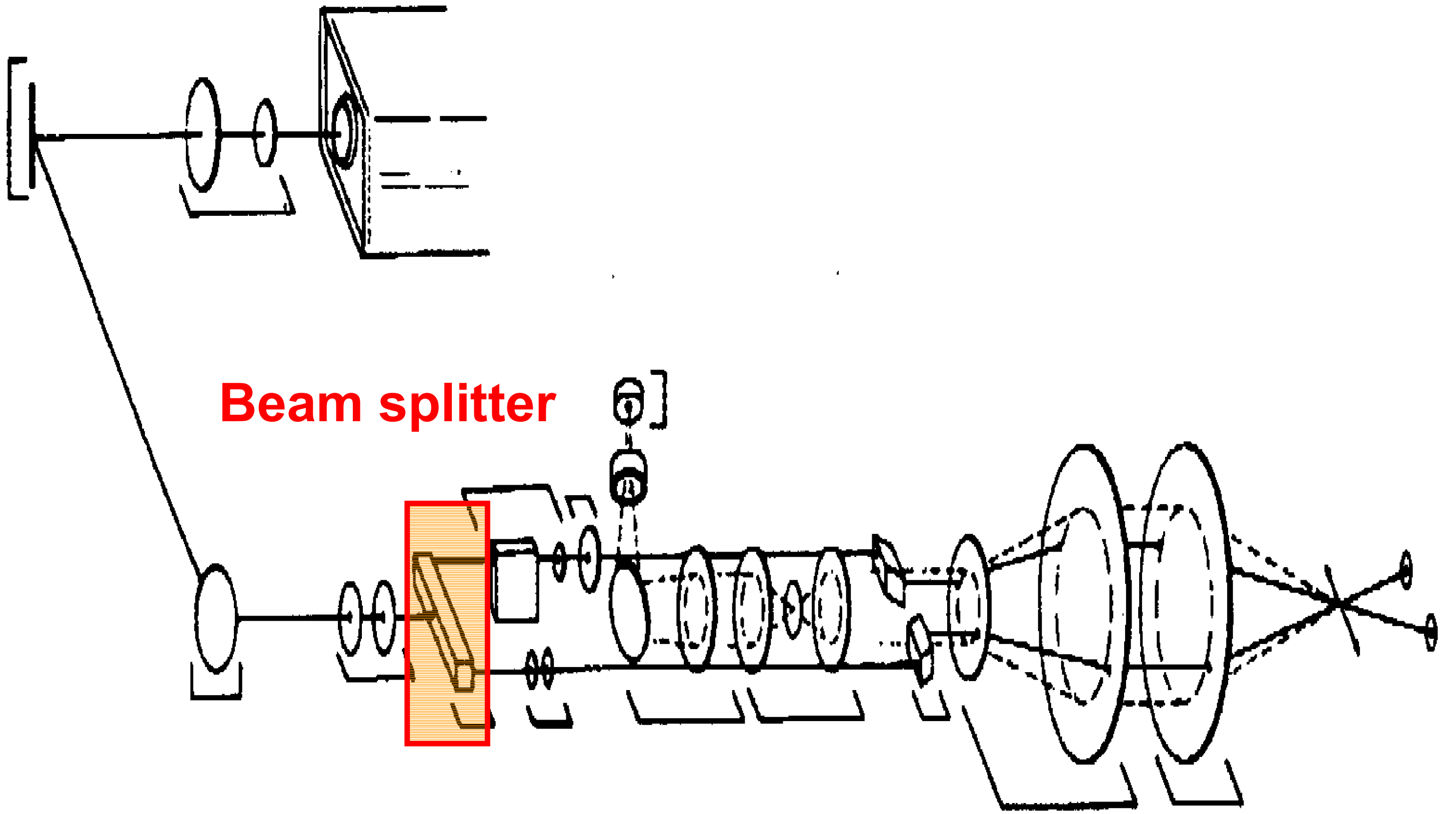
Beam Collimator

- **Beam collimator** is basically a pair of positive and negative lens, which is used to control the beam divergence of a given laser.
- Collimator is used to adjust the positions of both **laser-beam waists** located at the same place, to avoid **artificial turbulence** caused by fringe-spacing variations.
- The need of collimator increases as the optics become more complex.

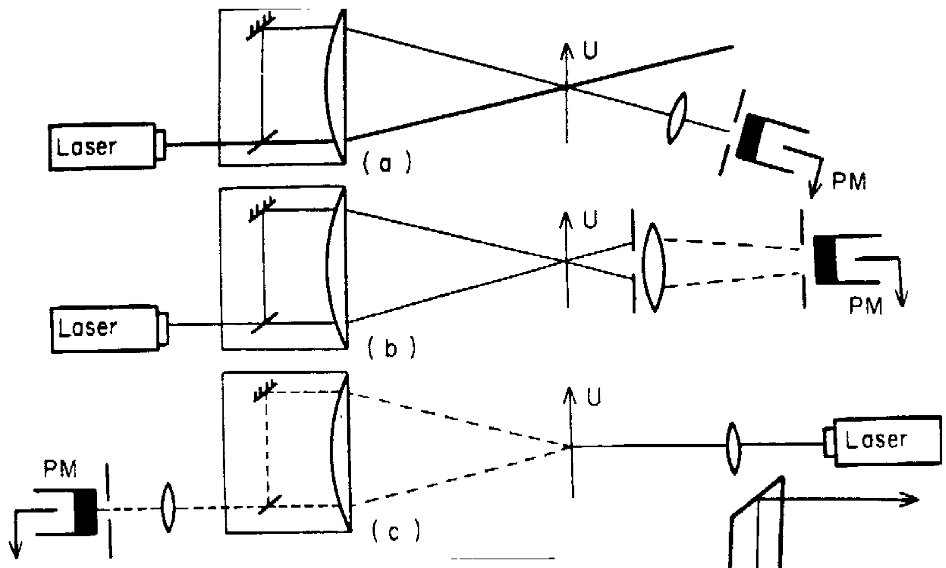




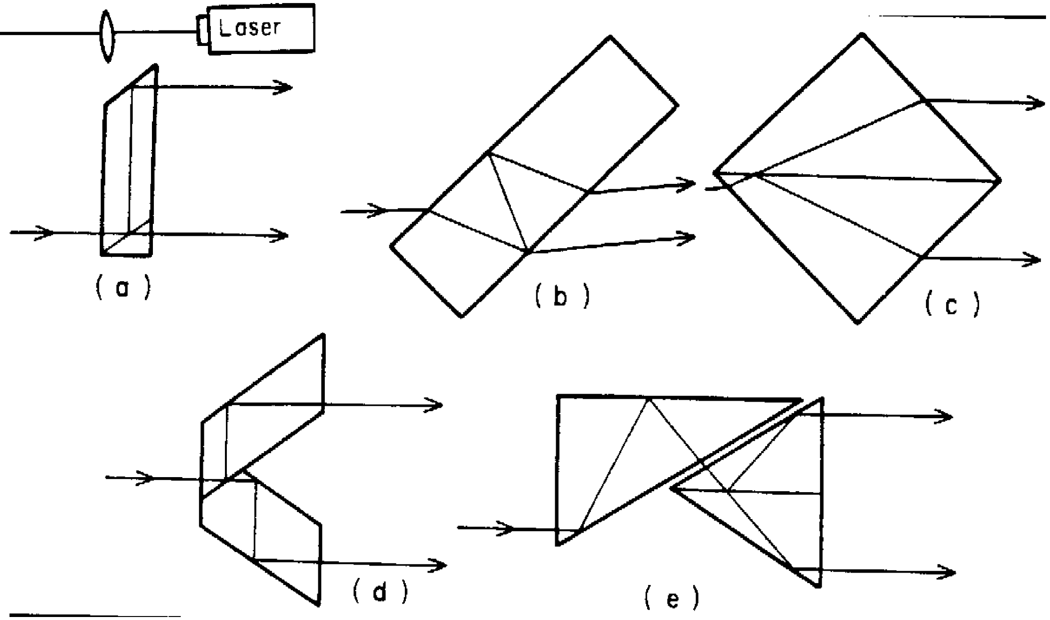
Principle and configuration of LDA



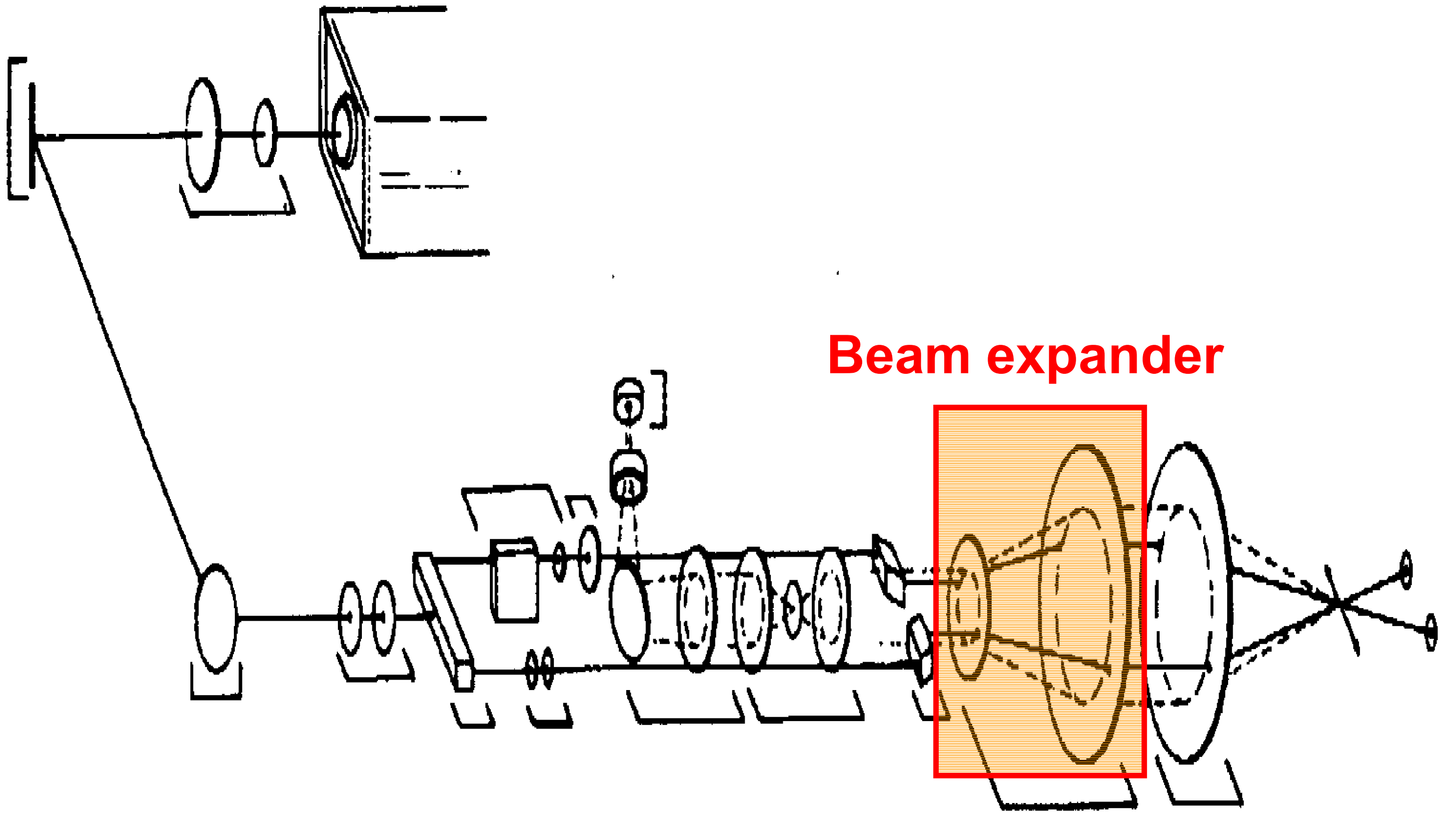
Beam Splitter



The integrated beamsplitters:

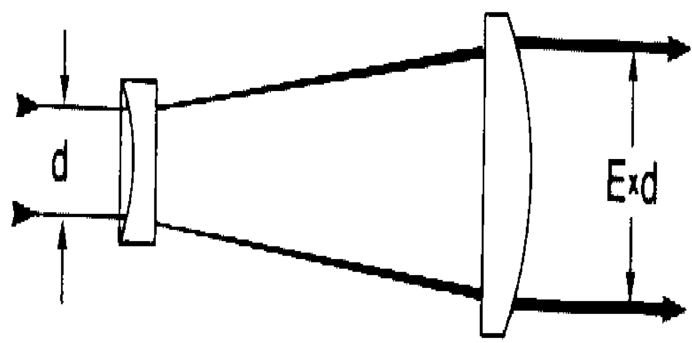


Principle and configuration of LDA



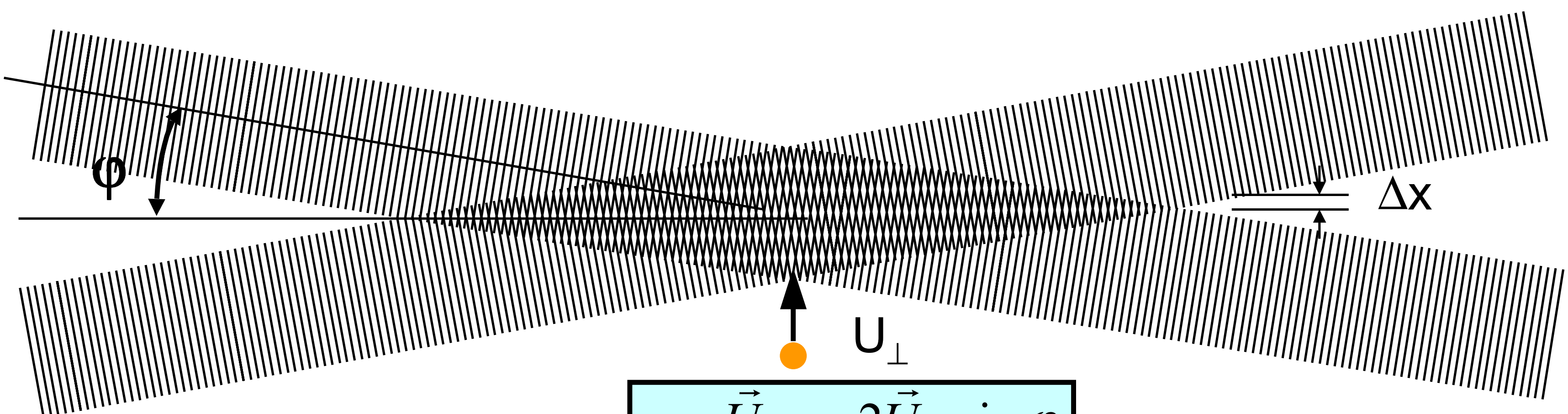
Beam Expander

- Beam expander is recommended when measuring for the case of large distance, or high velocity gradient or low SNR.
- Beam expanders are designed to increase (a) the input beam diameter and (b) the collection aperture (for backscatter). This results in a smaller mcv and better signal quality.
- A beam expander with expansion ratio E may decrease diameter of mcv ($d_m = \frac{4 f\lambda}{\pi D_{e^{-2}} \cos\varphi}$) by a factor of E, decrease measuring length by a factor of E^2, and improves estimated SNR.
- Commerial available: E = 2 ~ 8.5



Fringe Model

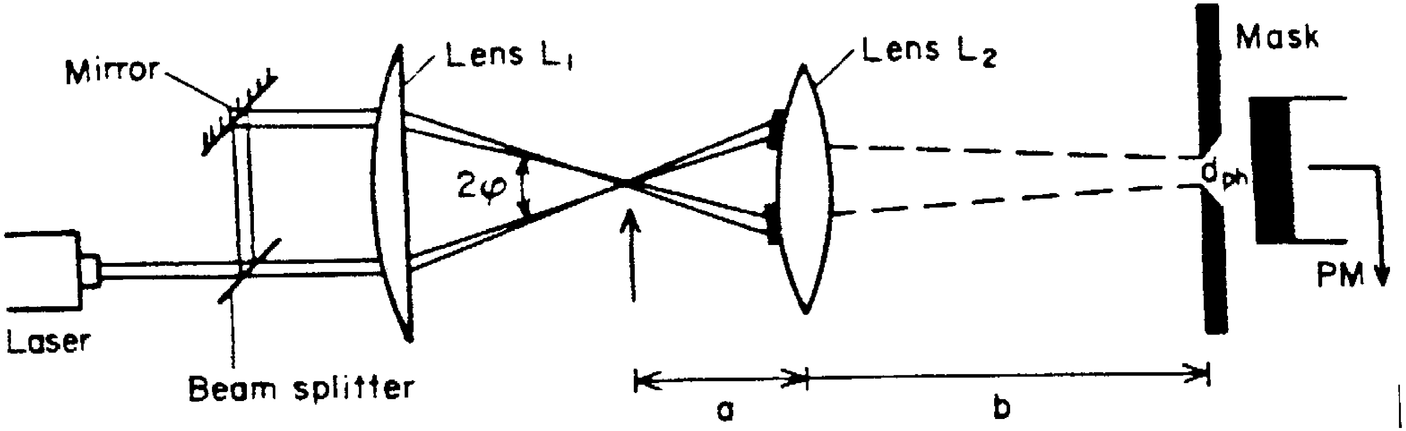
- ***Moire ri e*** can be used to illustrate the basic characteristics of an LDA. The ***resulting re uen*** agrees with that derived using the Doppler principle.



$$\nu = \frac{\vec{U}_{\perp}}{\Delta x} = \frac{2\vec{U}_{\perp}\sin\varphi}{\lambda}$$

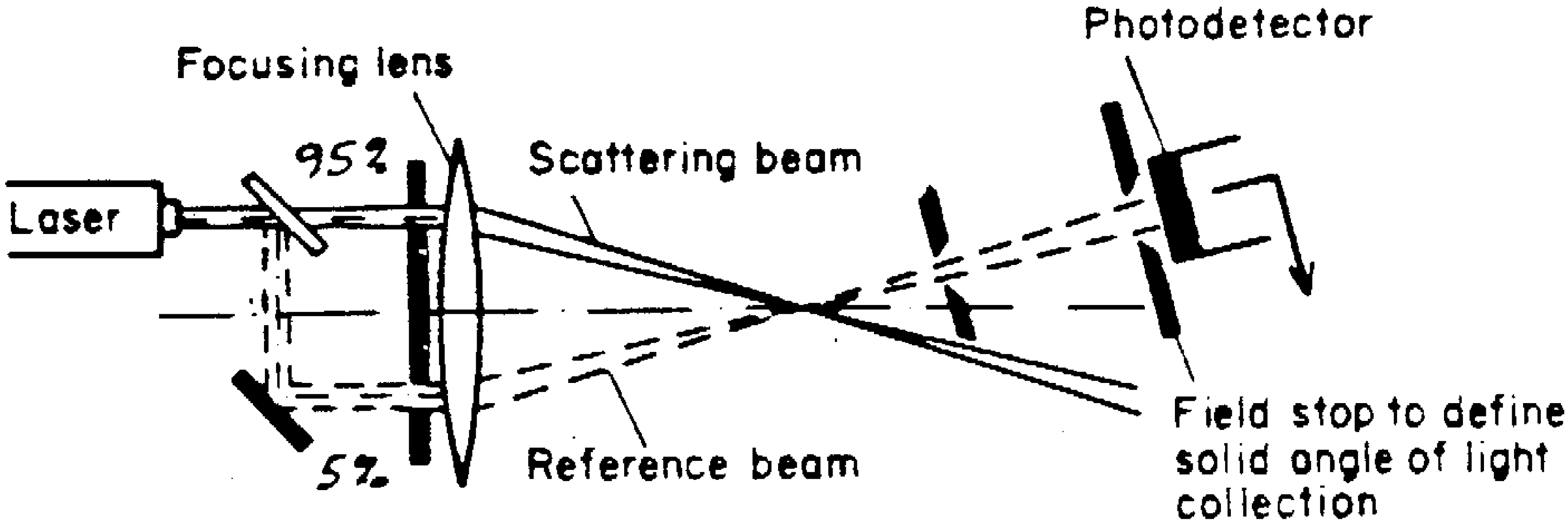
Dual Beam LDA

- Most commonly used arrangement



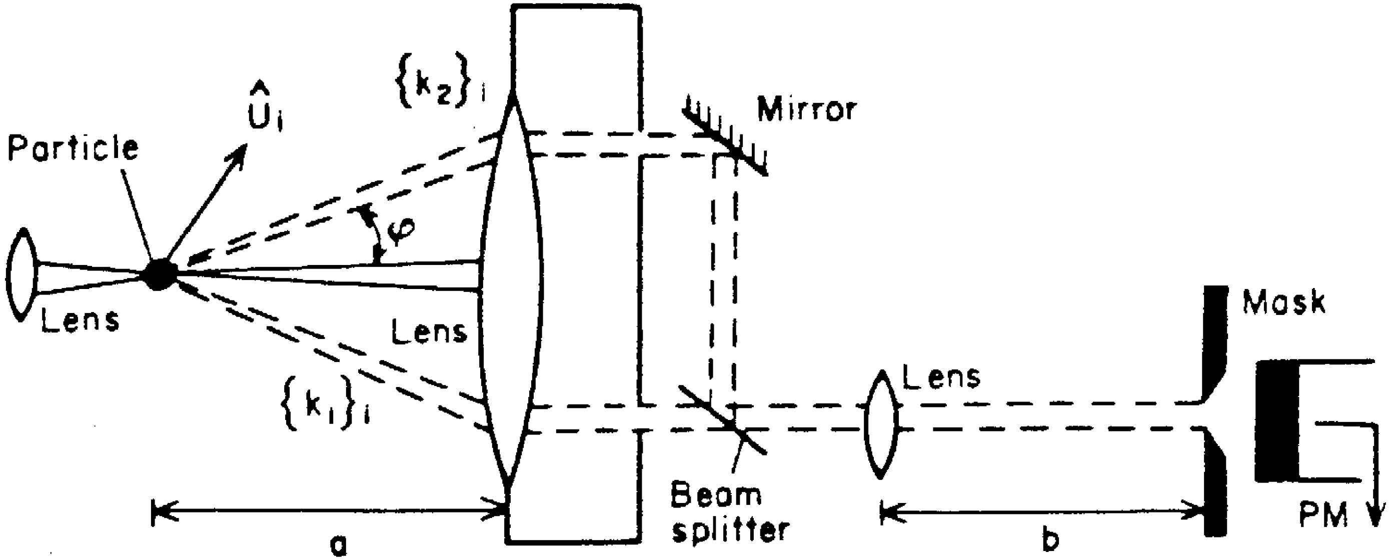
Reference Beam LDA

- advantageous in low-transparency ediu
- in general lower SNR



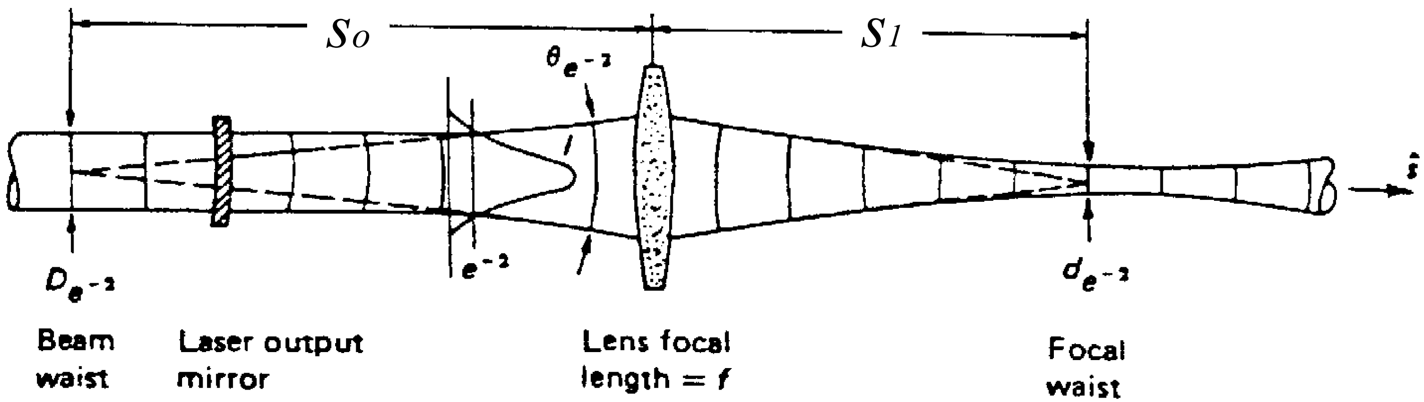
Two-Scattered Beam LDA

⊙



Measuring Control Volume (I)

- In describing the measuring control volume, the properties of ***Gaussian beams*** must be considered.



$$s_1 = \quad + \frac{s \quad -}{(s \quad -1 \quad + (\pi D_{e^{-2}} \quad \lambda}$$

$$\frac{1}{\quad_{e^{-2}}} = \frac{1}{\quad_{e^{-2}}}(1 - \frac{}{} \quad + (\frac{\pi D_{e^{-2}}}{\lambda}$$

For $s_o = f$,

then $s_1 = f$ and $d_{e^{-2}} = \frac{4 \quad \lambda}{\pi D_{e^{-2}}}$

Measuring Control Volume (II)

- The measuring control volume (mcv) is ellipsoidal in shape

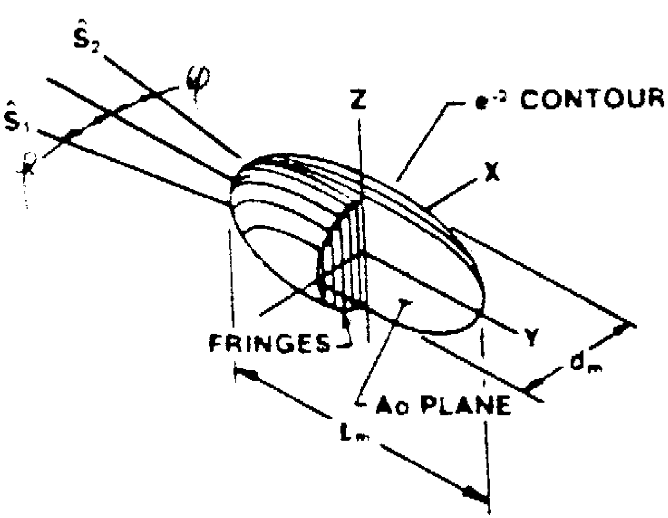
  diameter of mcv: $d_m = d_{-2} \quad \varphi$

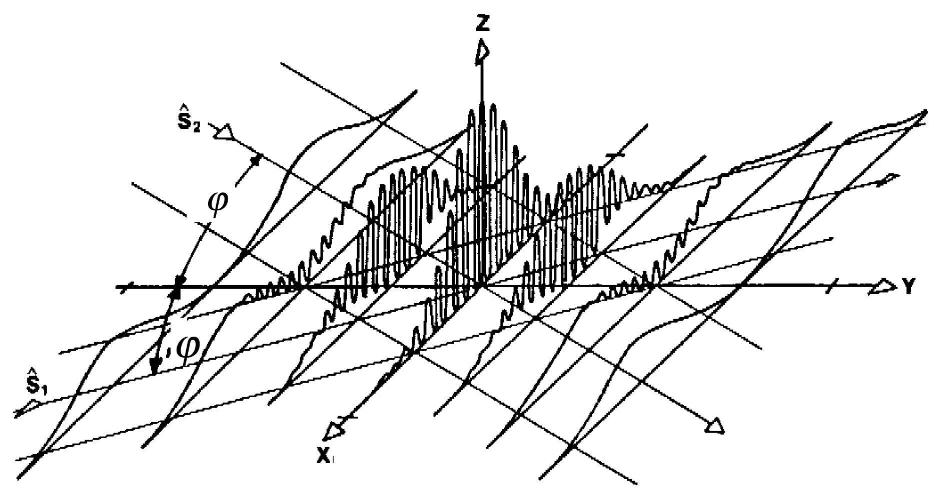
  length of mcv: $_m = d_{-2} \quad \varphi$

  no. of fringes: $= 1 \quad d \quad D_{-2}$

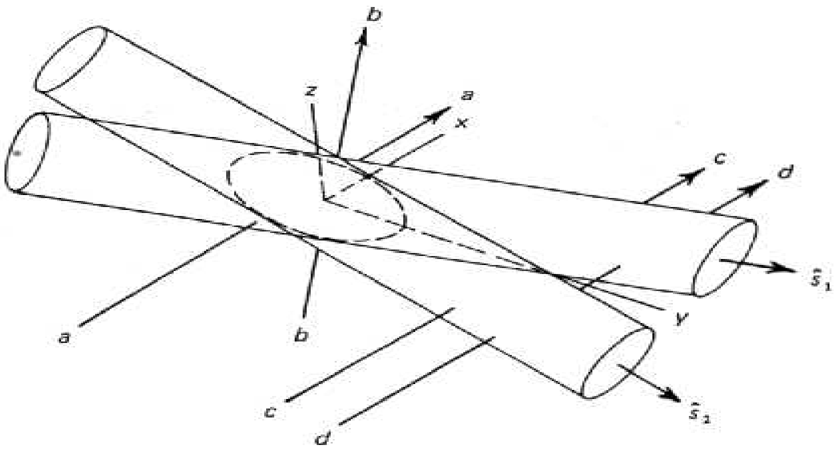
d: beam spacing before lens

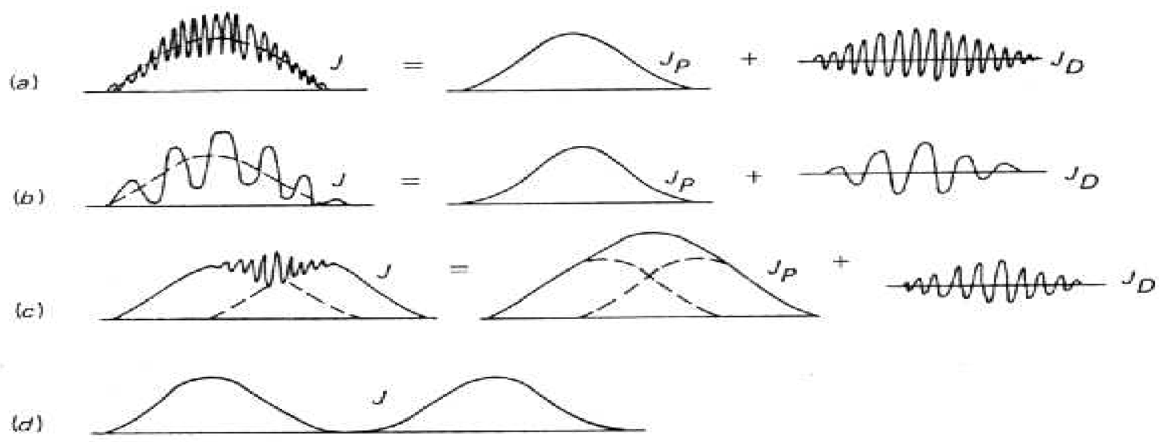
φ : intersection half-angle



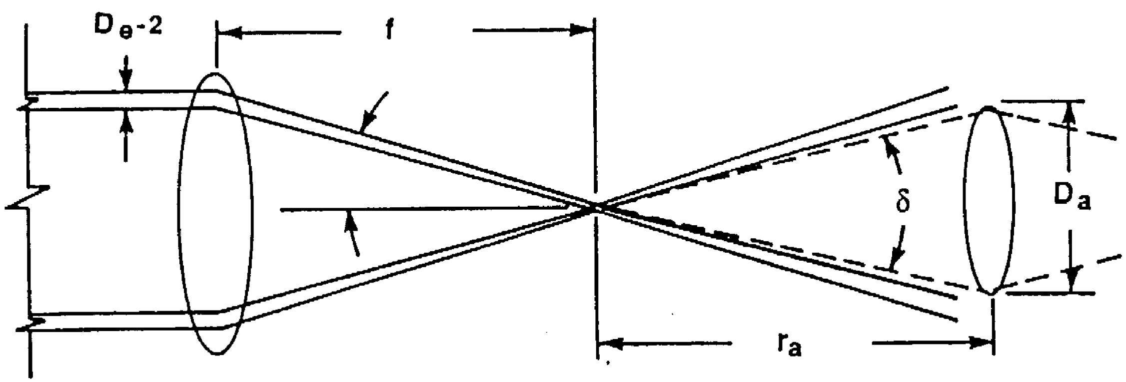


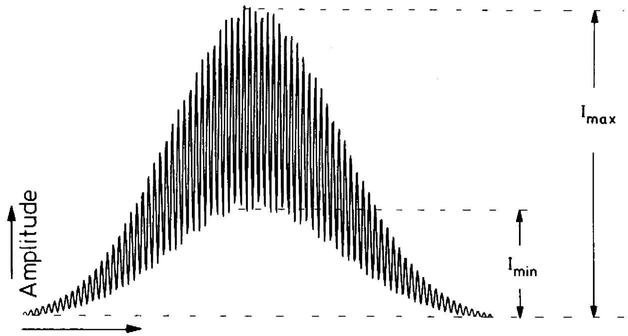
Characteristics of LDA Signals





Signal-to-Noise Ratio





SNR = signal to noise ratio (Power)

η_q = quantum efficiency of photodetector

P_o = power in each beam, watts

Δf = bandwidth, MHz

d_p = particle diameter, μm

$\bar{G}$ = scattering parameter

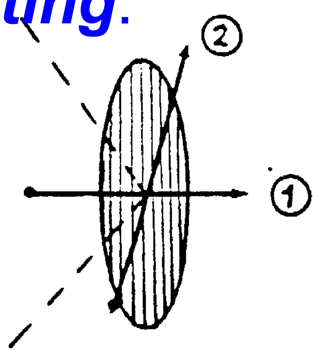
$\bar{V}$ = visibility

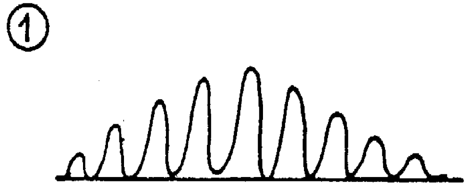
$$\bar{V} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

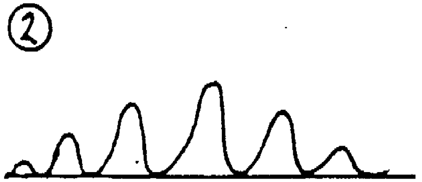
$$SNR = 4 \times 10^{11} \frac{\eta_q P_o}{\Delta f} \left[\frac{D_a}{r_a} \frac{D_{e^{-2}}}{f} \right]^2 d_p^2 \bar{G} \bar{V}^2$$

Directional Sensitivity

- The ***simple*** dual-beam LDA does ***not*** allow the direction of the particle to be determined. In addition the ***measurable turbulence level*** is very ***low***.
- The signal processor may require a ***minimum number of Doppler periods for validation***, thus restricting particle trajectories to a certain range of angles. This leads to a biased velocity.
- Both the directional sensitivity and the problem of measuring high turbulence levels can be resolved using ***frequency shifting***.

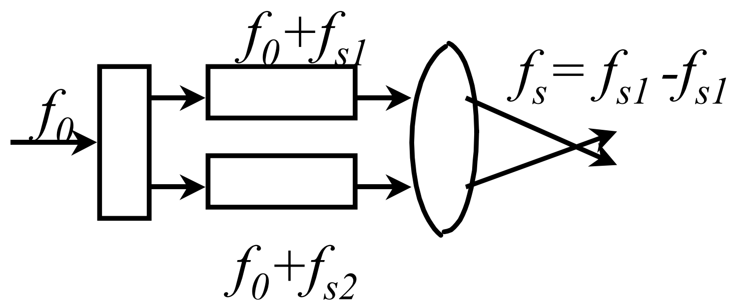






Frequency Shifting (I)

- The concept of frequency shifting involves producing a ***frequency difference*** between two LDA beams. This can be achieved by shifting the frequency of one beam or of both but different amounts.
- The detected frequency will be larger or smaller than f_s depending on the sign of the velocity.



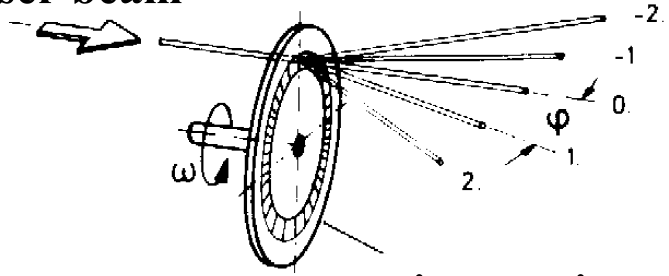
the detector sees

1

$$f = \frac{2\sin\varphi}{\lambda} U_{\perp} + f_s \frac{1 - \frac{\vec{U}\cdot\vec{l}_1}{c}}{1 - \frac{\vec{U}\cdot\vec{l}_2}{c}}$$

Frequency Shifting (II)

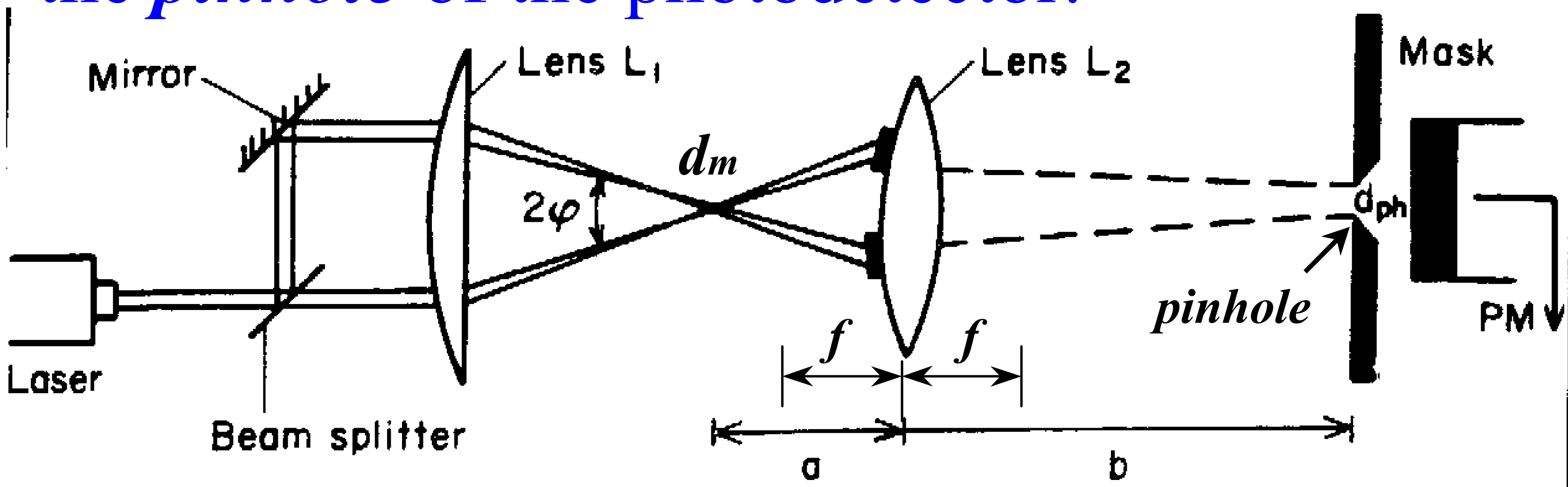
- Particle with $U_{\perp}=0$ still produce signals. By choosing f_s correctly there will be sufficient Doppler periods to allow validation by the processor.



- **Methods of frequency shifting**:
  - **rotating grating** (mechanical): simple, inexpensive, moderate accurate,$f_s=n\quad N$ <15MHz
  - **Pockel's cell**: produces transient shift magnitude
  - **Kerr cell** (electro-optical): processing complex
  - **Bragg cell** (AOM): highly stable, accurate, relative high shift frequency

Receiving Optics (I)

⦿ The ***effective mcv*** is the volume imaged onto the ***pinhole*** of the photodetector.



Magnification factor : ***M= b/a***

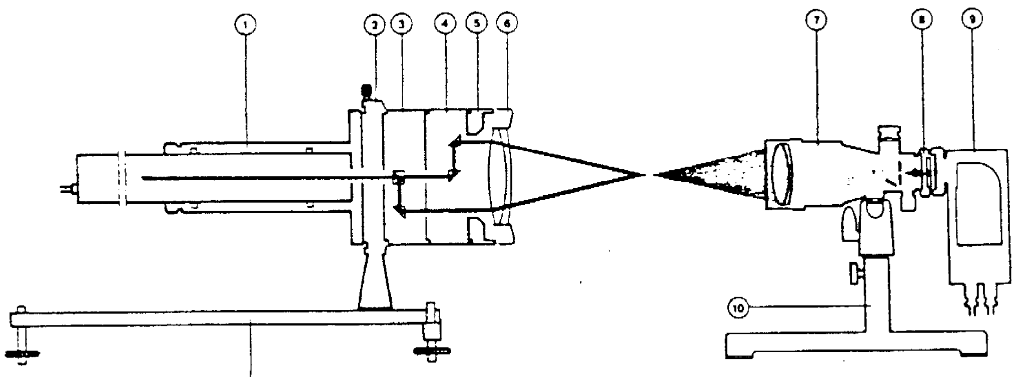
Effective no. of fringes :

$$N_{fr}' = N_{fr}\, d_{ph} / (M\, d_m)$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

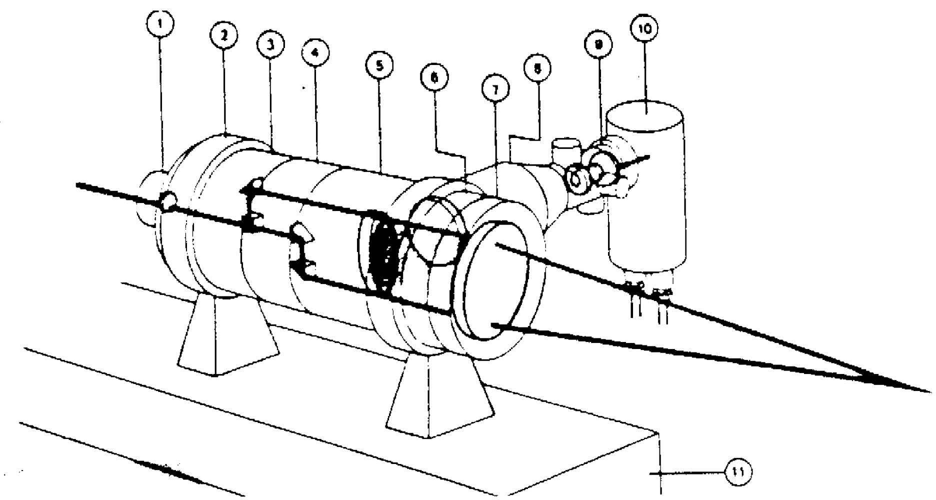
Receiving Optics (II)

⦿ *Forward Scattering*



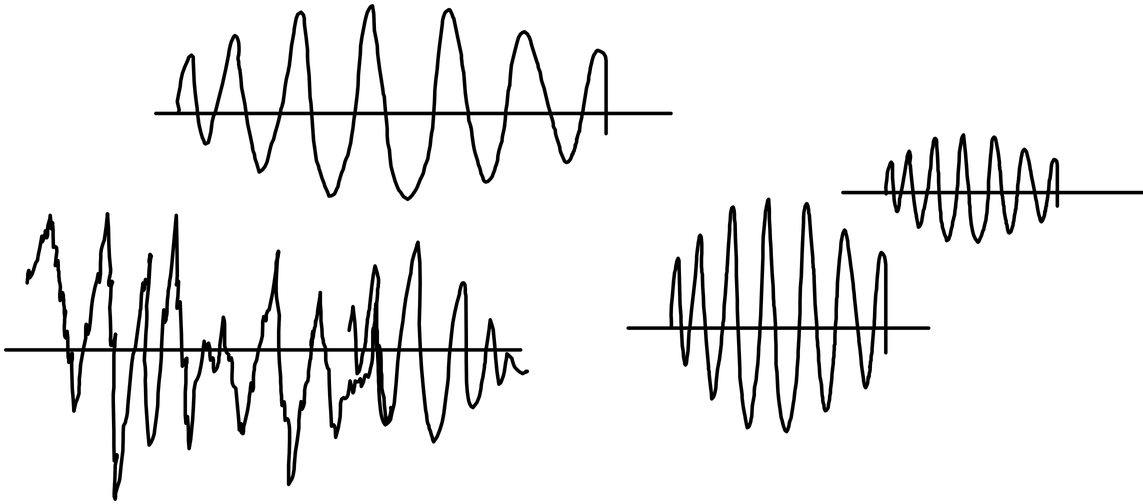
- Good ***SNR***
- Low laser power is needed
- more complex traversing rig

⦿ *Backward scattering*



- Only one optical acess necessary
- Self-adjusting
- more laser power needed

Light scattering, Signal processing
&
LDA Applications



Required Properties of Particles

Suitable tracer particles used for LDA measurements should have the following properties:

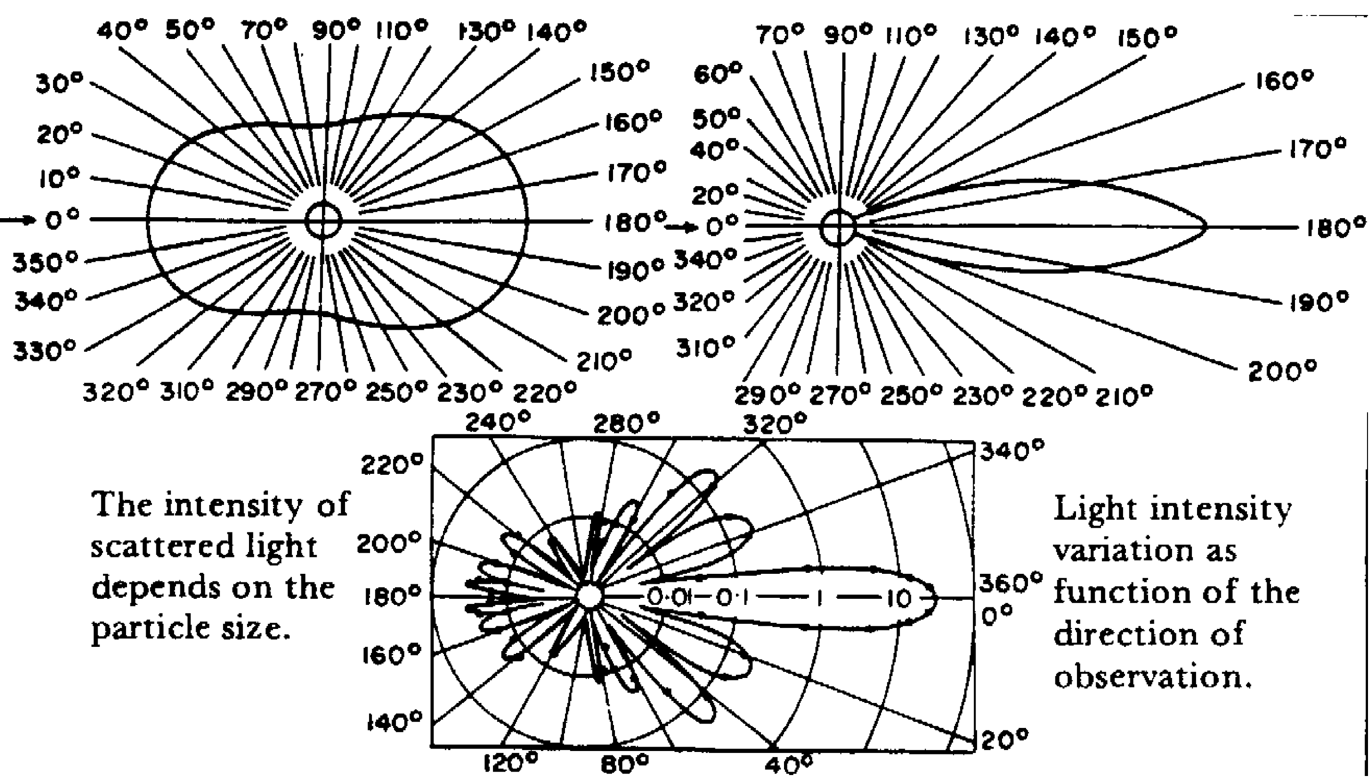
- small slip velocity
- good scattering properties to yield high signal strength
- good produceability of particles
- cheap
- chemically inactive
- non-toxic

Light Scattering from particles (I)

- The light scattering phenomena is described by the *Mie-scattering theory* for spherical particles.
- *Mie parameter:* $q = \dfrac{2 r_p \pi}{\lambda}$ and *m* (refractive index)
- The *intensity of scattered light* depends on:
  - incident intensity
  - wavelength (λ)
  - particle shape , particle size (r_p), particle concentration and particle distributions
  - index of refraction of particle
  - scattering angle

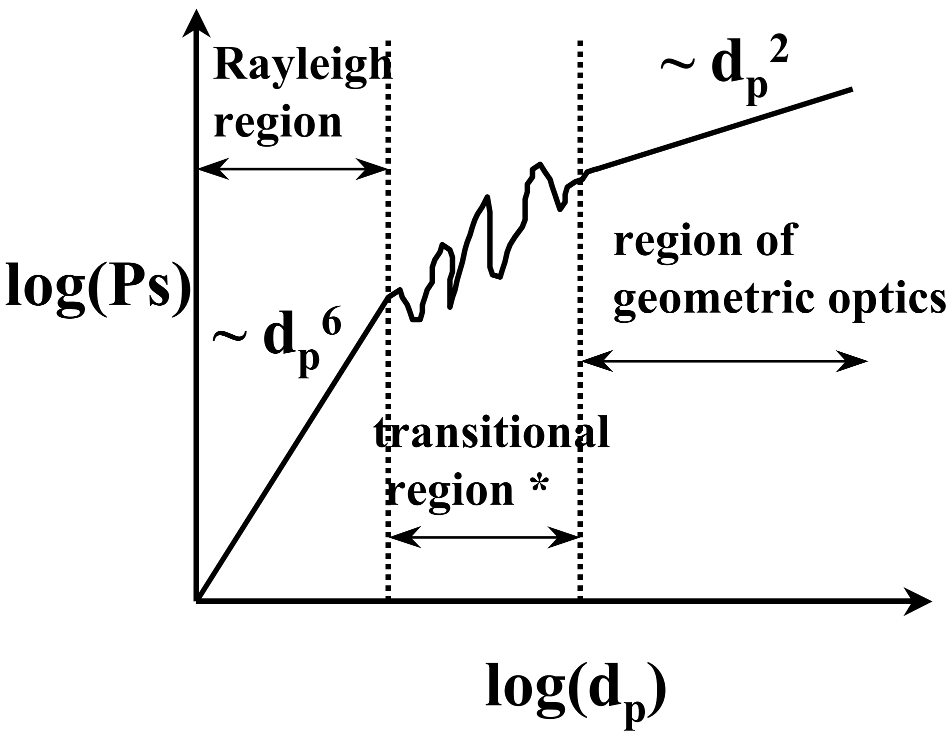
Light Scattering from particles (II)

$$q = \frac{2r_p\pi}{\lambda}$$

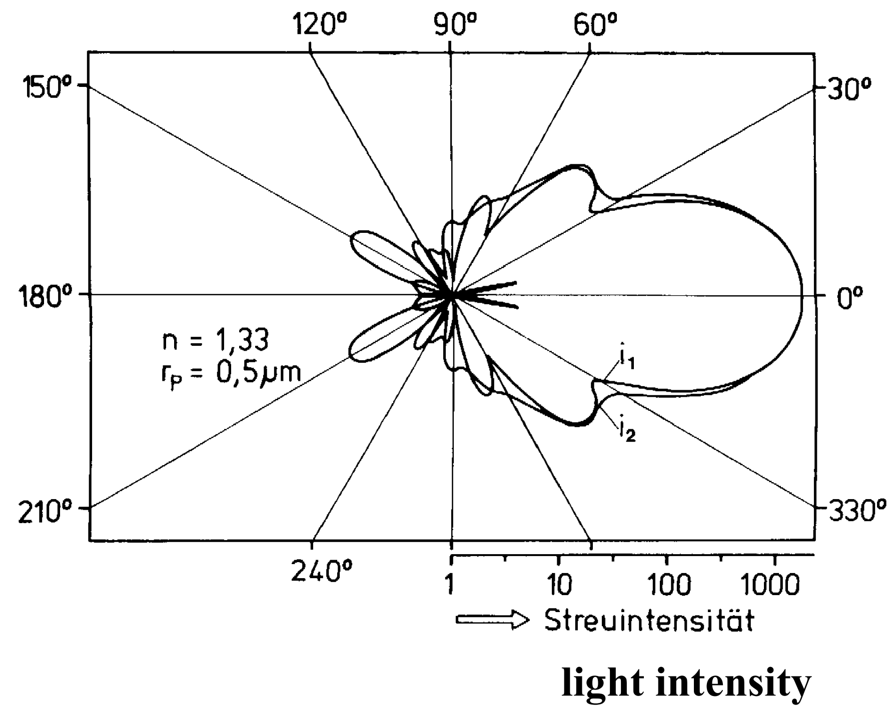


For a given system, the signal intensity is several orders of magnitude larger in forward scattering arrangement.

Light Scattering from particles (III)



(* for water $\mathbf{d_p}$ ~ 0.5 - 3µm)



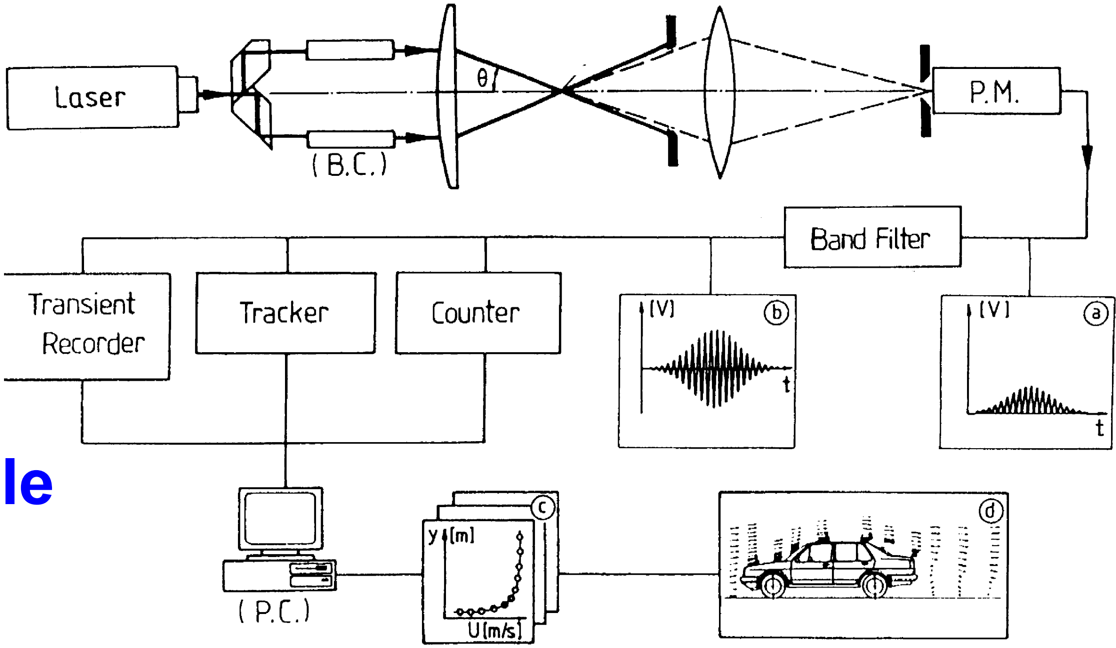
(for water droplet $\mathbf{d_p}$ =1 µm)

Signal Processing Tasks (I)

- The signal processing has the broad task of extracting fluid mechanics information from the Doppler signals. This entails
  - signal conditioning
  - determination of Doppler frequency

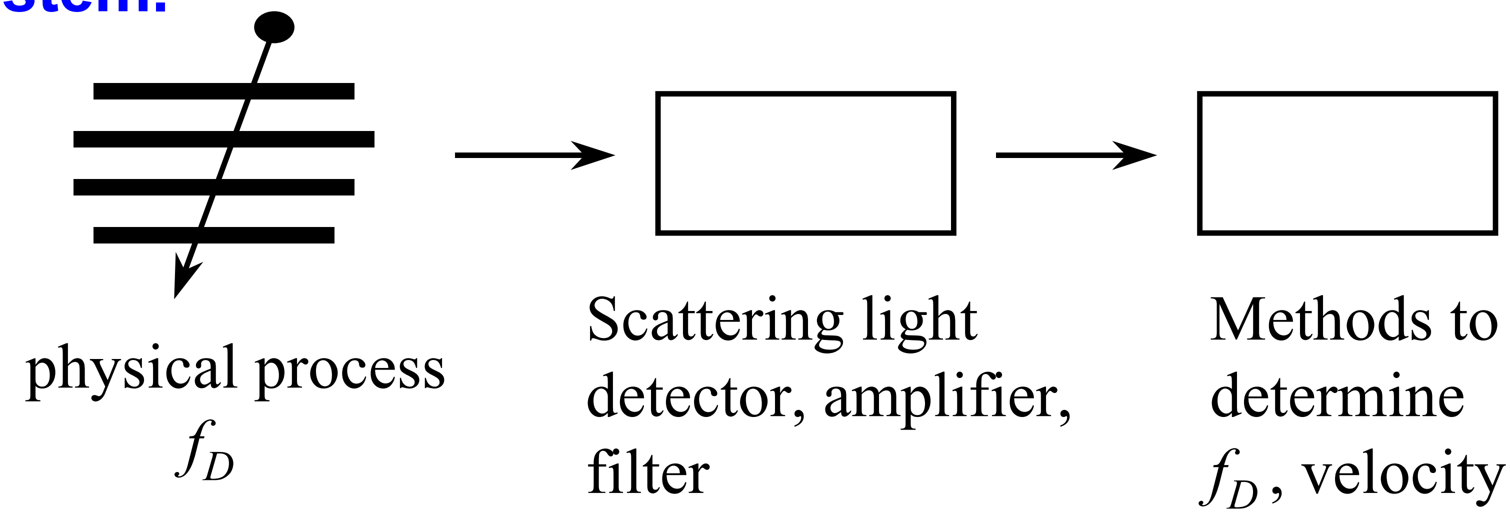
$$U_{\perp} = \frac{\nu_D \lambda}{2 \sin \varphi}$$

  - computation of statistics and possible coordinate transformations.



Signal Processing Tasks (II)

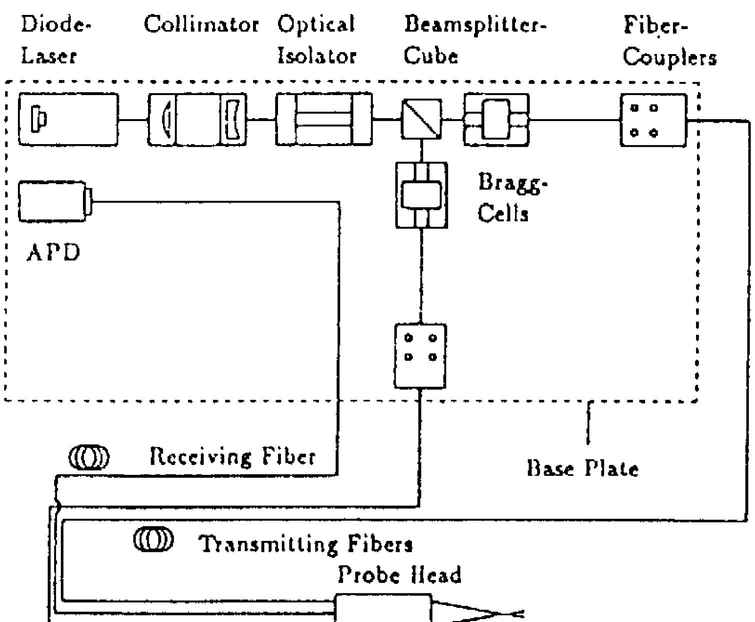
- **The signal processor is *not* an *independent* part of a LDA system.**



- ***Don't* expect to *improve* your signal by signal processor.**
- **There are several (either time domain-based or frequency domain-based) instruments available to process the LDA-signal.**

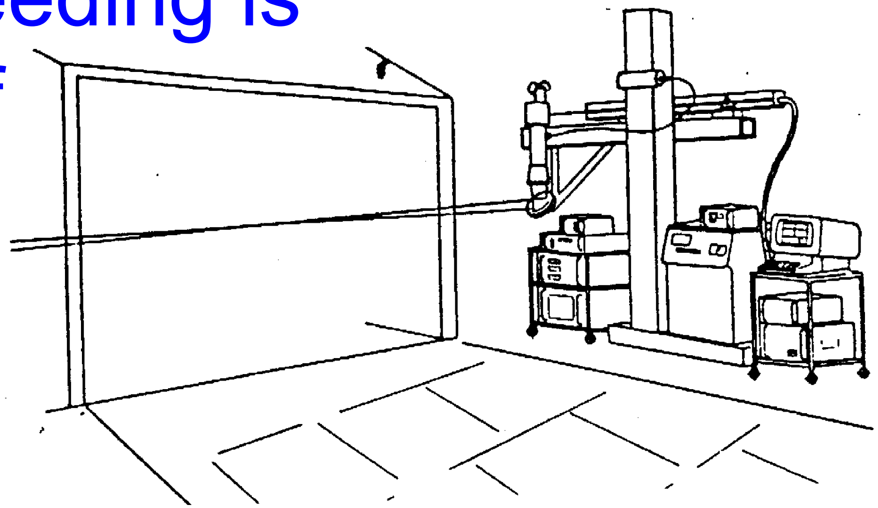
Diode Laser Fiber-optic LDA

- The whole system is handy, compact and robust with minimum adjustment necessary.
- The entire optical system: 500mm x 175mm (LxW)
- 100mW, 830nm, low power consumption (~250W)



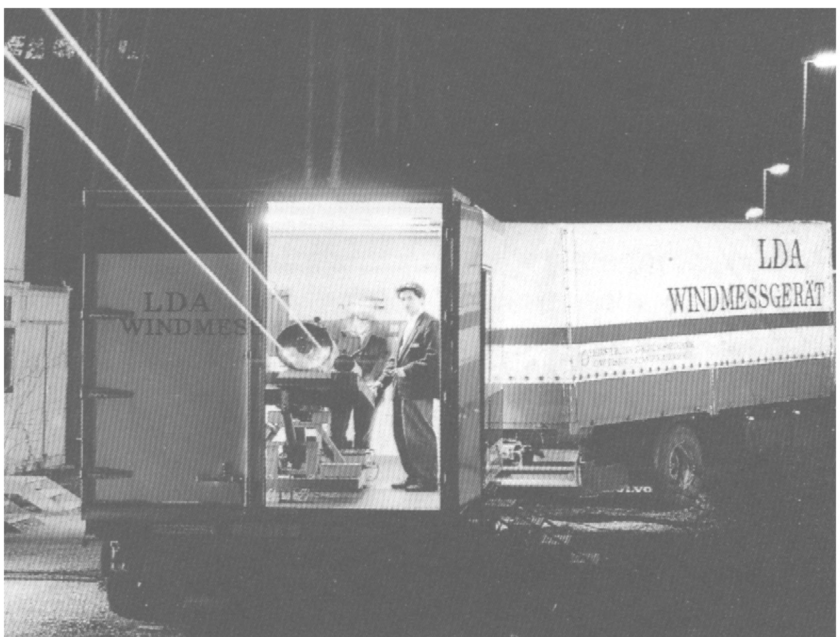
Wind Tunnel LDA

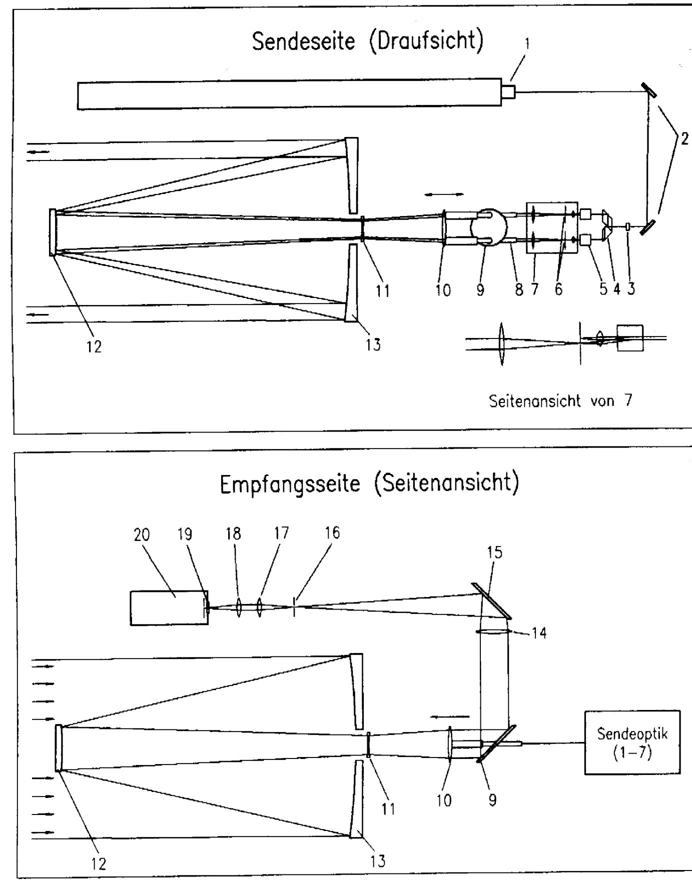
- There are generally additional constraints and demands, such as: **seedings, long focal length, special traversing mechanism** ...etc. for LDA used in wind tunnel.
- Little or no particle seeding is expected, because of
  - tunnel contamination
  - flow disturbance
  - mcv is too large



Applications of LDA (I)

- **Long range wind velocity measurements (LSTM, FAU)**
- **150~300m Zoom-optics, 10W (0.514nm) SP-laser 2030**

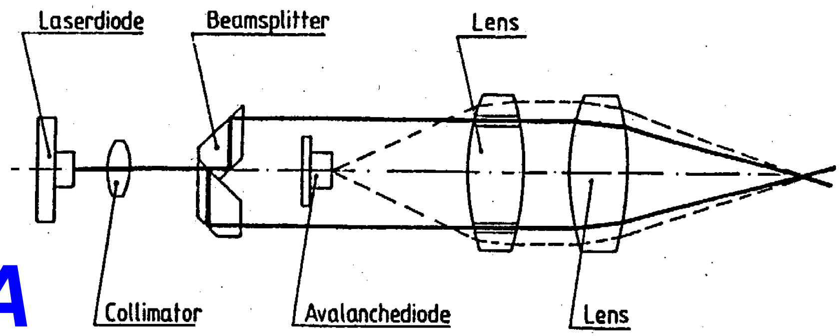


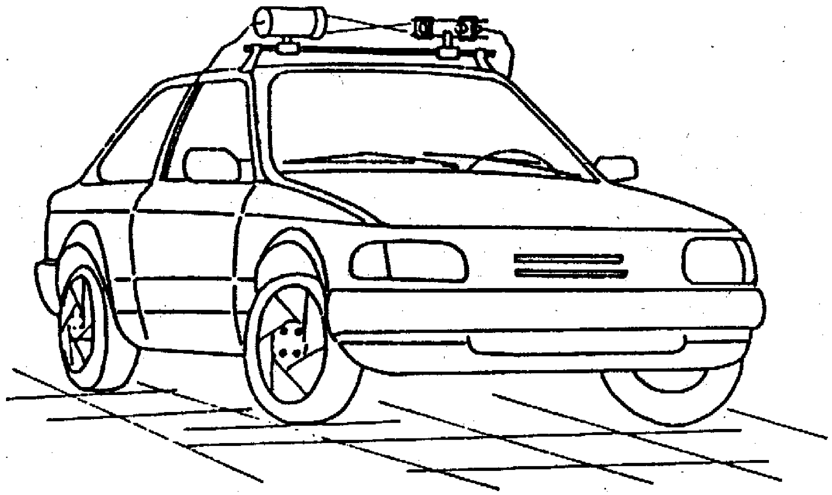


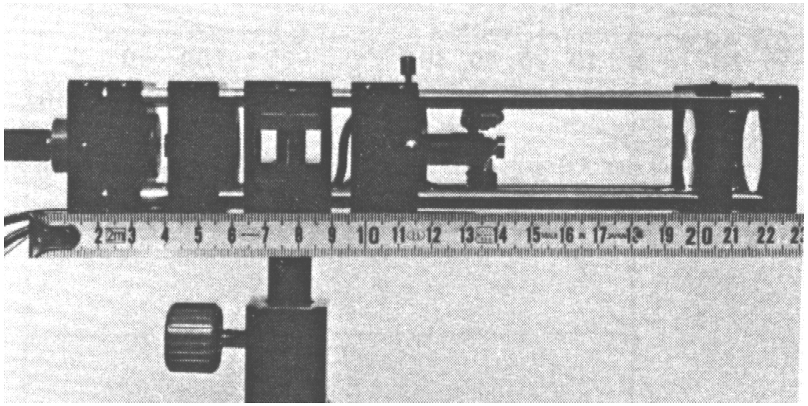
1 Argon-Ionen-Laser
2 Umlenkspiegel
3 λ/2-Plättchen
4 Strahlteiler
5 Bragg-Zellen
6 Lochblenden
7 Strahlaufweitung
8 Teleskopleitrohre
9 durchbohrter Spiegel
10 Zoomlinse
11 Negativlinse
12 Fangspiegel
13 Hauptspiegel
14 Abbildungslinse
15 Umlenkspiegel
16 Lochblende
17 Justierlinse
18 Justierlinse
19 Interferenzfilter
20 Photomultiplier

Applications of LDA (II)

- ⊙ *wind velocity measurements (LSTM, FAU)*
- ⊙ *Semiconductor LDA*

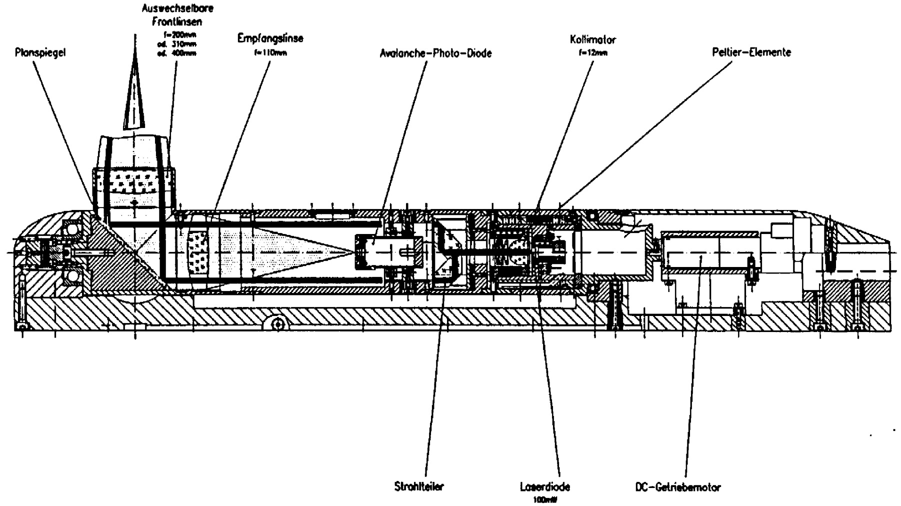


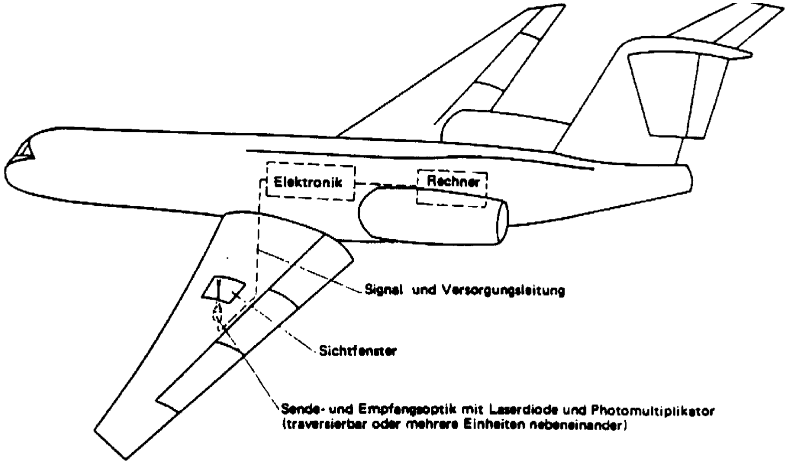


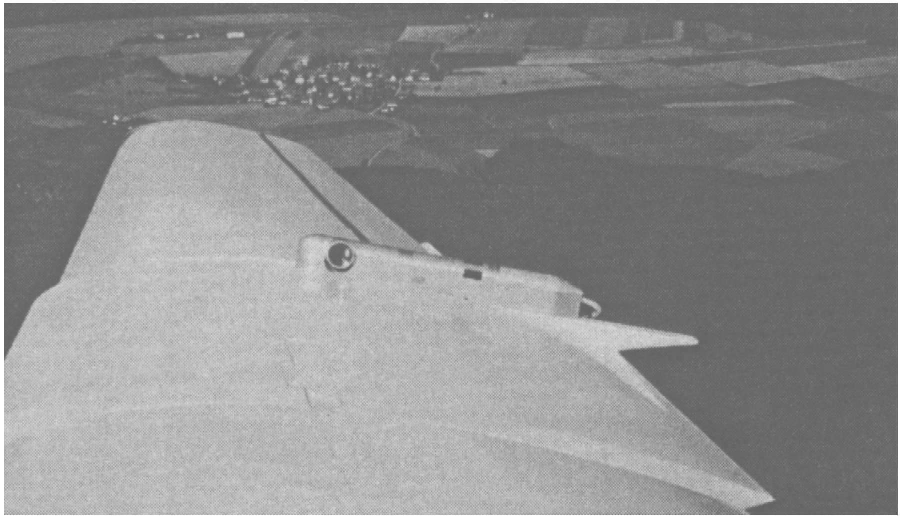


Applications of LDA (III)

- ⊙ ***Wind measurements in the sky*** ***(LSTM, FAU)***

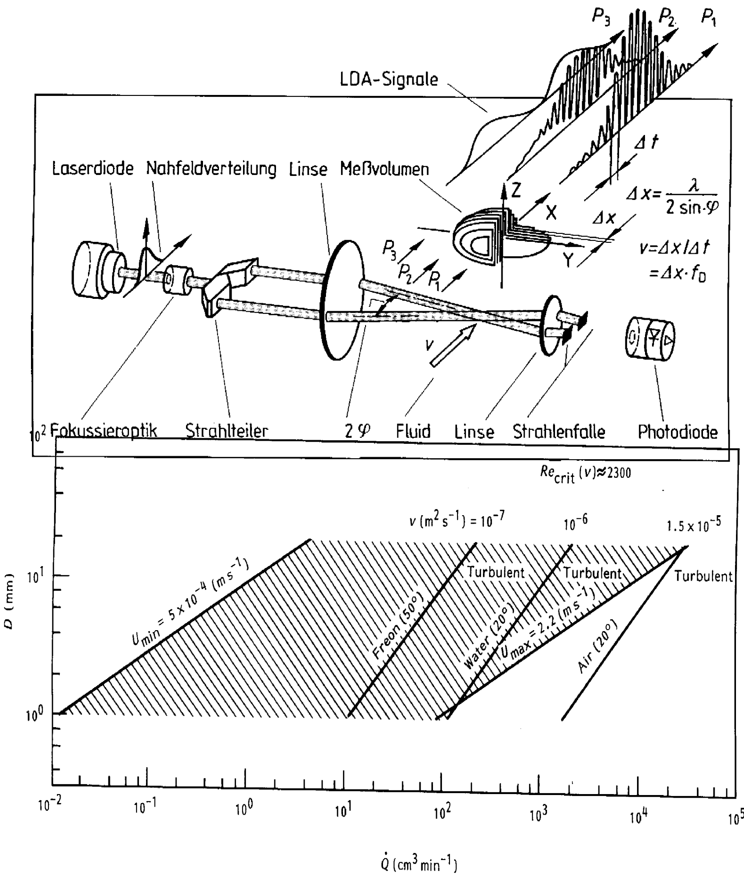


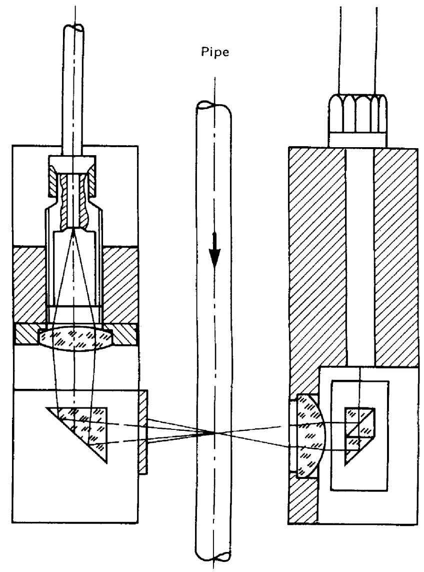


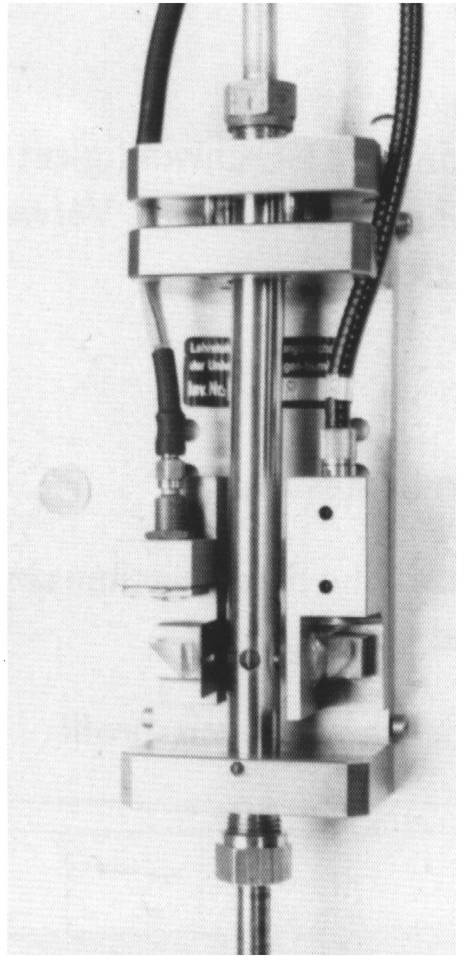


Applications of LDA (IV)

- Q = 0.05 cm³/min, range: 1:6500

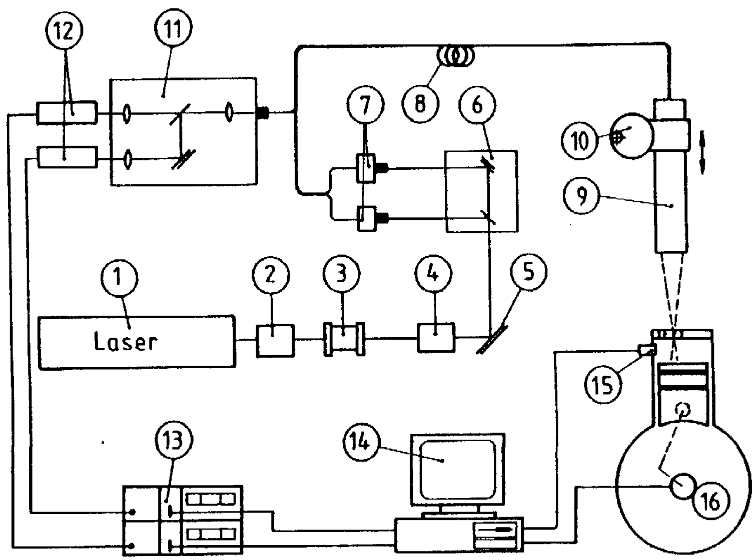




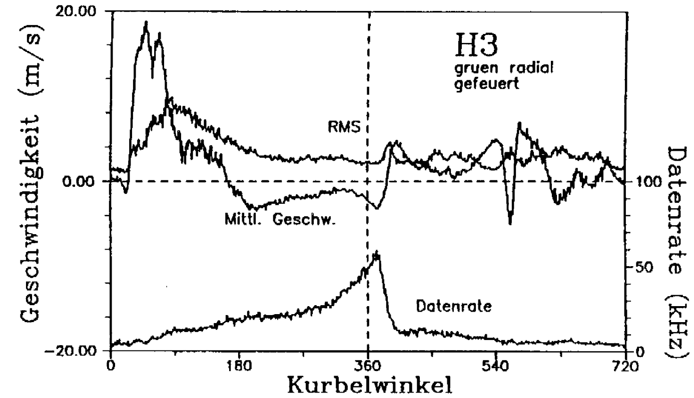


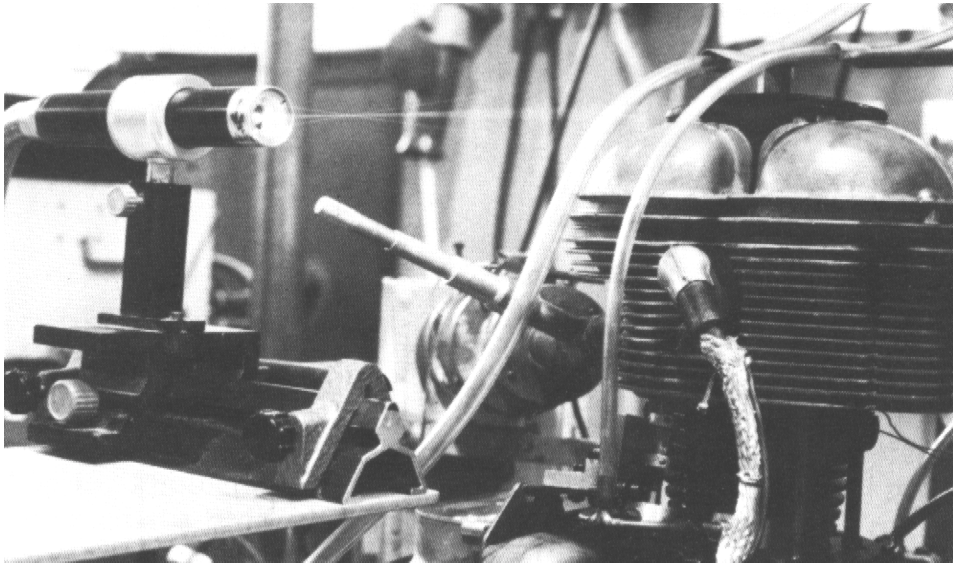
Applications of LDA (V)

⊙ *BMW-403 motor measurements*



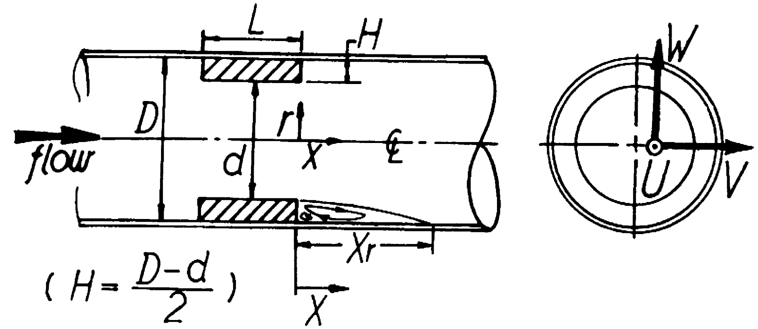
① Laser, ② Braggzelle, ③ Kollimator, ④ Farbtrennung (AMICI-Prisma), ⑤ Spiegel, ⑥ Umlenkung ⑦ Einkopplung, ⑧ Glasfasern, ⑨ Meßkopf ⑩ Traversierung, ⑪ Farbtrennung des Streulichtes, ⑫ Avalanche Photodioden, ⑬ Frequenzzähler, ⑭ Computer, ⑮ Drucksensor, ⑯ Kurbelwinkelgeber

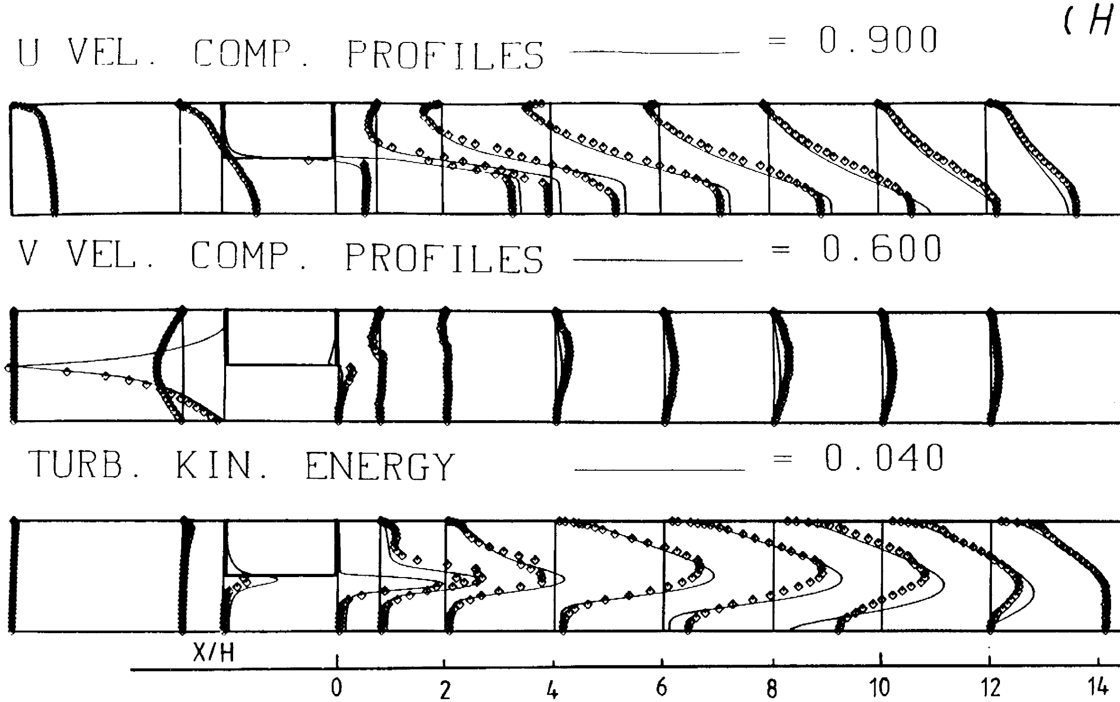


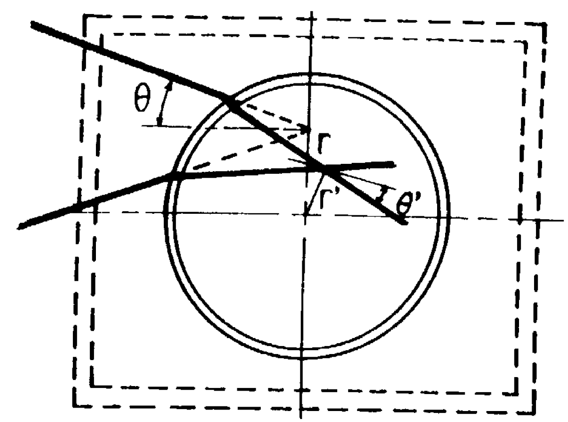


Applications of LDA (VII)

- **Basic research of separated flows**

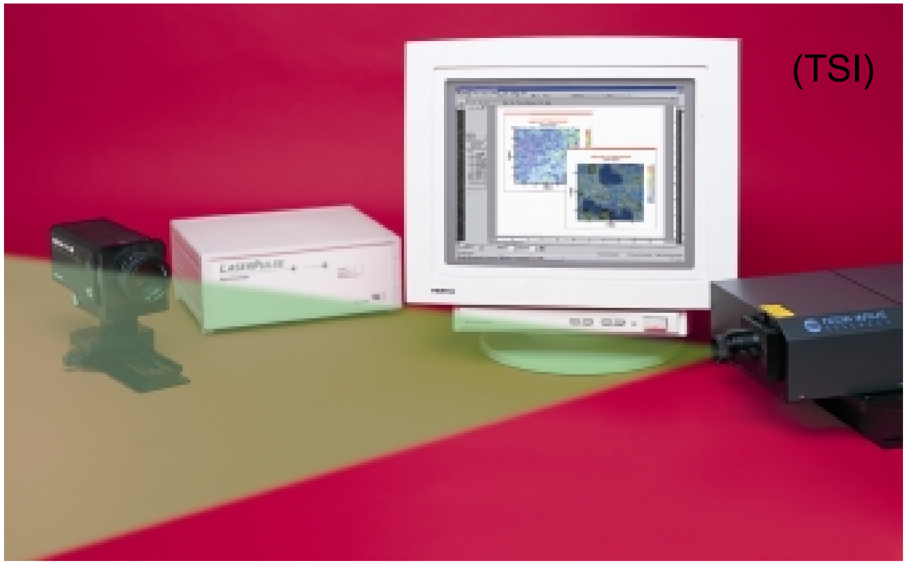


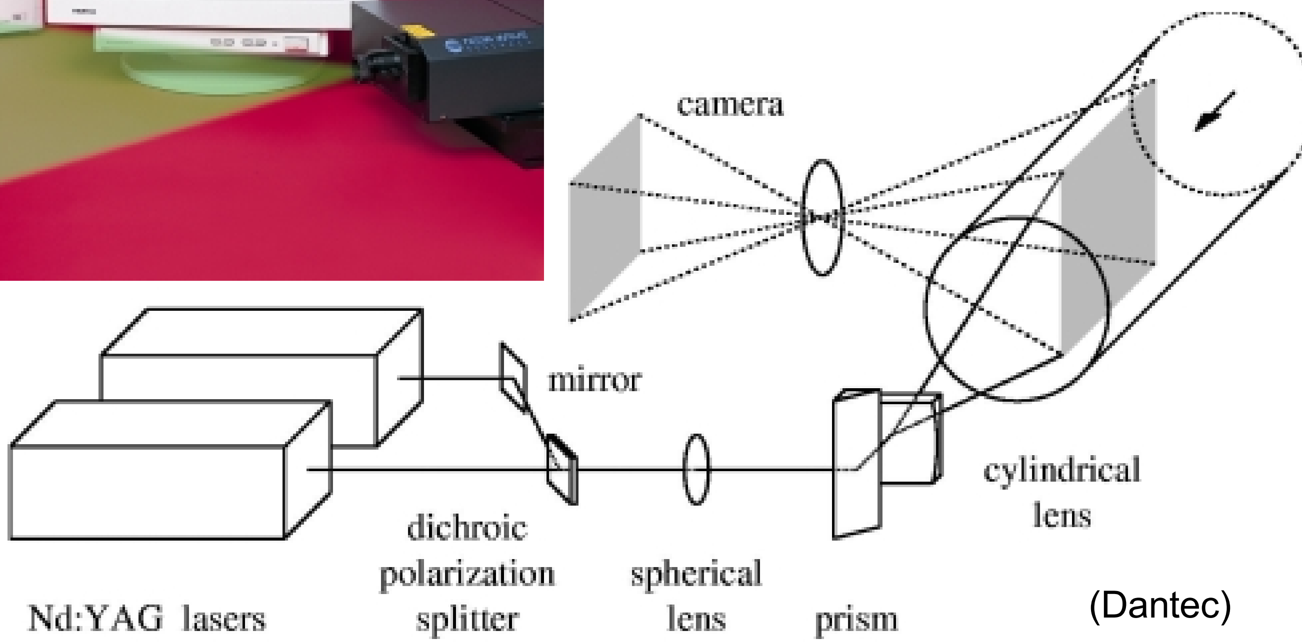




Introduction to Particle Image Velocimetry (PIV)

PIV system





(Dantec)

Introduction to PIV

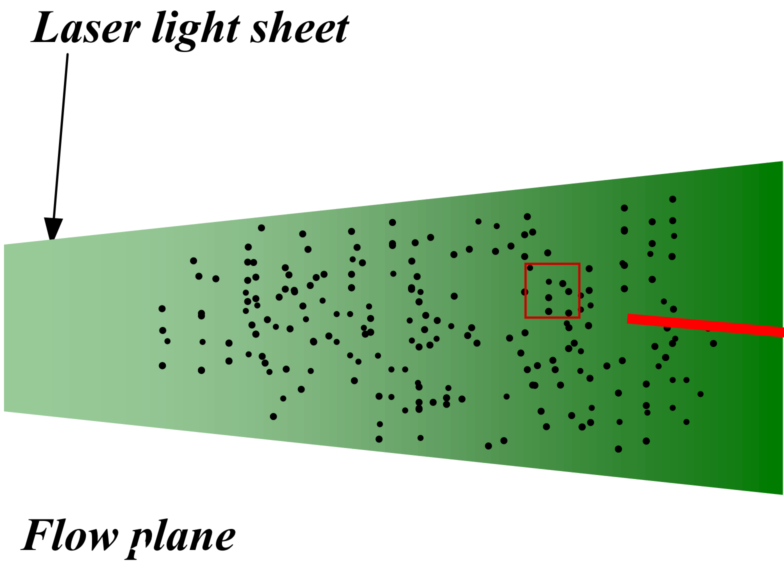
- First commercial PIV in 1988
- PIV is a quantitative flow visualization by using an optical method to measure fluid velocity at many points in a flow field simultaneously.
- Similar techniques:
  PTV: particle tracking velocimetry, offers lower
  accuracy and resolution for low seeding density.
  LSV: laser speckle velocimetry,

Characteristics of PIV

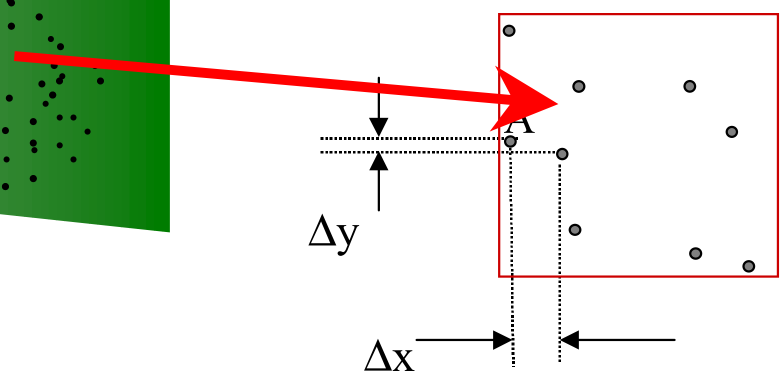
- **Advantages:**
  - provides instantaneous velocity vectors in detail (for flow structure, especially for turbulence).
  - provides spatial gradients of instantaneous and average flow properties for many points
  - Ideal for unsteady or periodic flows
  - obtain global nature of flows

- **Disadvantages of PIV:**
  - expensive cost
  - seeding
  - small measuring region

Principle of PIV



- ➔ Illuminate a seeded flow twice in succession
- ➔ Record the images using a CCD or film camera
- ➔ Digitally process the images to obtain a 2D velocity field frozen in time



Δt - time between two pulses

Δx - particle displacement in x direction

Δy - particle displacement in y direction

Velocity of particle A

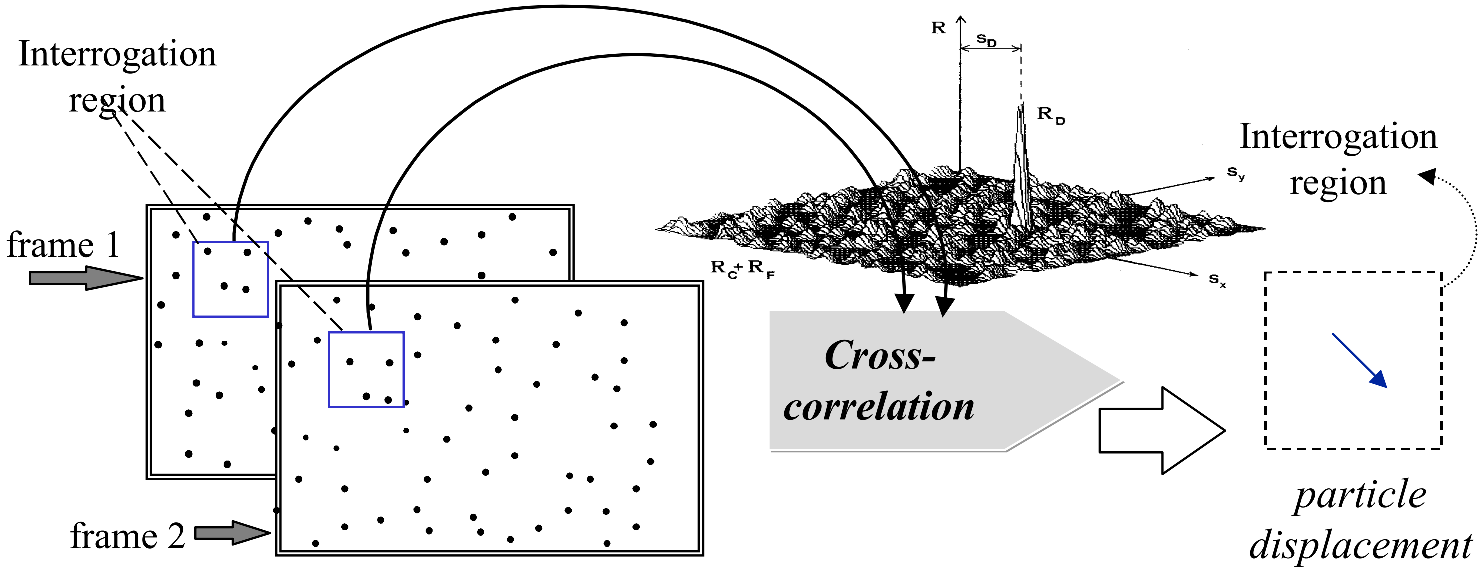
$u_x = \Delta x/\Delta t$ as $\Delta t \rightarrow 0$

$u_y = \Delta y/\Delta t$ as $\Delta t \rightarrow 0$

Data processing of PIV

- **The image displacements are obtained by doing the spatial crosscorrection or spatial autocorrection of the image intensity field.**
- **Spatial cross-correlation:**
  - **- Particle images from each laser pulse is on separate frames**
  - **- No directional ambiguity (because sequence of frames is known)**
  - **- Dynamic range can be greater than 100 to 1**
  - **-** Robust algorithm - can detect lower signal quality
- **Spatial Autocorrelation**
  - **- Double or multiple pulses on each frame**
  - **- Directional ambiguity**
  - **- Dynamic range may be up to 10 to 1**

Crosscorrelation Processing



Each frame contains particle images from one laser pulse.

Analysis by correlating the two image fields from separate video frames.

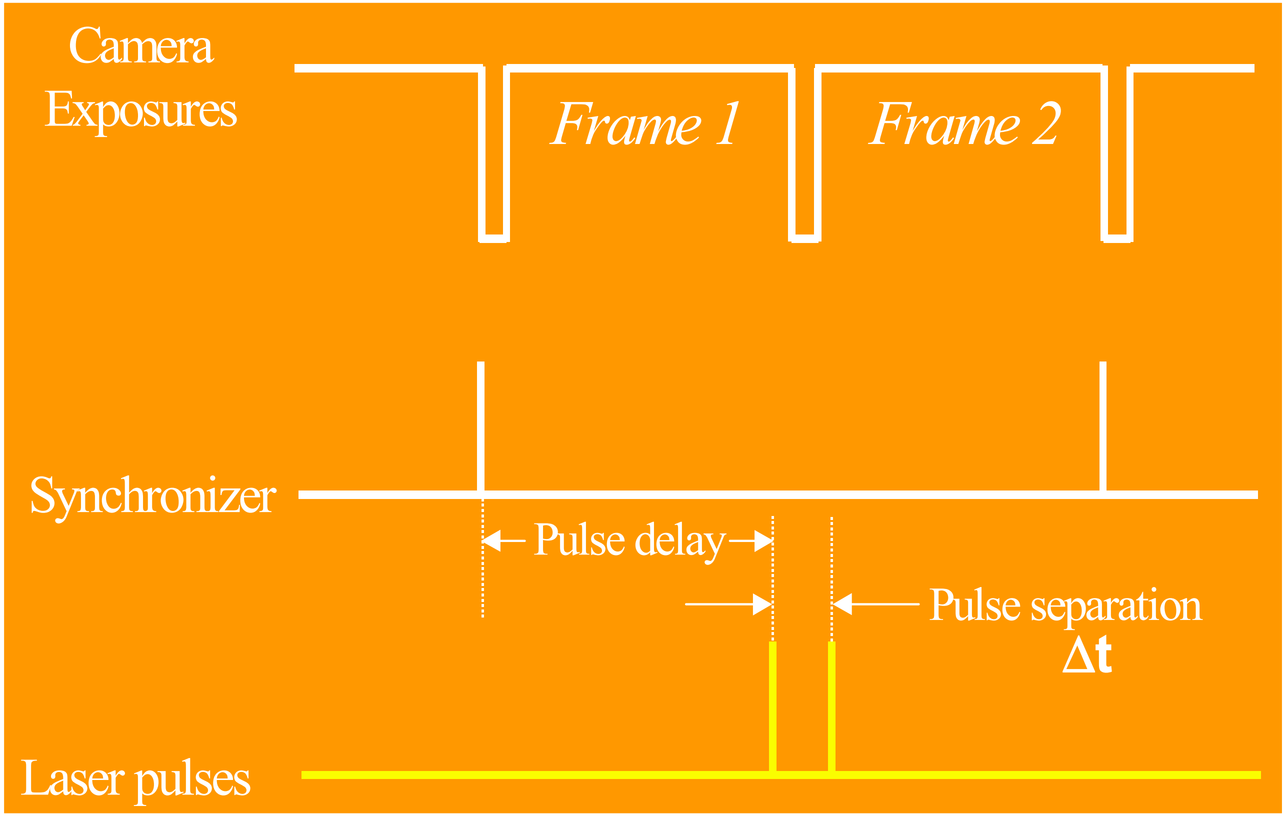
Advantages:

works very well to >400 m/sec (with specially developed cameras and frame straddling technique)

no additional hardware required to resolve flow direction

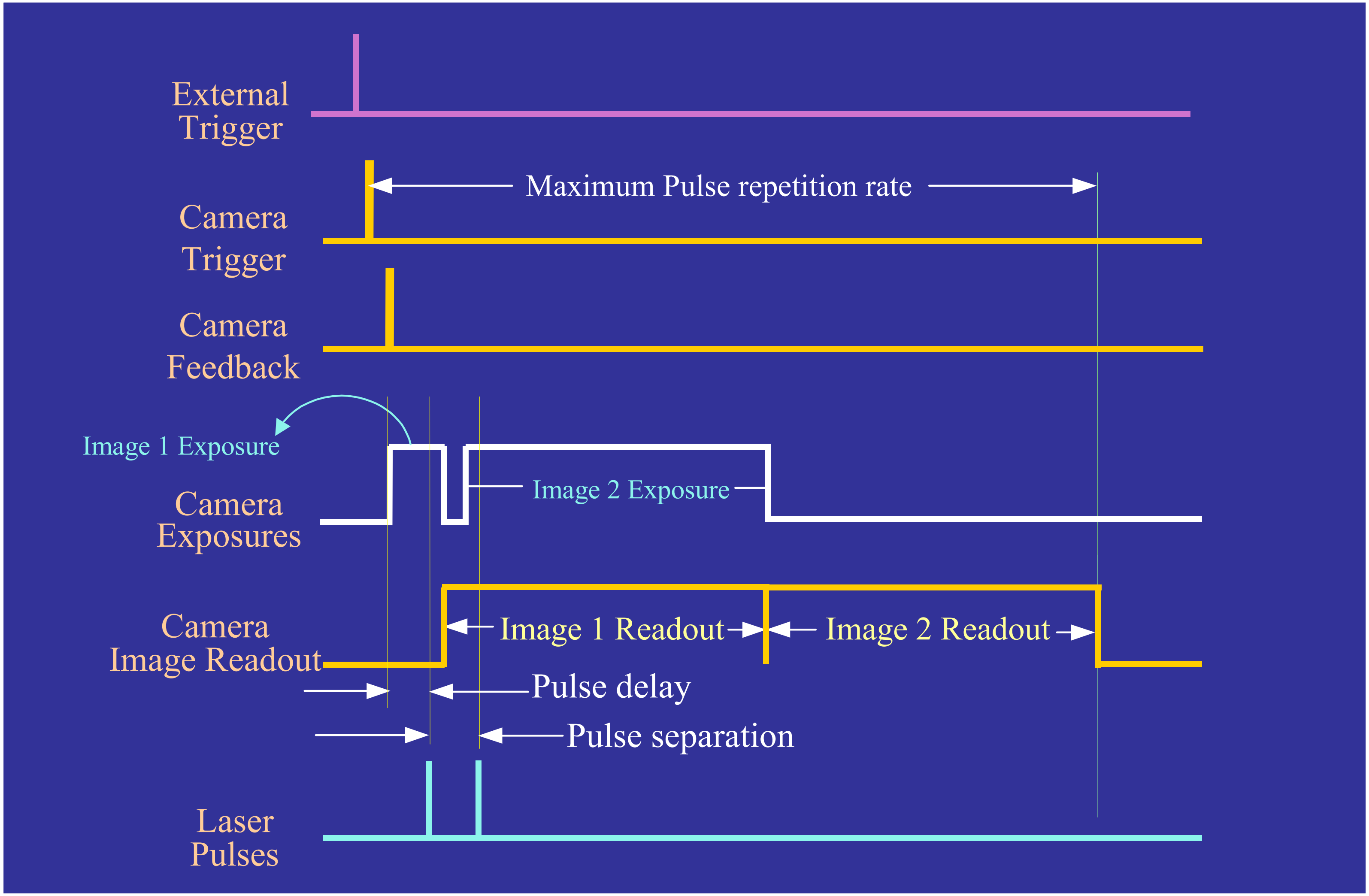
frames need not be successive (especially for measuring very low speed flows)

Frame straddle with free-run camera

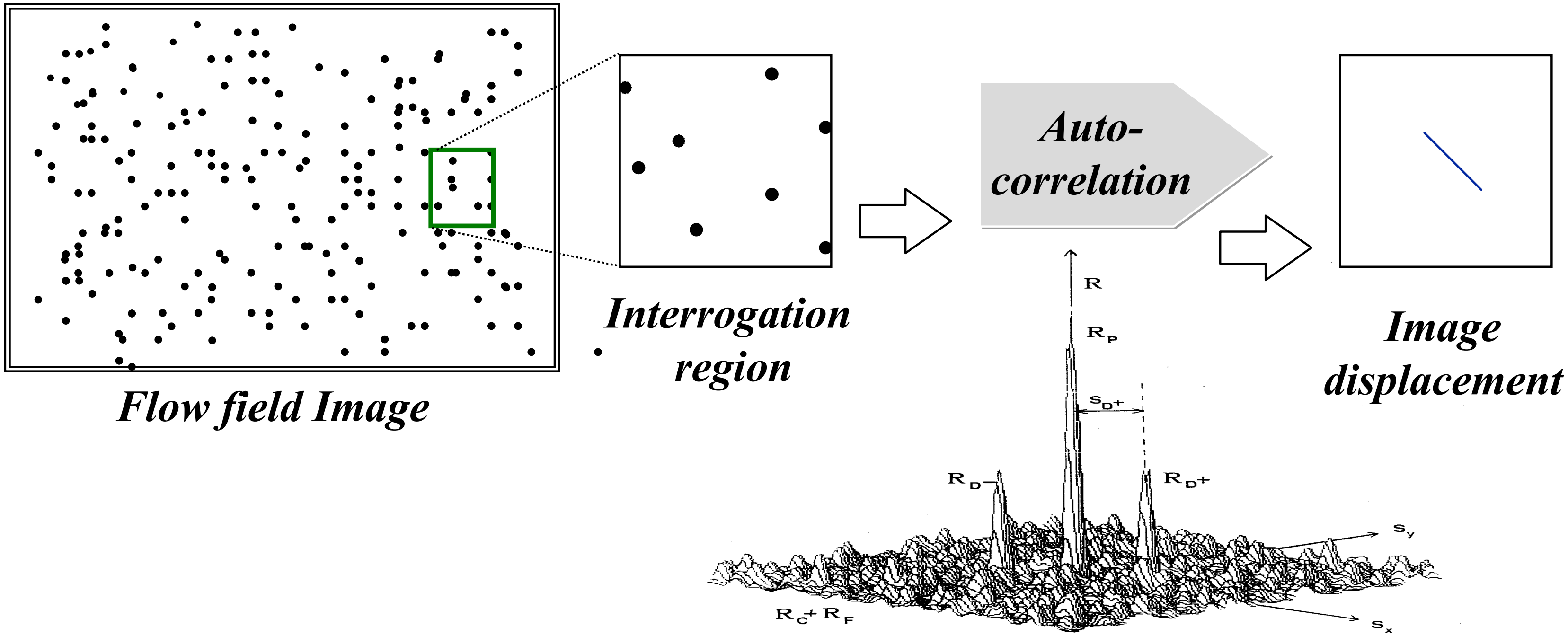


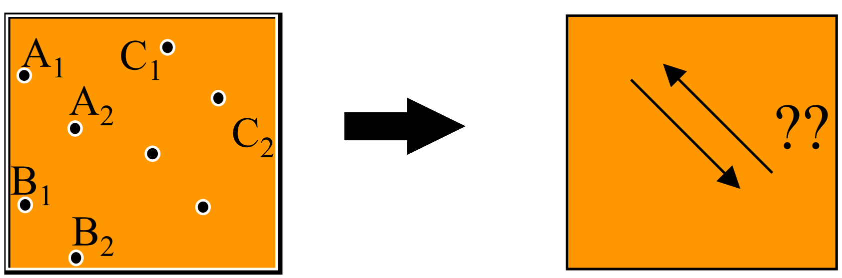
max. measuring velocity is function of image size and camera type

Frame straddle with external trigger



Autocorrelation Processing (I)





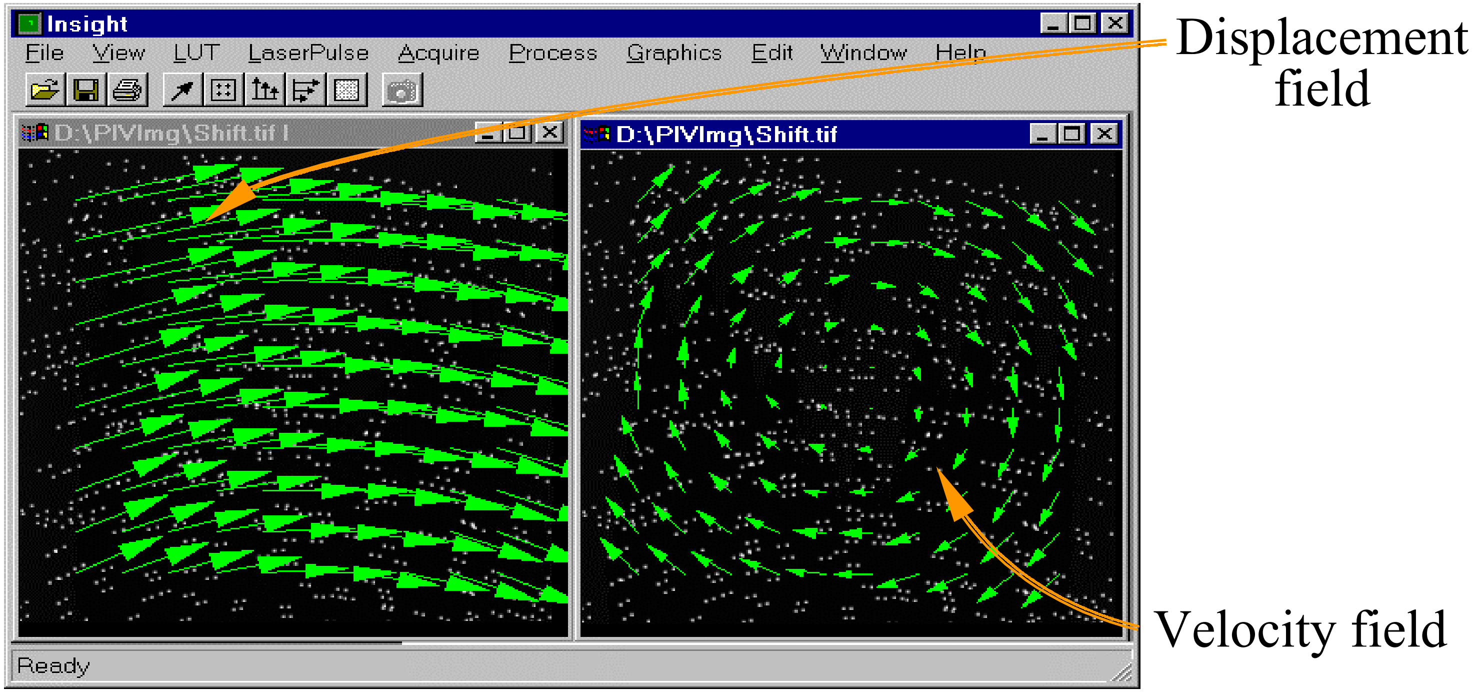
Directional ambiguity:

Is the flow direction from C_1 to C_2 or from C_2 to C_1 ?

Autocorrelation Processing (II)

Solution: using Image shifting to make all displacements to be positive

$$\text{Velocity} = \frac{\text{image displacement- image shift}}{\Delta t}$$



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