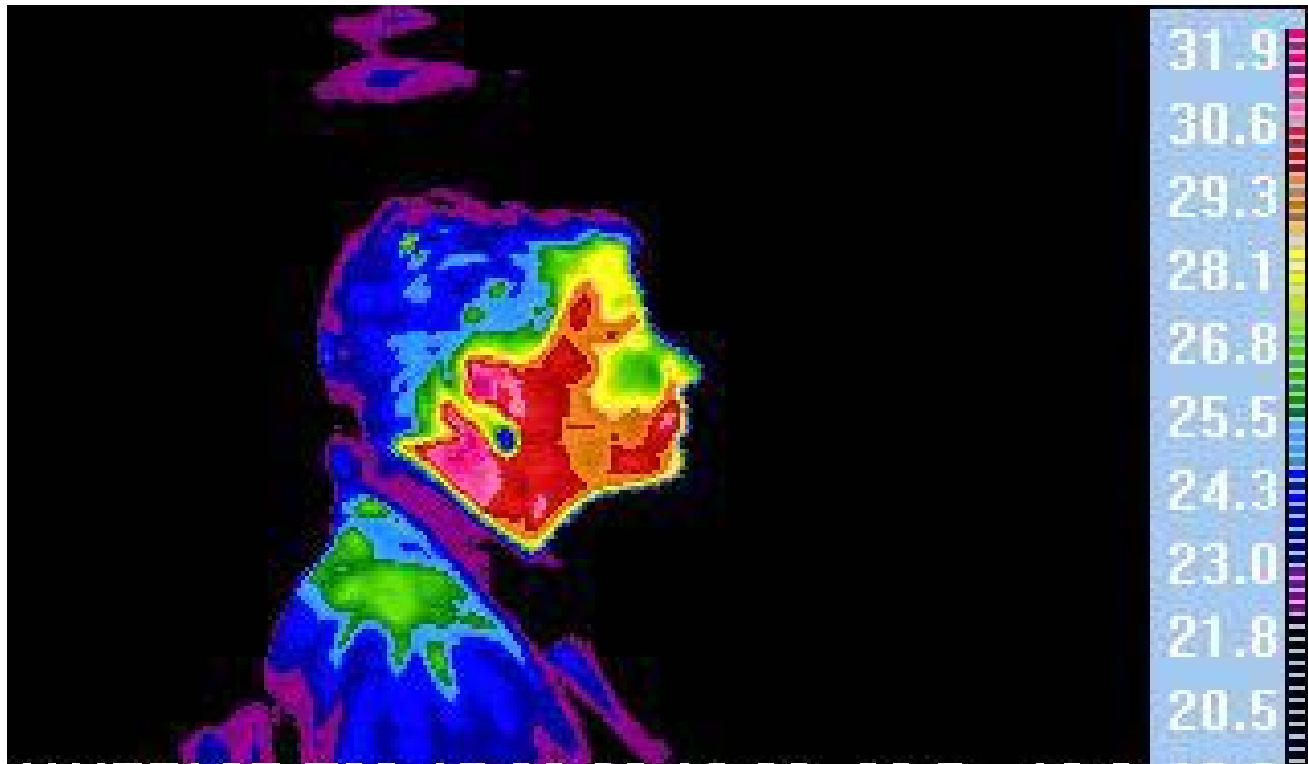


Thermal Infrared Systems

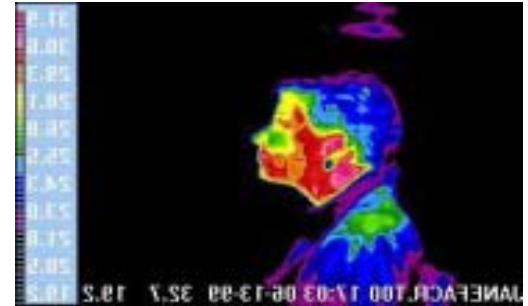
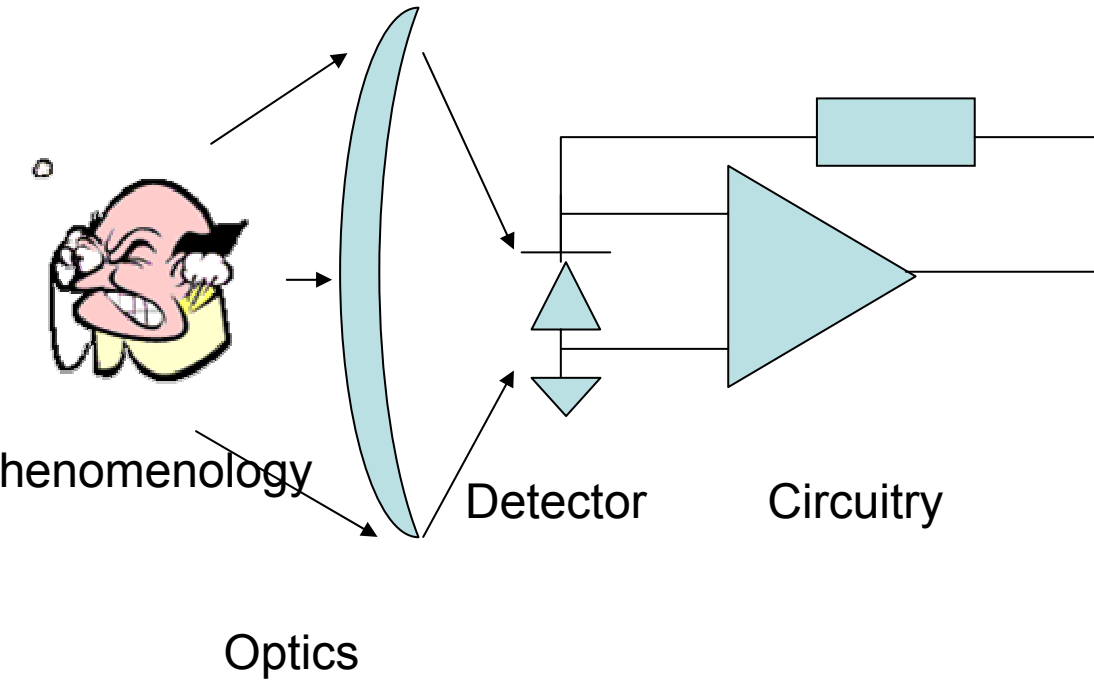
Lecturer: B T Yang 楊丙邨 March 2005 NTU



Lecture Outline

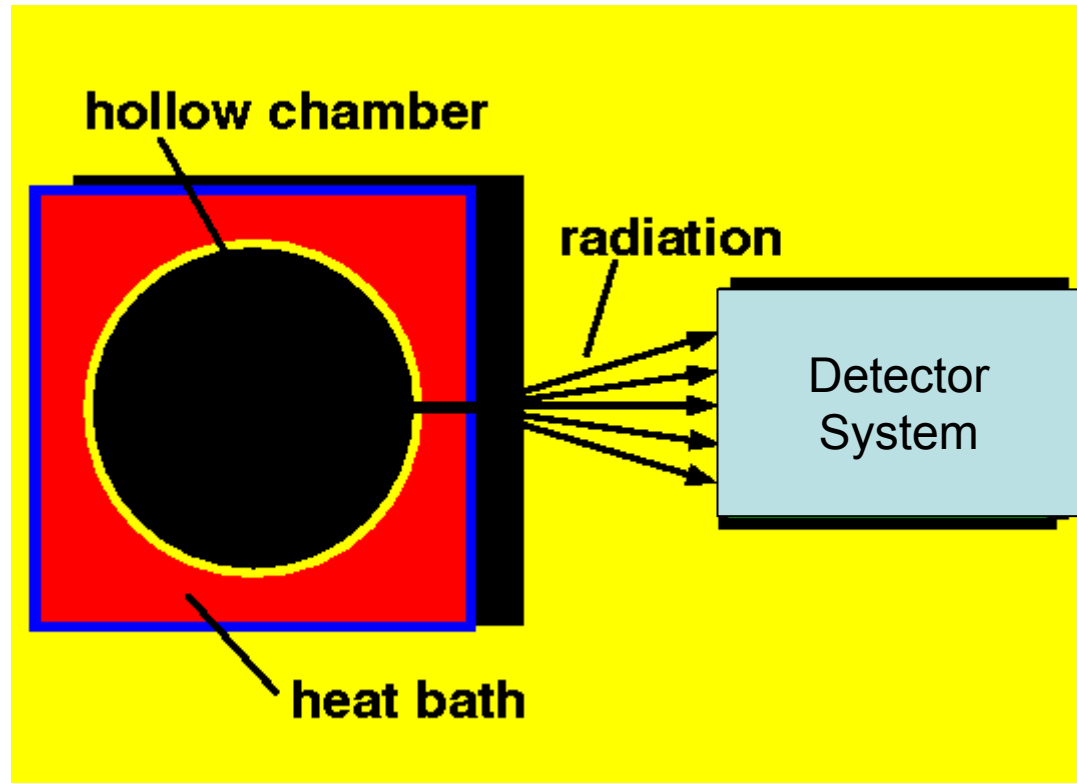
1. Phenomenology: “What is”
2. Optics
3. IR Detectors: Thermal, PC, PV
4. IR Detector Circuitry and Noises
5. IR Systems and Applications

Typical IR System



Signal Processing
and Display

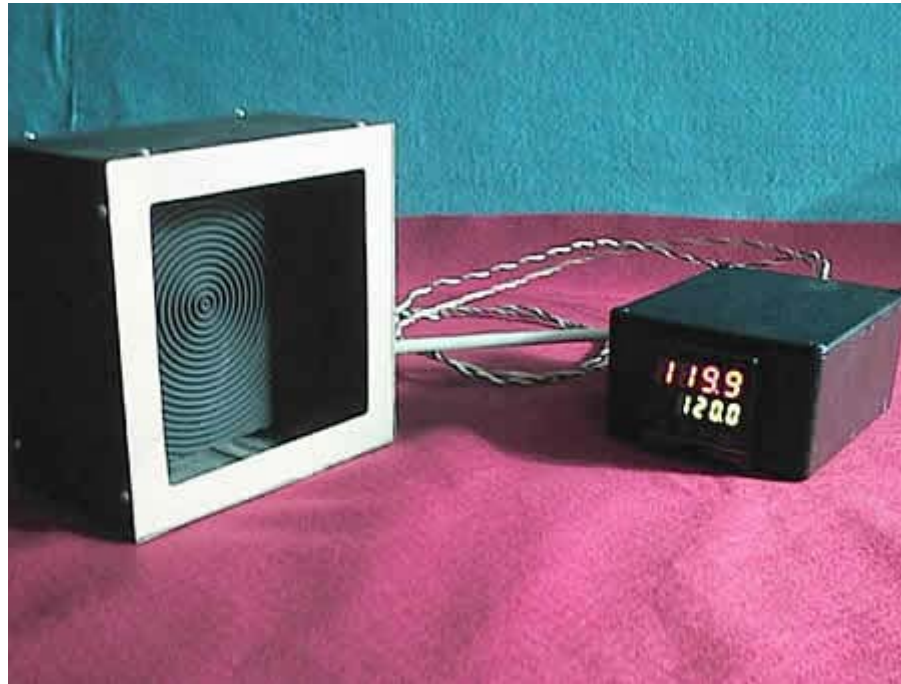
IR is Never Complete without Introducing the “Blackbody”



“Black” means No Light is Reflected, but “Light” can be emitted!

Grooved Planar Black Body Source

- “Grooved surface enhancing the emissivity



Absolute Blackbody Radiance Calibration Standard: Metal Freeze Temperature

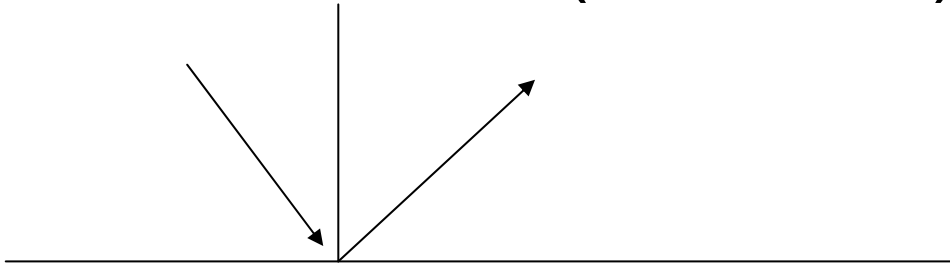
Pure Metal	Freeze Temp* (°C)	Pure Metal	Freeze Temp* (°C)
Gallium	29.7646**	Aluminum	660.323
Indium	156.5985	Silver	961.78
Tin	231.928	Gold	1064.18
Zinc	419.527	Copper	1084.62

Definition of a Black Body

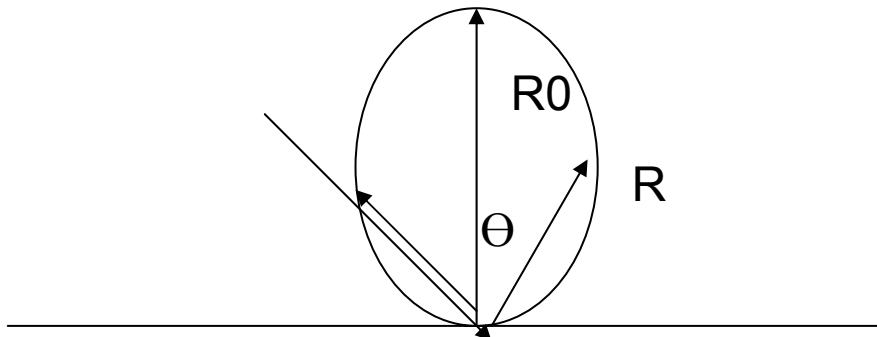
- **A blackbody absorbs all incident radiation; $r=0$**
- **At a given temperature, no surface can emit more energy than a blackbody**
- **A blackbody is a “diffuse” emitter that follows the “Lambertian Laws”**

Lambertian Law

- Specular Surface (reflective)



- Lambertian Surface (diffuse surface)

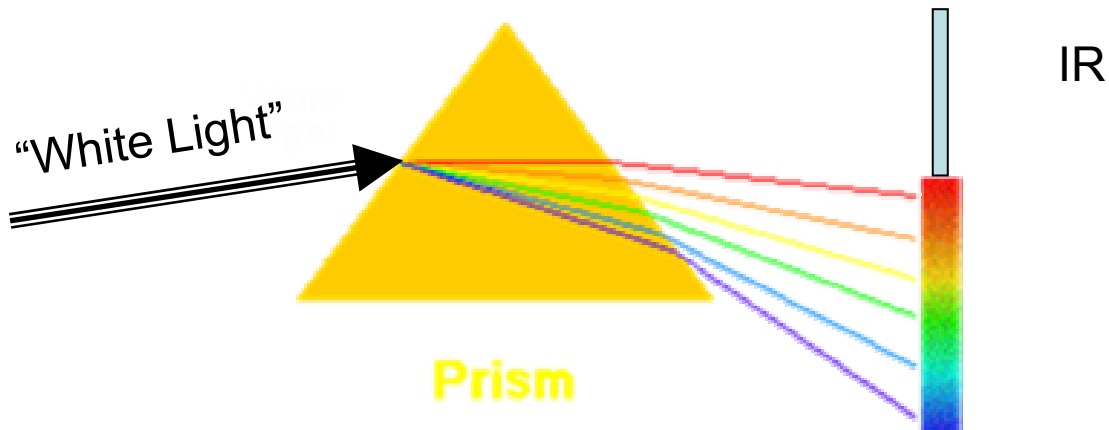


$$R = R_0 \cos\theta$$

The beginning of Infrared

Infra= Ln. below

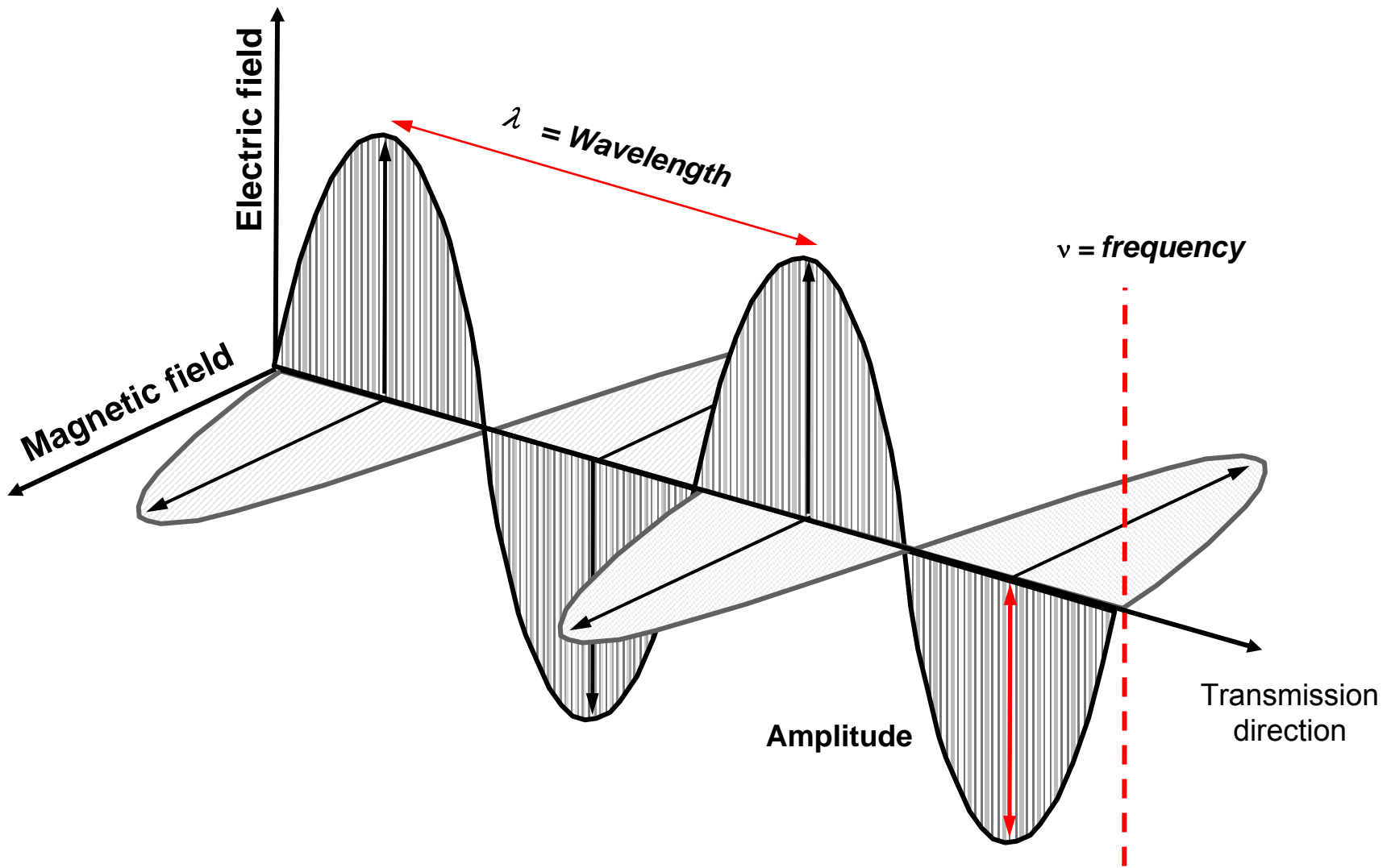
- In 1800, Sir William **Herschel**, using a prism to spread sunlight, observed the heating “beyond the red end” of the visible light spectrum



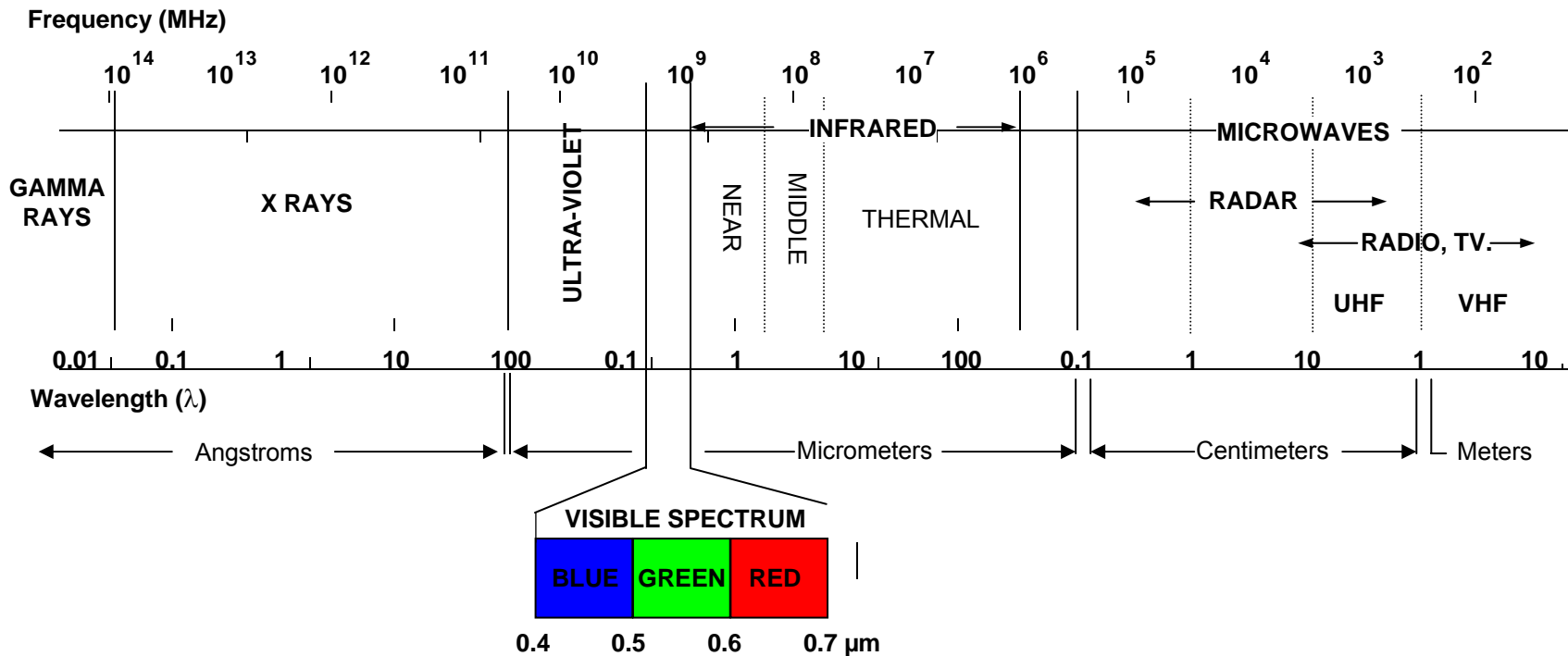
IR: Heat?

- A known effect of infrared light on skin is dilation of blood vessels that transport blood to and from the skin for cooling=> sensation of heat!
- According to Kirchhoff Law, if $r=0$
$$\varepsilon(\text{absorptivity}) = \sigma(\text{emissivity})$$
- Since skin is a good IR emitter then it must be a good IR absorber!

Light: An Electromagnetic wave



The Electromagnetic Spectrum



IR Frequency and Energy

- Frequencies: $.003 \times 10^{14}$ to 4.3×10^{14} Hz
- Wavelengths: 1 mm – $0.7 \mu\text{m}$
- Quantum energies: 0.0012 - 1.16 eV

$$\Delta E [\text{eV}] = 1.24 / \lambda [\mu\text{m}]$$

For Si, $\Delta E = 1.13 \text{eV}$; $\lambda_{\text{cutoff}} = 1.1 \mu\text{m}$

Planck's Equation

M_λ : Spectral Exitance [$\text{W} \cdot \text{cm}^{-2} \cdot \mu\text{m}^{-1}$]

λ : wavelength [μm]

T: absolute temperature [K]

h = Planck's constant = $6.63 \times 10^{-34} \text{ W sec}^2$

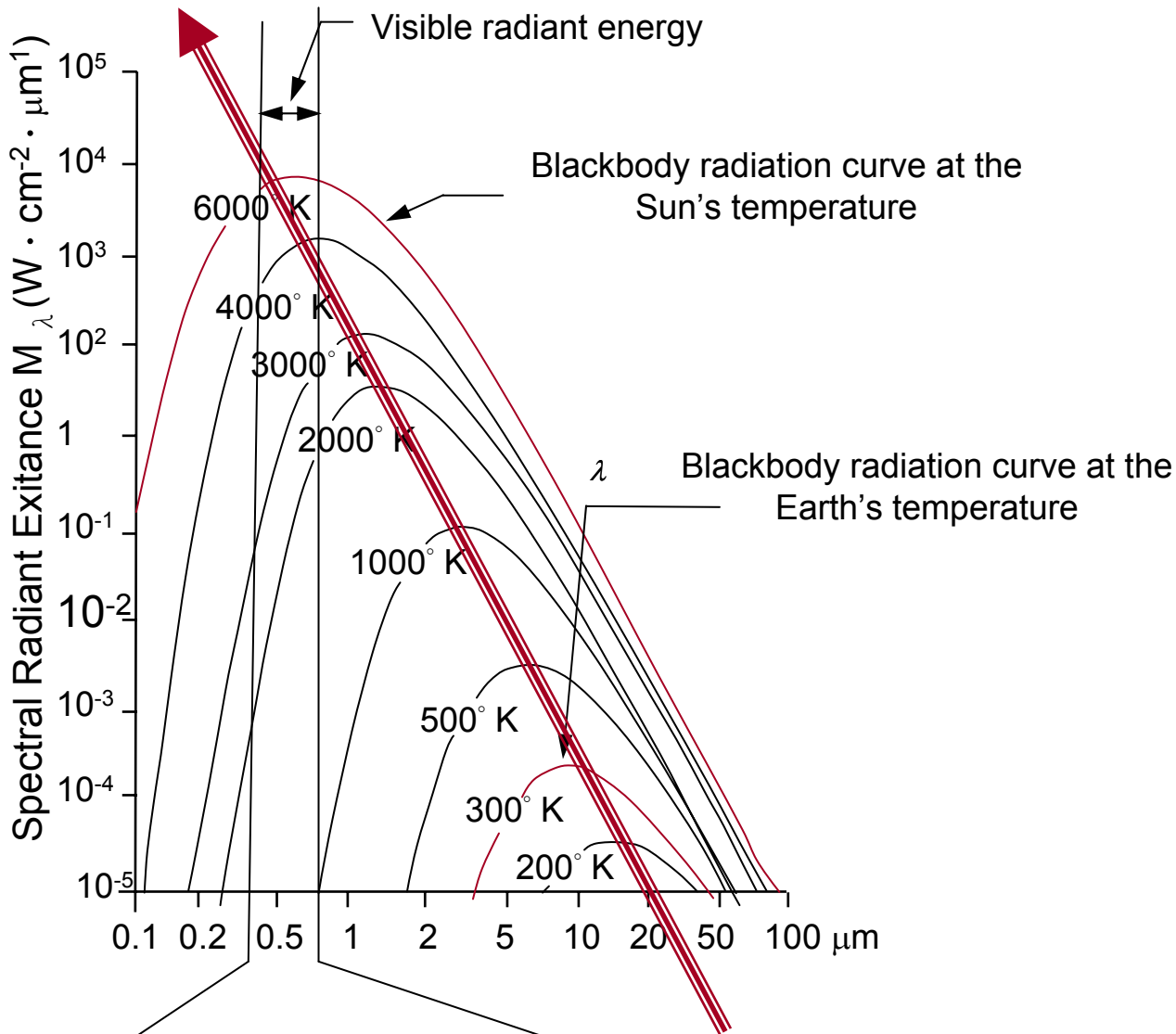
C = $3 \times 10^{14} \mu\text{m /sec}$

k = Boltzmann's constant $1.38 \times 10^{-23} \text{ W sec K}^{-1}$

$$M_\lambda(\lambda, T) = \frac{2 \pi h c^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k T}} - 1}$$

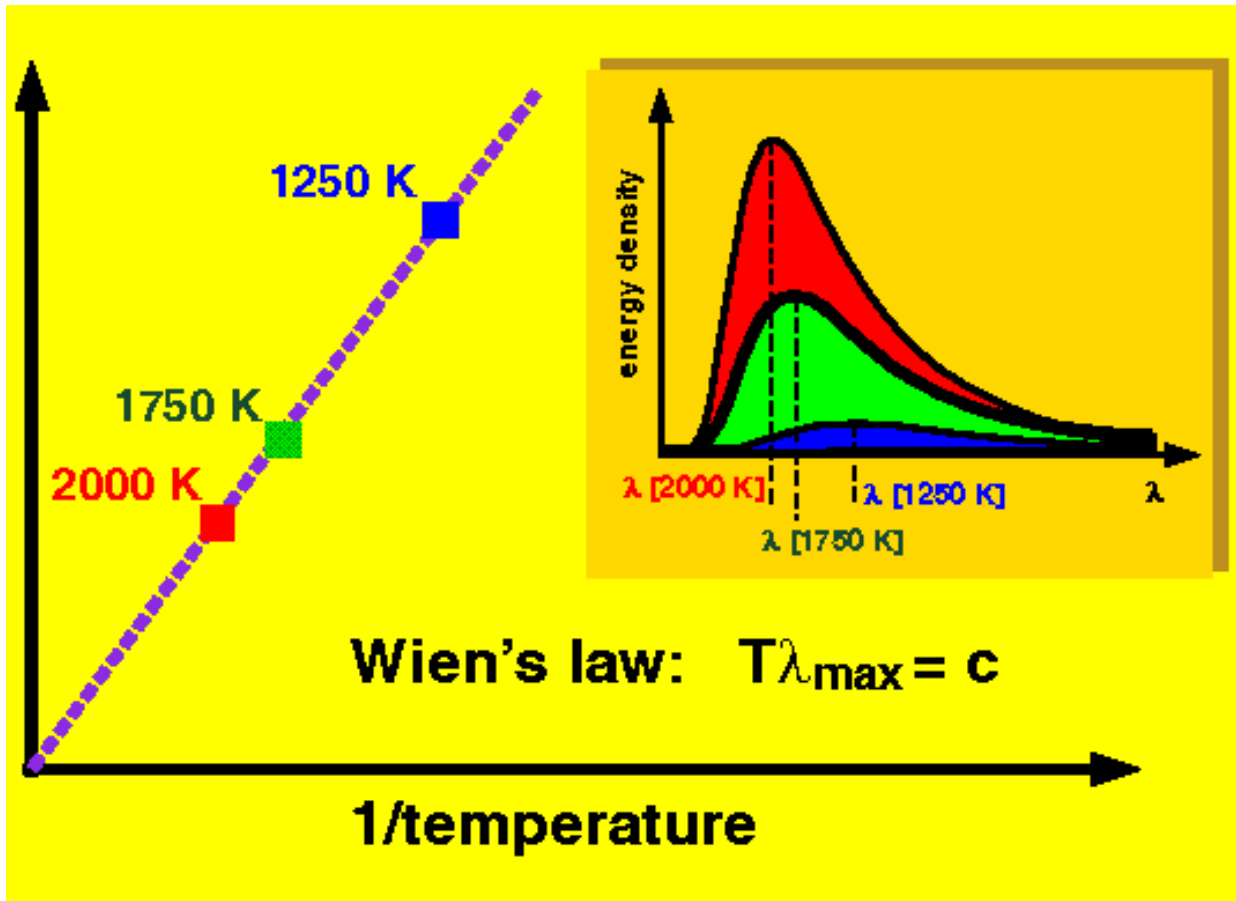
$$= \frac{3.74 \times 10^4}{\lambda^5 \left[e^{1.44 \times 10^4 / \lambda T} - 1 \right]}$$

Spectral exitance of a blackbody



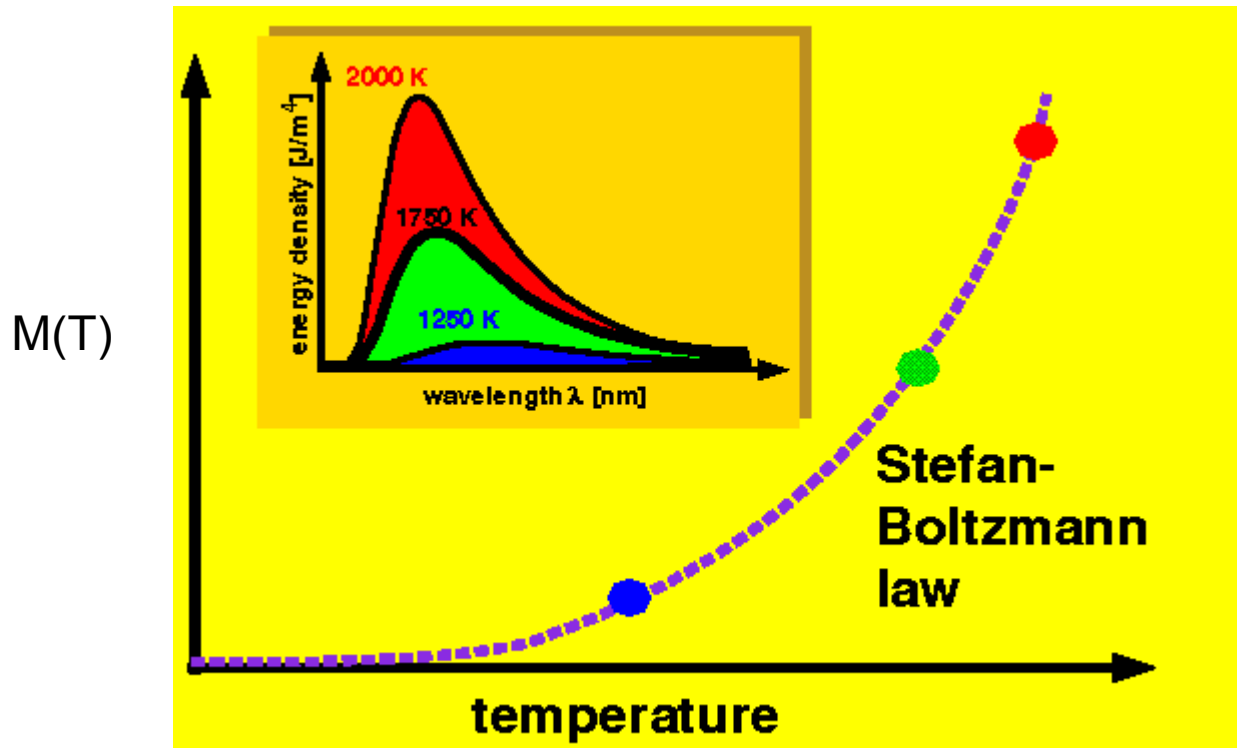
Wien's Law

- $\lambda_{\max} T \sim 3000 \mu\text{m}\cdot\text{K}$



Stefan-Boltzmann's Equation of Radiation

- $M(T) = \int M_{\lambda}(\lambda, T) d\lambda = \sigma T^4$ [W·cm⁻²]
- $M(T)$: Exitance (not Spectral Exitance)
- σ : Stefan-Boltzmann's constant 5.67×10^{-12} W · cm⁻² · K⁻⁴



Grey Body?

- When emissivity ε is not unity
- Most physical surfaces are grey bodies
 $\varepsilon_{\text{skin}} \sim 0.95$, then it must be “Approximated as a Blackbody

$$M_{\lambda} = \varepsilon M_{\lambda}$$

$$M = \varepsilon \sigma T^4$$

Atmospheric Transmission Spectra

UV

VNIR

SWIR

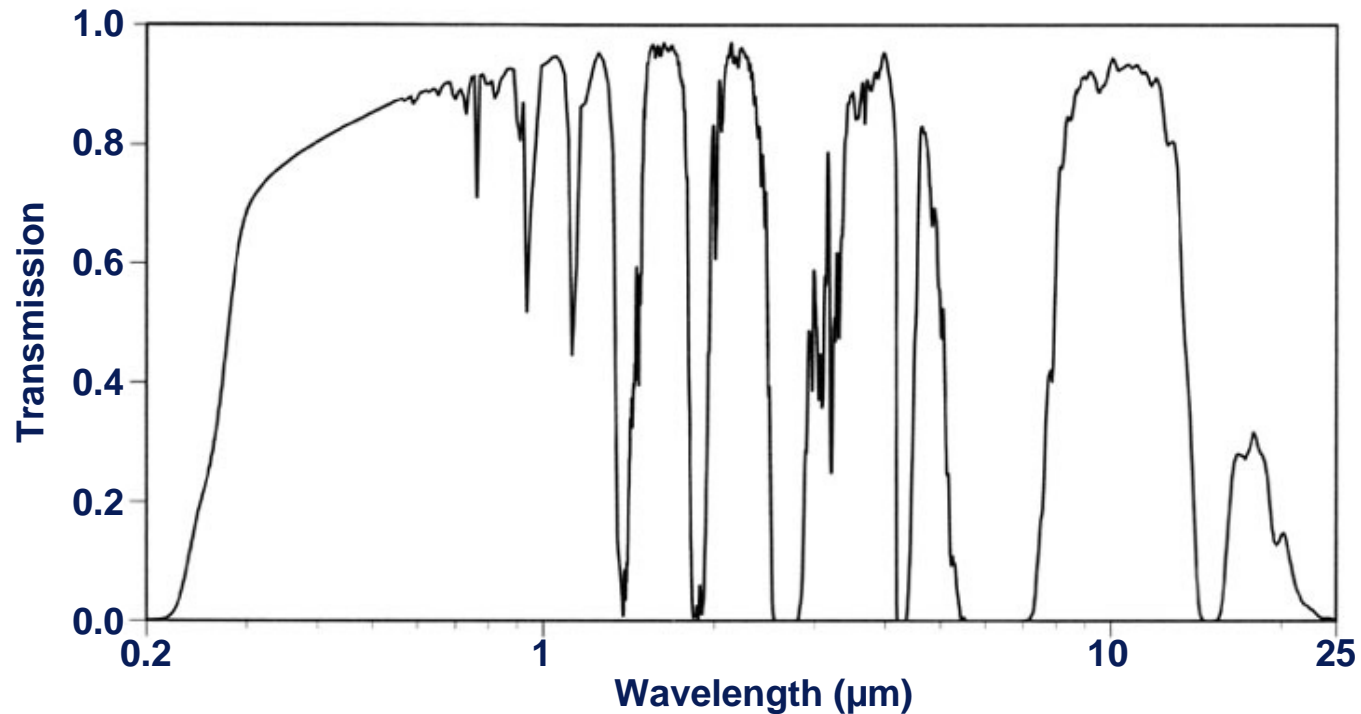
MWIR

LWIR

For Ref:

NIR: 0.7-1.1 μm ; SWIR: 1.1-3.0 μm ; M Ω IP: 3-5 μm ;

LWIR: 8-14 μm ; VLIR: > 14 μm



Infrared Interactions

<http://hyperphysics.phy-astr.gsu.edu/hbase/mod3.html#c3>

- . The result of infrared absorption is heating of the tissue since it increases molecular vibrational activity..



•

Discrete Energy State

- Planck's 1900's "lucky Guess" $\Delta E = h\nu$

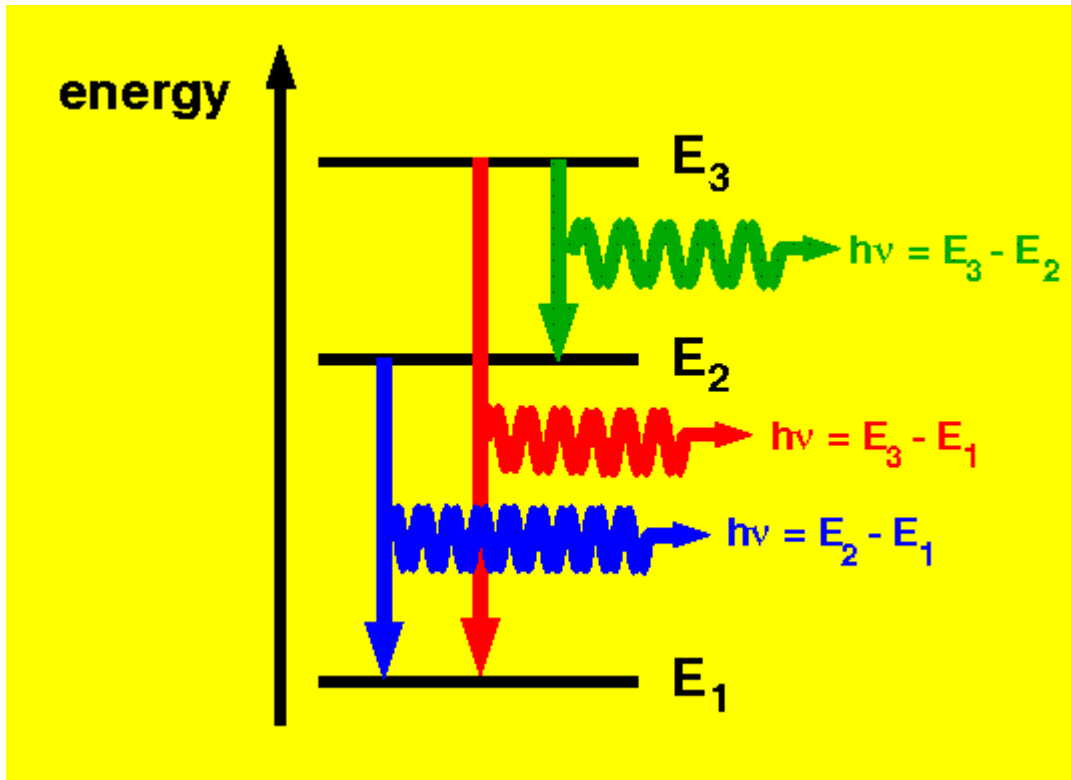
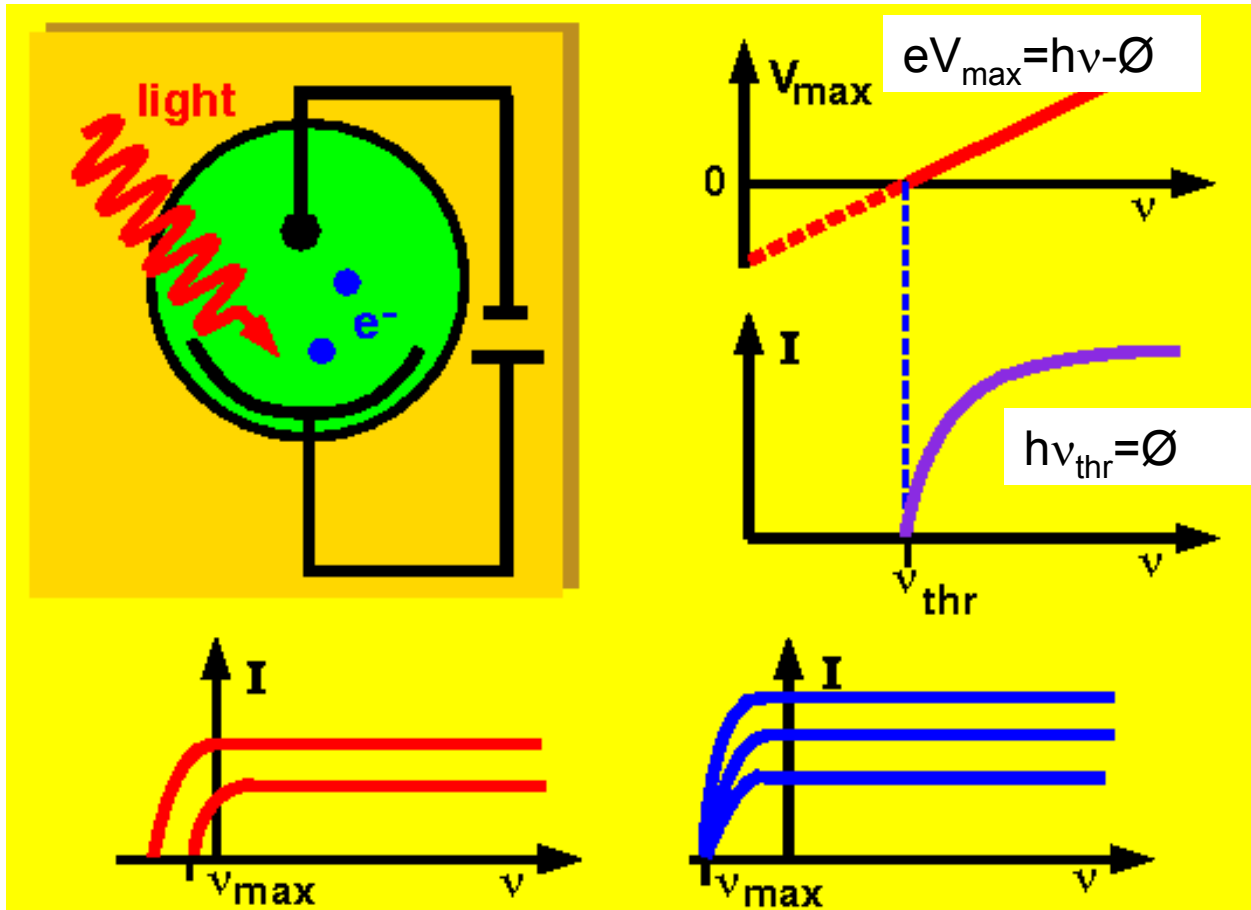


Photo-Electric Effect

- Einstein 1905's Paper Confirming the Discrete Energy

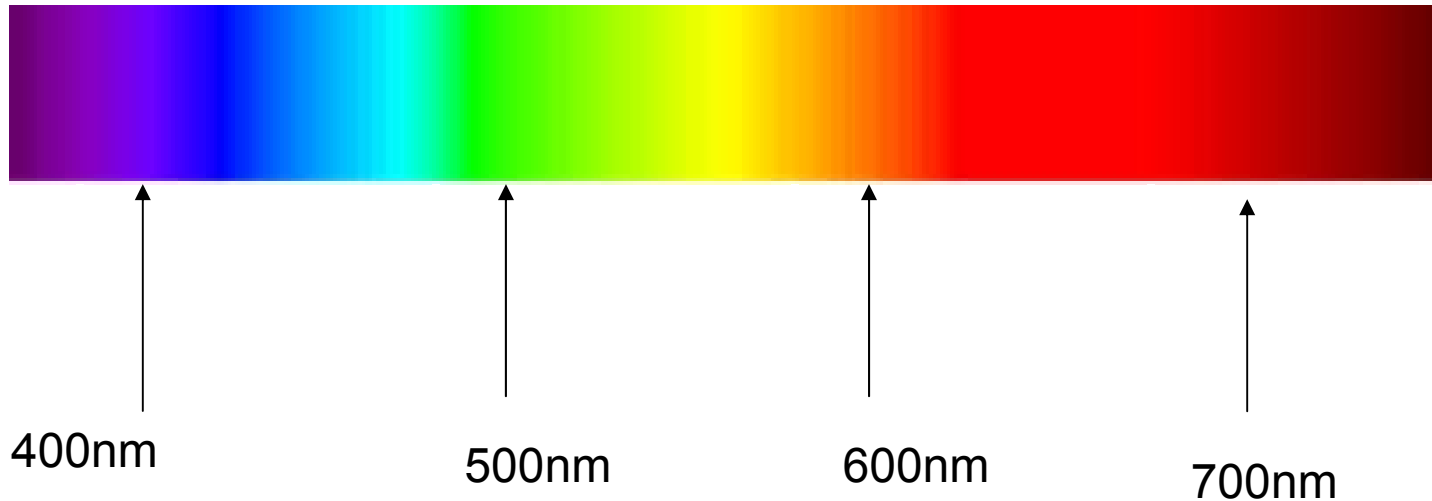


Varying ν

Same ν , varying intensity

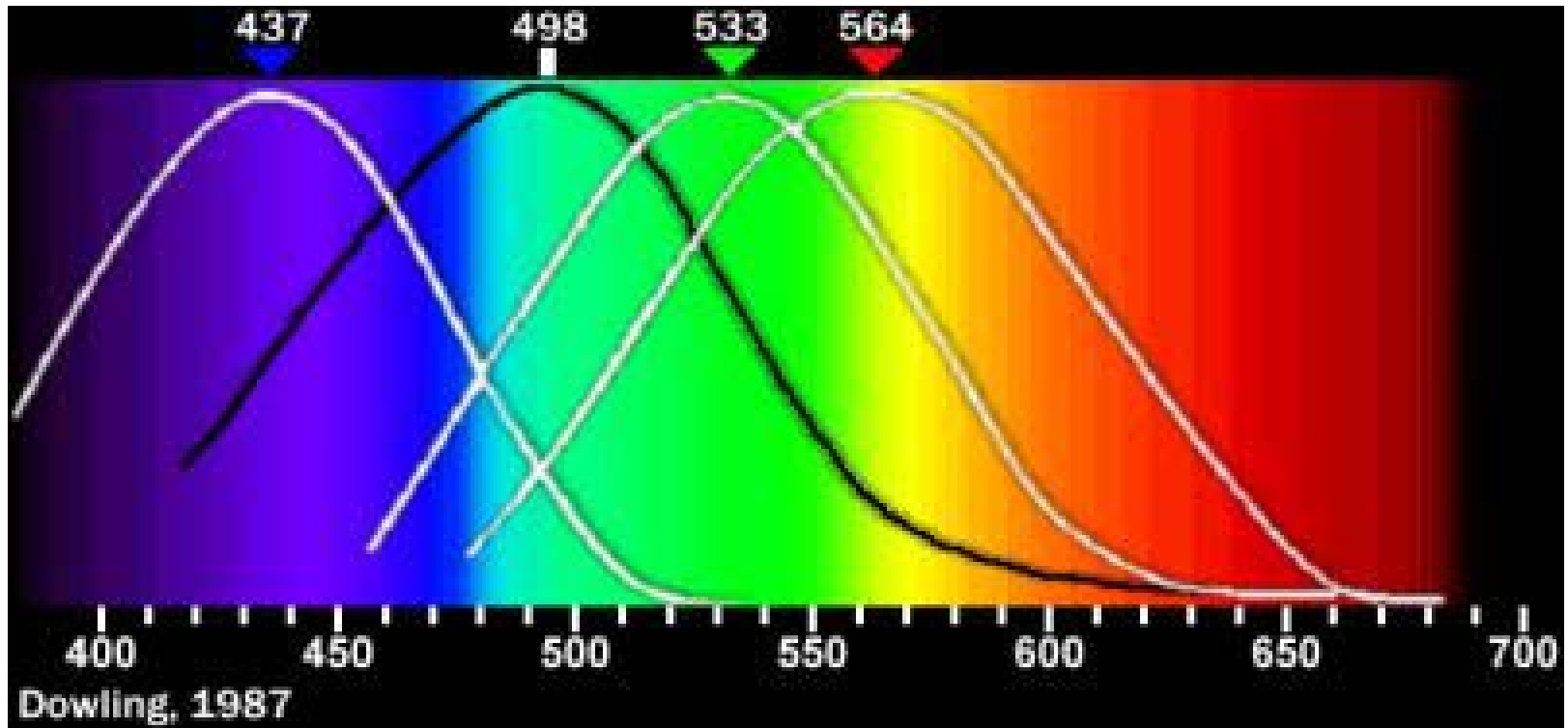
Visible Spectral Range

- Visible Band: 400nm to 700nm



Eye's Cones' (3) and Rods' Responses

- Rods for night vision (more sensitive)
- Cones for color day vision

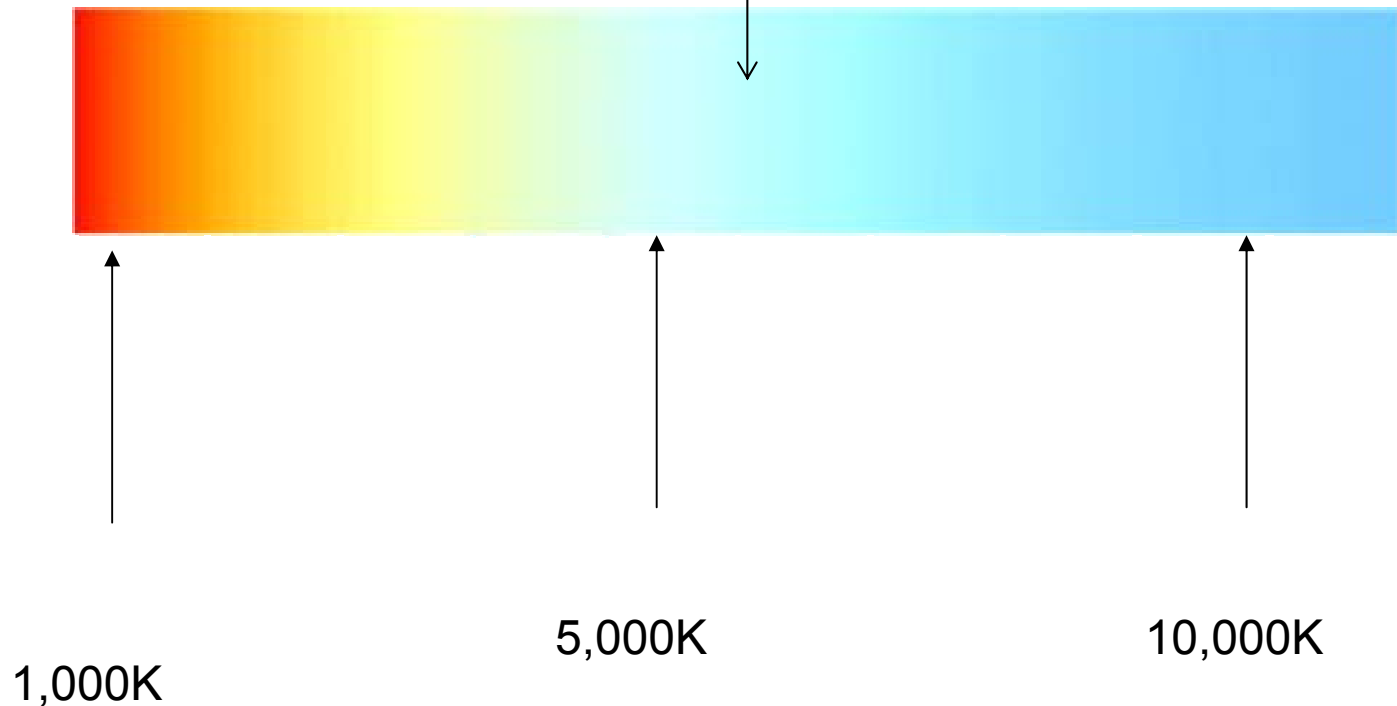


“Color” Temperature

An Apparent Visible Color of a Blackbody at T

At 800K, or Draper Point,
Blackbody Begins to be Visible

Sun~
6000K



Night Goggles are “not” true Thermal Images

- Night Goggle Images are “Reflected NIR Images”, not “Emitted Thermal images”

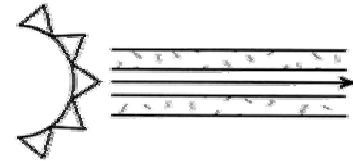
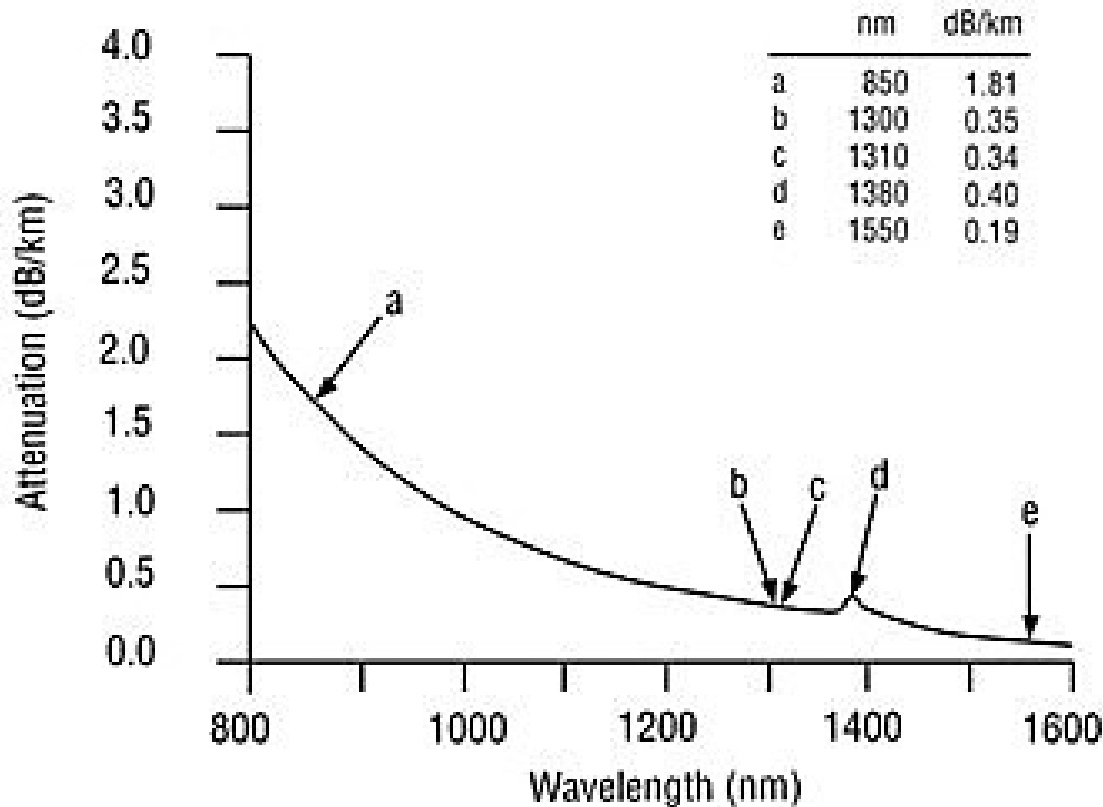


Many Low-Cost Low-Light Detection Systems are NIR Systems

“Near IR Wavelength Used for Optical Communications”

“Single mode fiber”
single path through the fiber

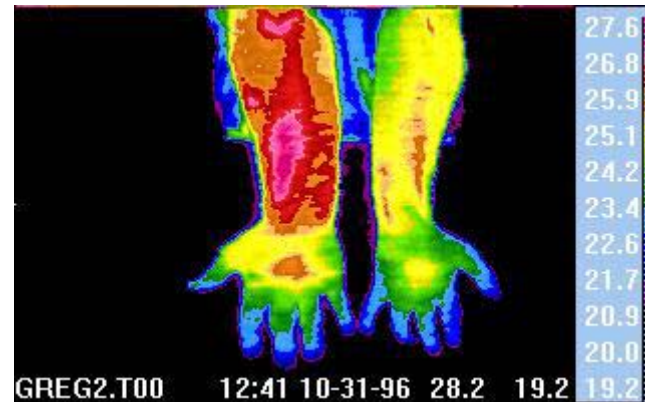
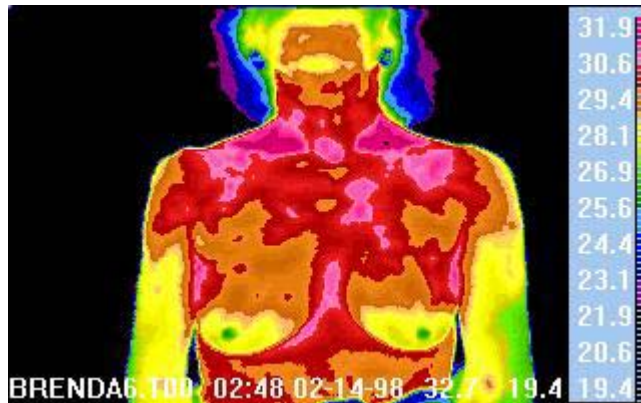
Spectral Attenuation (typical fiber):



Singlemode Fibre 10/125

Human Thermal Images

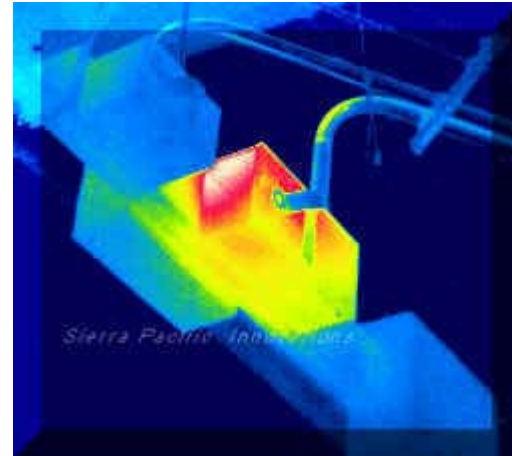
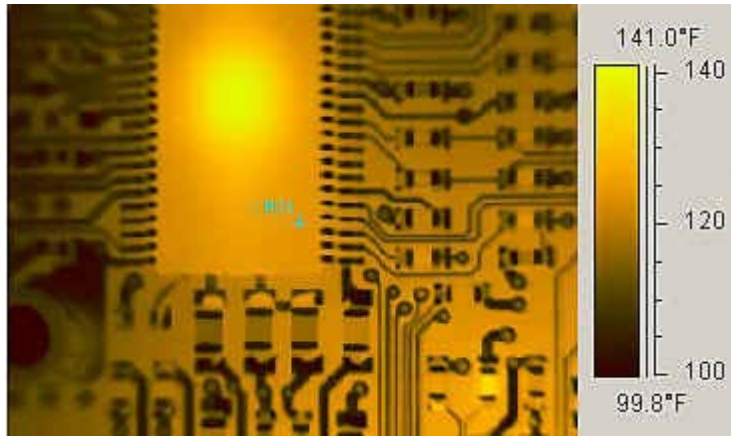
- http://www.ir55.com/infrared_IR_camera.html



PC Board Localized Heating

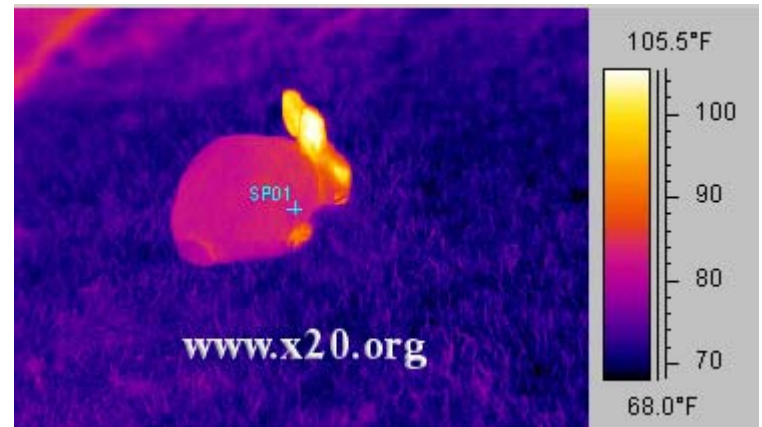


Localized IC Chip Detection



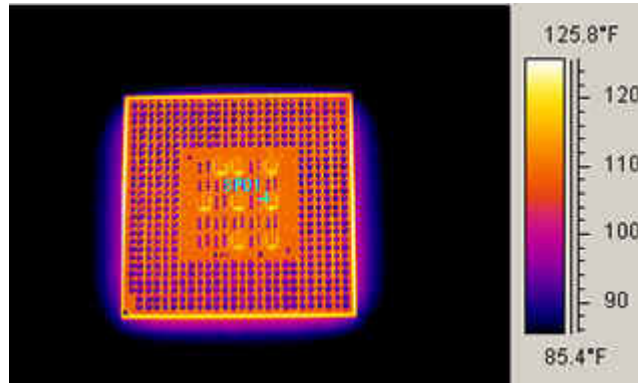
Burglar Detection

•



Underside Celeron Chip

•



SARS Temperature Screening

-



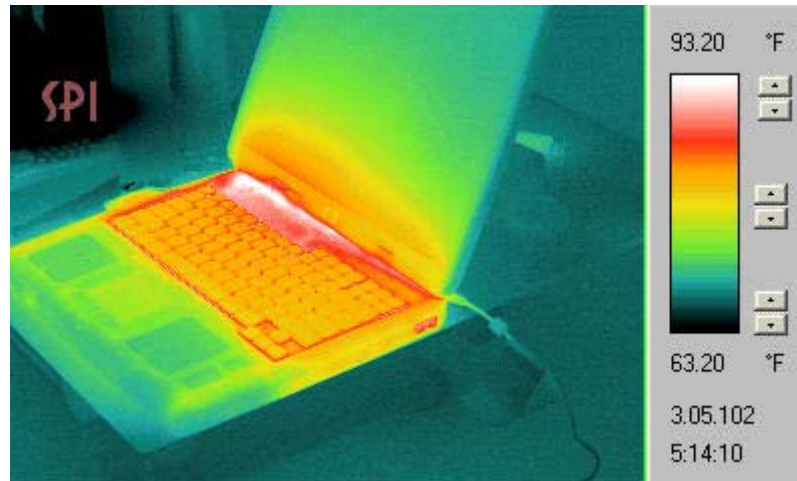
Preventive Maintenance

- Electrical Fuse Thermal Image

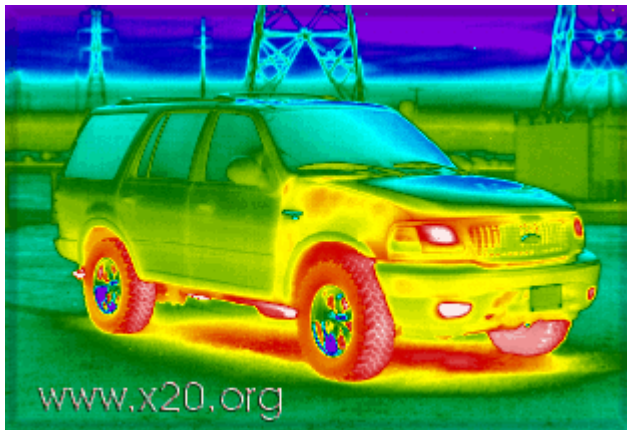


Thermal Management

-

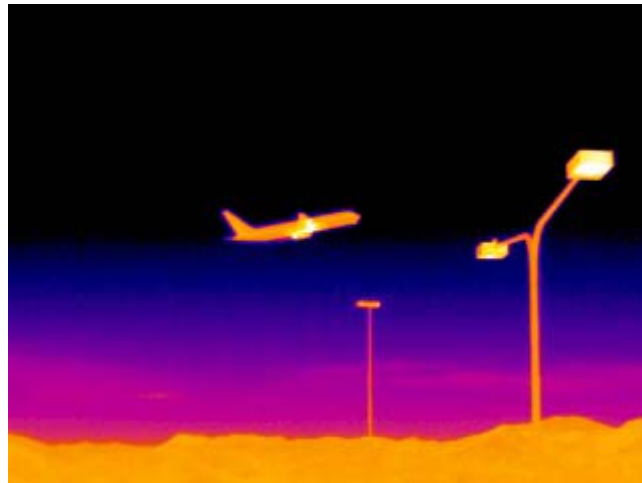


Defense Applications



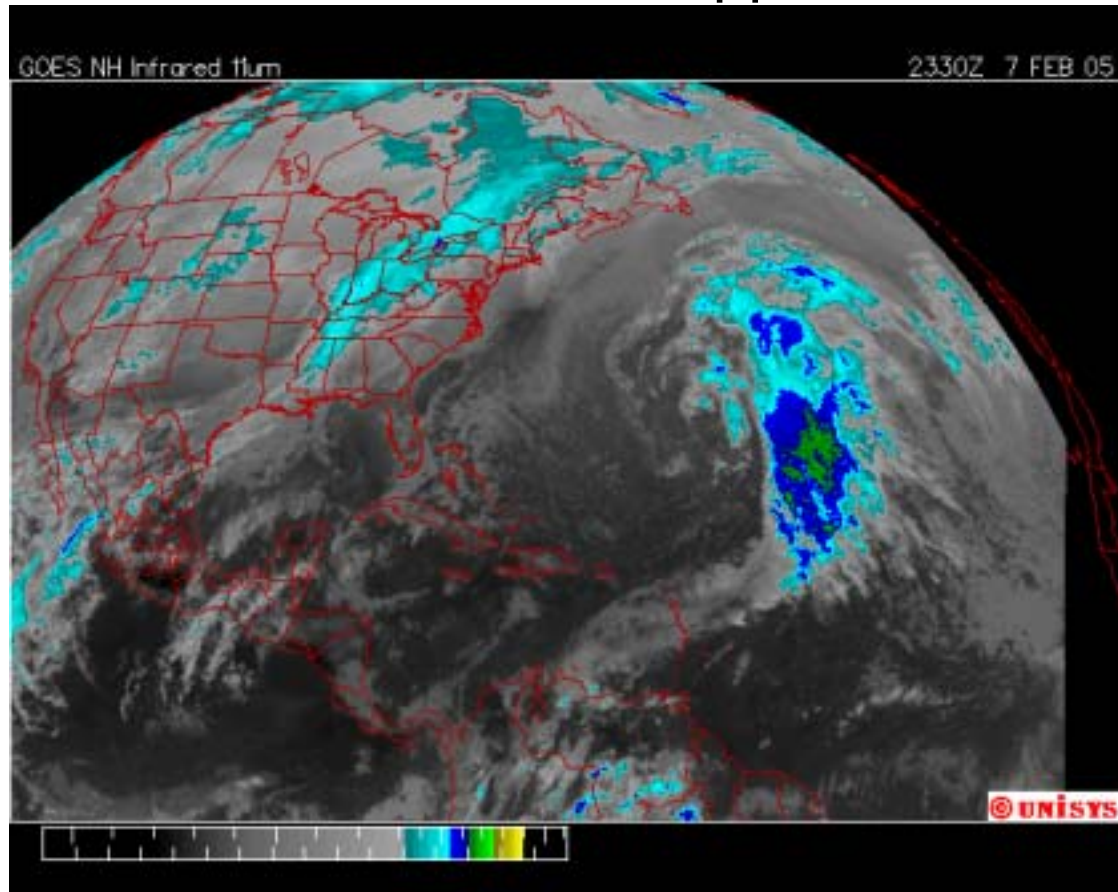
Sky Surveillance

- Collision Prevention



Weather Monitoring

- Geosynchronous Weather Satellite Application



What “Limits” Your Measurements?

1. Spatial (How Small an Area Can the System Resolved?):

Optics

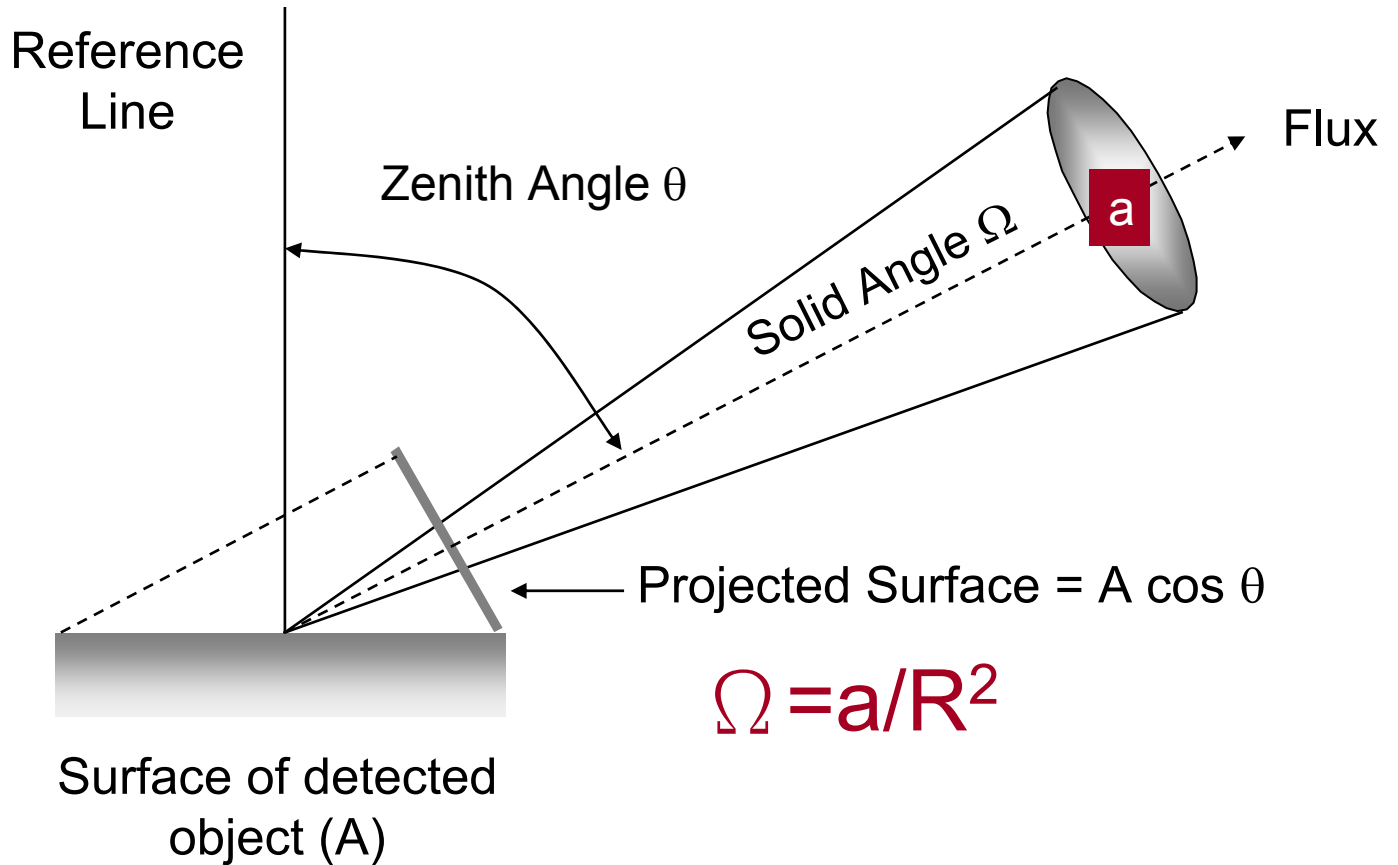
2. Temporal (How Fast Can the System Do?):

Detector and Electronics Responses

3. Resolution of the System (What is the Smallest Temperature the System can resolve?):

NEP

Solid Angle Concept



Radiance L

- Radiance is Defined as the Power per Unit Area per Steradian(Sr)

$$L[\text{W m}^{-2} \text{Sr}^{-1}] = M(T) / \pi$$

And

The so is the “Spectral Radiance”

$$L_{\lambda}[\text{W m}^{-2} \text{Sr}^{-1}] = M_{\lambda}(T) / \pi$$

Solar Constant K_{solar} (Example)

- Solar disk “subtends” $1/2^\circ$ (or 9 mRadians) in view, the solar constant is the total Radiance Power per unit area
- Since the Radiance is
 $L = 1/\pi M(6000\text{K}) = (\sigma/\pi) \times 6000^4 = 2.34 \times 10^7 \text{ W} \cdot \text{m}^{-2} \text{ Sr}^{-1}$
- The solid angle of the sun is
- $\Omega = (\pi/4)(0.009/2)^2 \sim 6.4 \times 10^{-5} \text{ Sr}$
- The Solar Constant is then:
- $K_{\text{solar}} = L \cdot \Omega \sim 1.5 \text{ KW/M}^2$
- σ : Stefan-Boltzmann’s constant $5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$

Equilibrium Temperature Concept

- The Total Power Absorbed by a 1M^2 Plate Perpendicular to Sun Rays is a Solar Constant K_{solar} of 1.5KW
- The Radiated Power is
- The **Equilibrium Thermodynamic** Condition Stipulates:
- $\sigma T_{\text{plate}}^4 = K_{\text{solar}}$
- $T_{\text{plate}} = (1500 / \sigma)^{1/4} \sim 403\text{K} = 130^\circ\text{C}$

How to Manipulate the Equilibrium Temperature T_{equi}

- By varying Surfaces Solar Absorption Coefficient α and ϵ
- For α of 0.2 and ϵ of 0.9, $T_{equi} \sim 277K \Rightarrow 4^\circ C!$
- $\alpha K_{solar} = \epsilon \sigma T_{plate}^4$

$$T_{equi} = \sqrt[4]{\frac{\alpha K_{solar}}{\epsilon\sigma}}$$

- For α of 0.2 and ϵ of 0.9, $T_{equi} \sim 277K \Rightarrow 4^\circ C!$

Why is a Metal Surface so Warm in the Sun?

Polished Metal Surfaces have low α and ε

Assume $\alpha = \varepsilon = 0.2$

$$T_{equi} = \sqrt[4]{\frac{\alpha K_{solar}}{\varepsilon \sigma}} = \sqrt[4]{\frac{0.2 \times 1500}{0.2 \times 5.67 \times 10^{-8}}} = 403K!$$

• σ : Stefan-Boltzmann's constant $5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$

Does “Absolute Temperature” Have to Do with Heat Transfer?

- Conduction

$$\Delta Q \sim \Delta T$$

- Convection

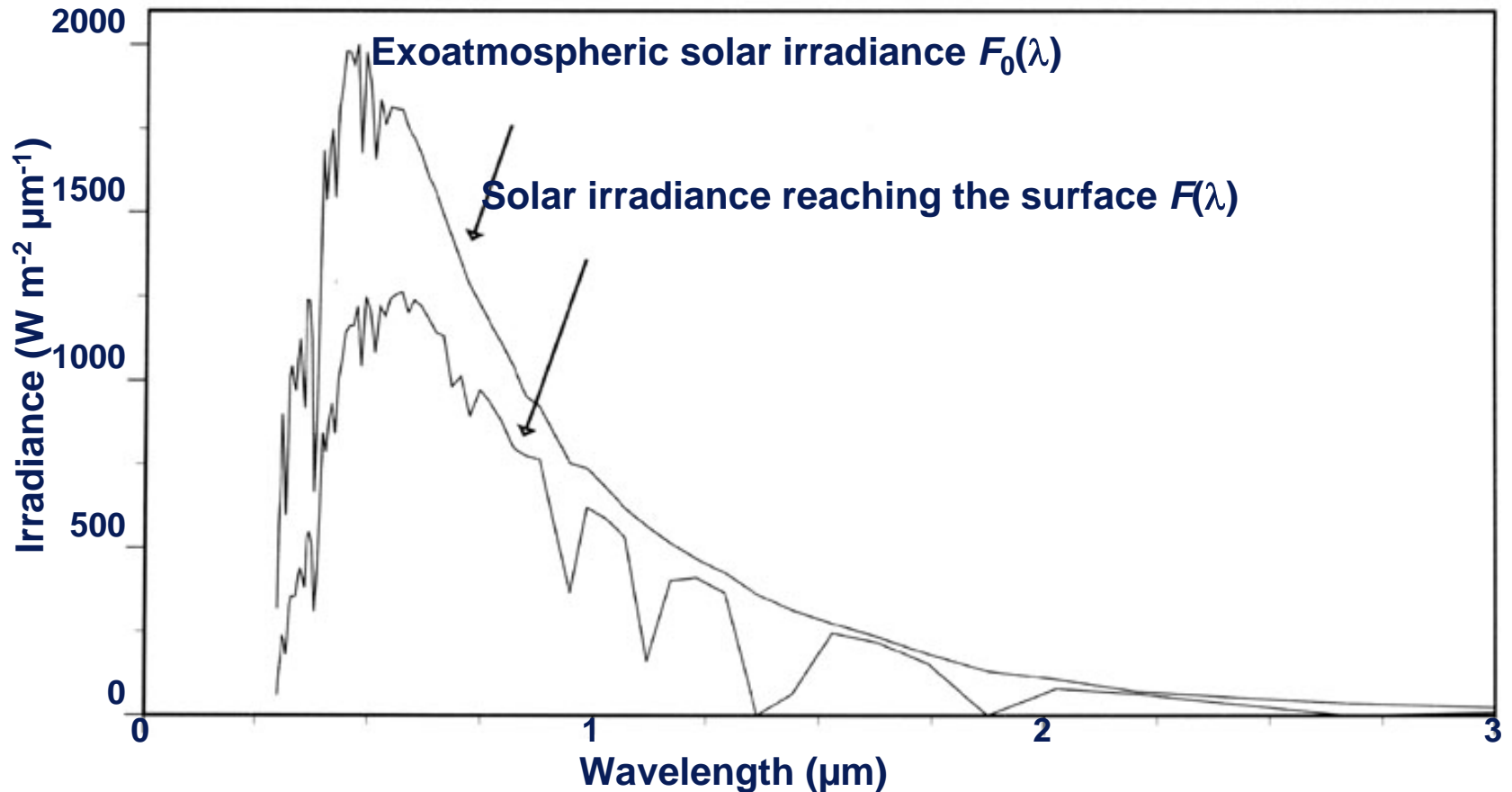
$$\Delta Q \sim \Delta T^n; n \neq 1$$

- Radiation

$$\Delta Q \sim \Delta (T_1^4 - T_2^4)$$

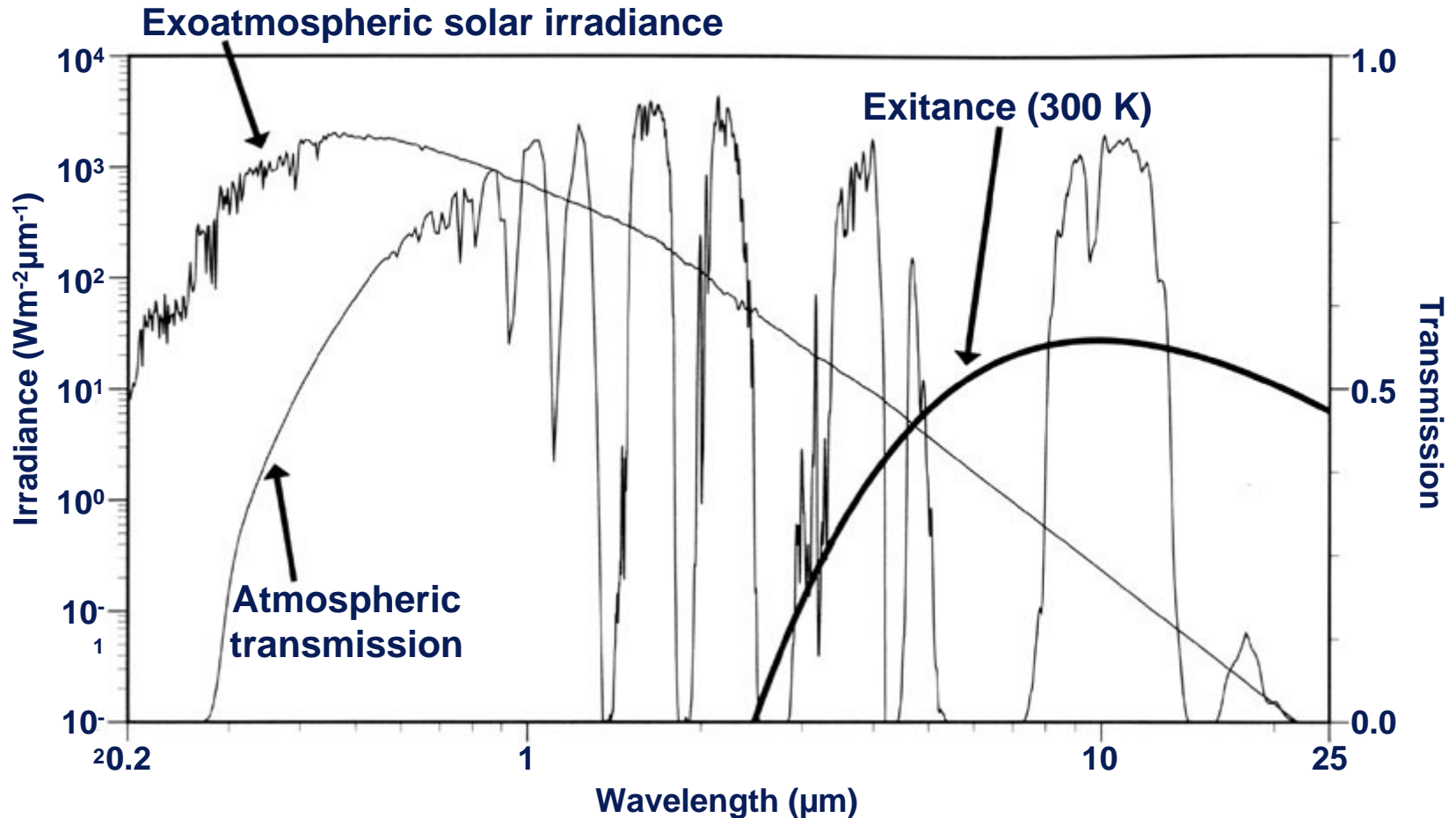
Radiative Heat Transfer is the Only Form of Heat Transfer that requires the knowledge of Absolute Temperature instead of Temperature Difference

Scattering of Sunlight by the Earth- Atmosphere-Surface System



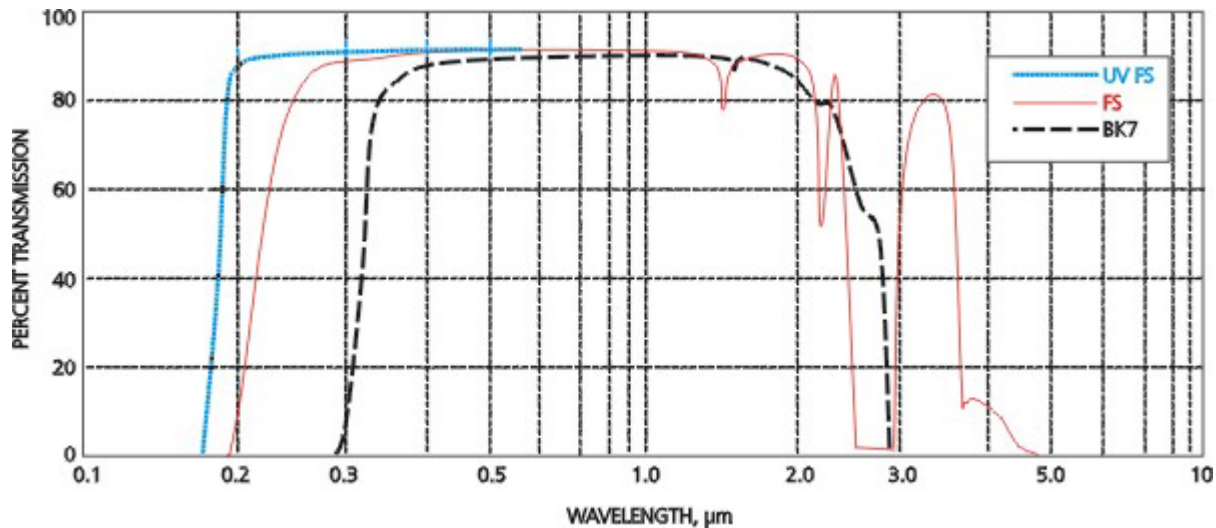
Atmospheric Transmission and Greenhouse Effects

http://tbrs.arizona.edu/education/553-2004/2004/Lect083104_Ch2.ppt-link.ppt#21



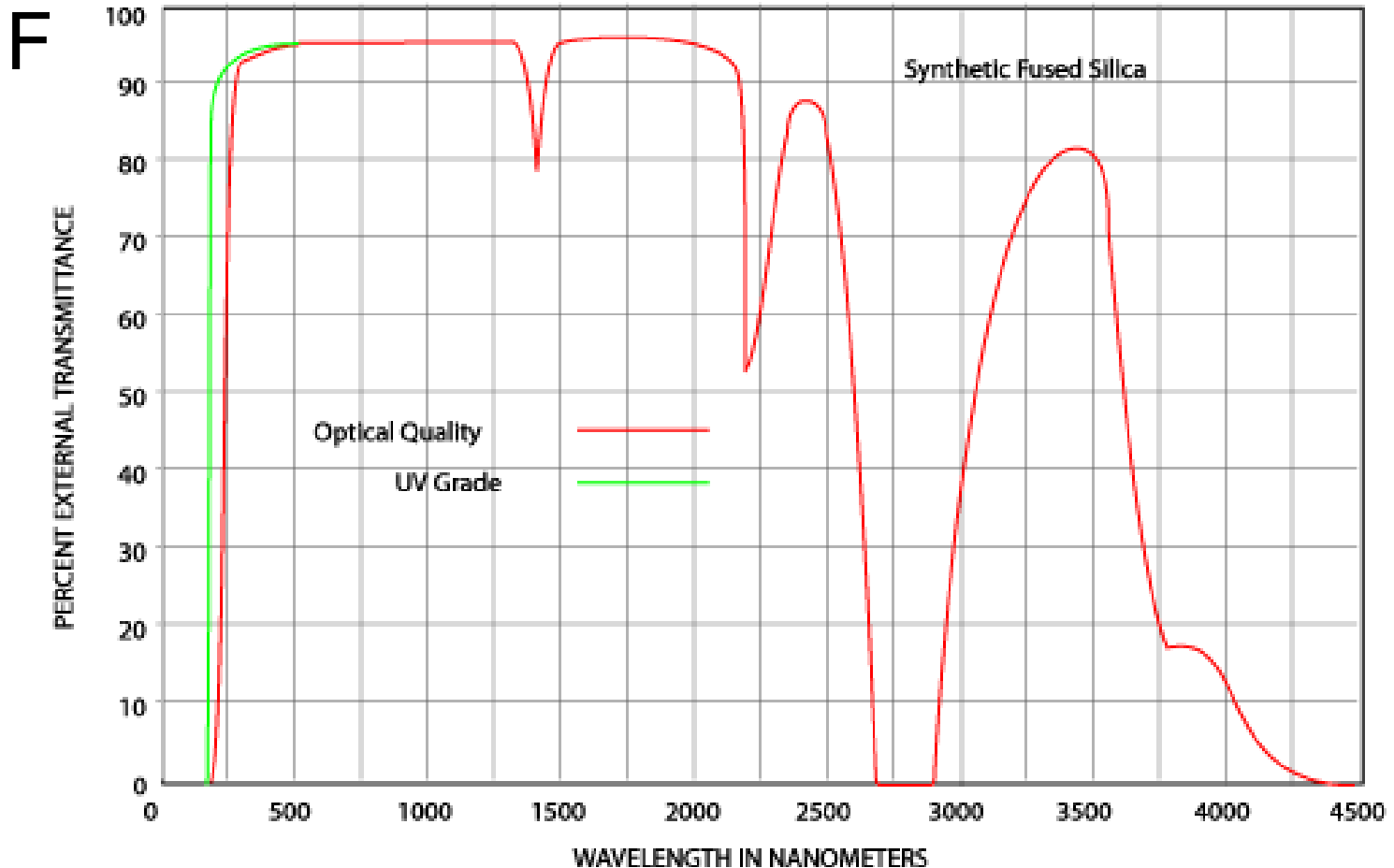
BK-7 Transmission Curve

- Most Plate Glass, Similar to BK7
- Plate Glass is Opaque to LWIR

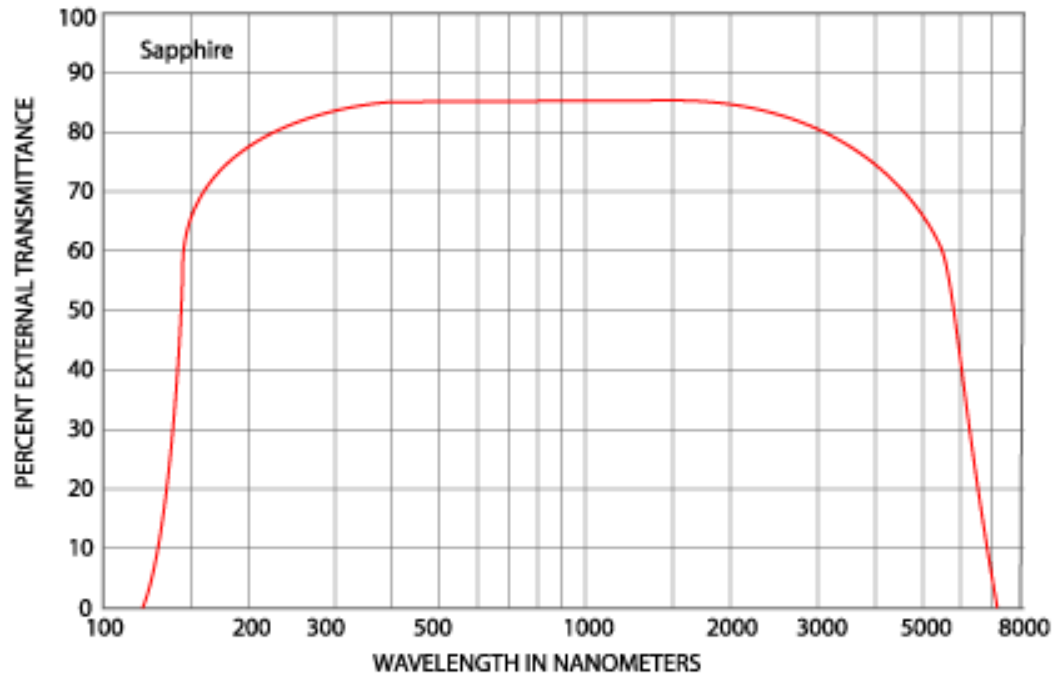


Fuse Silica (quartz) Transmission

-



Sapphire Transmission

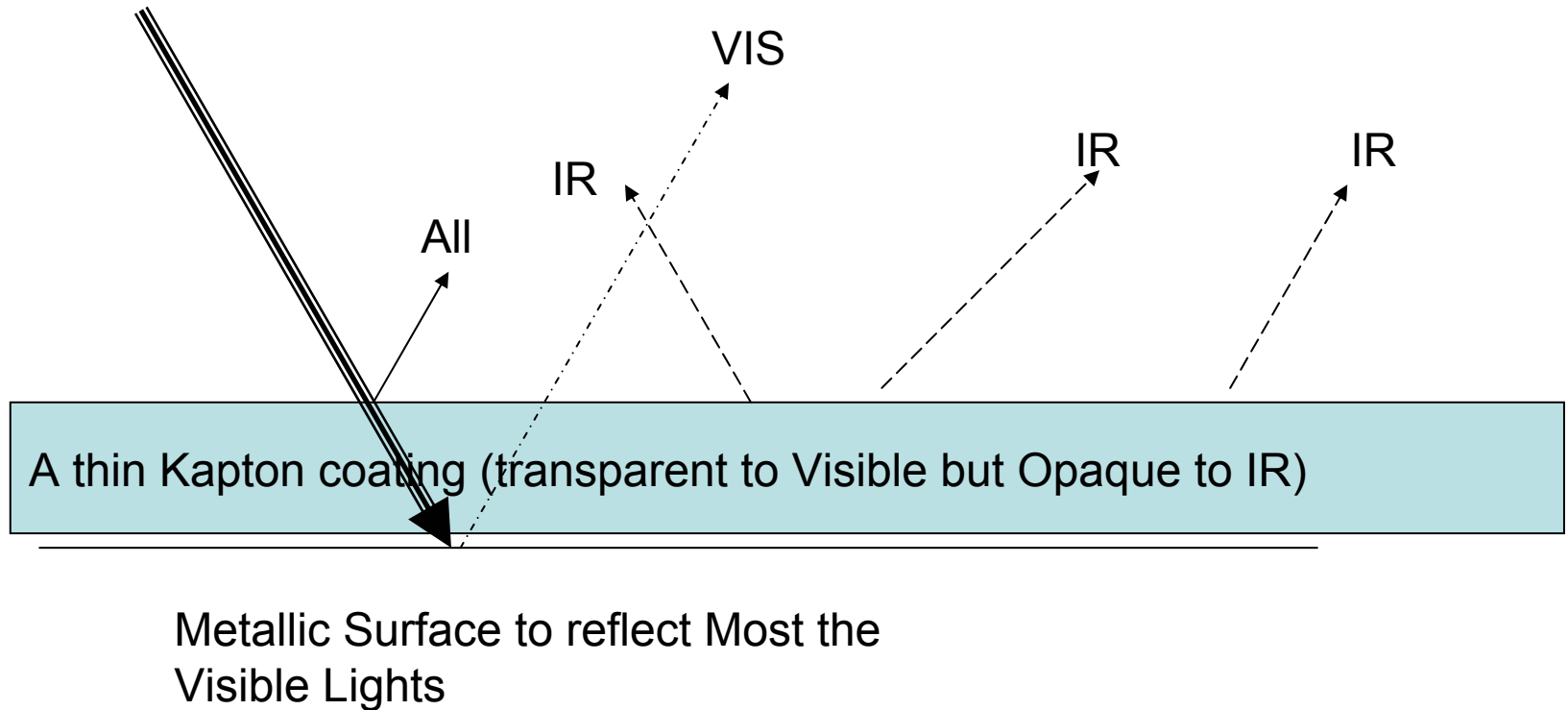


Why is the Interior of a Car so Warm in the Sun?

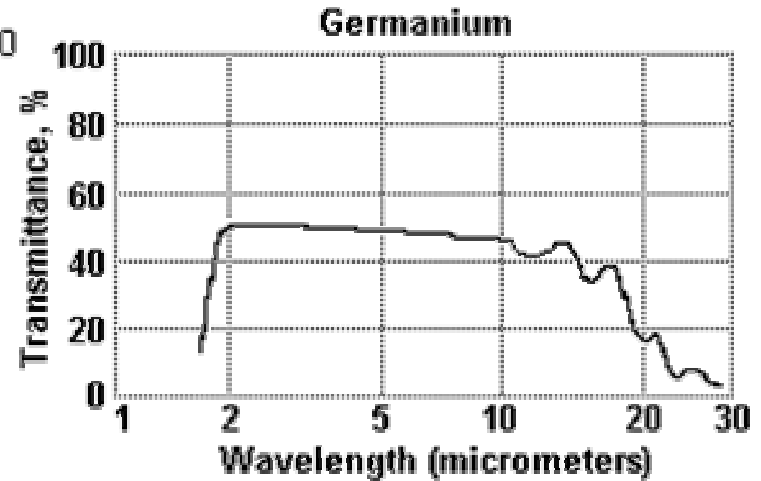
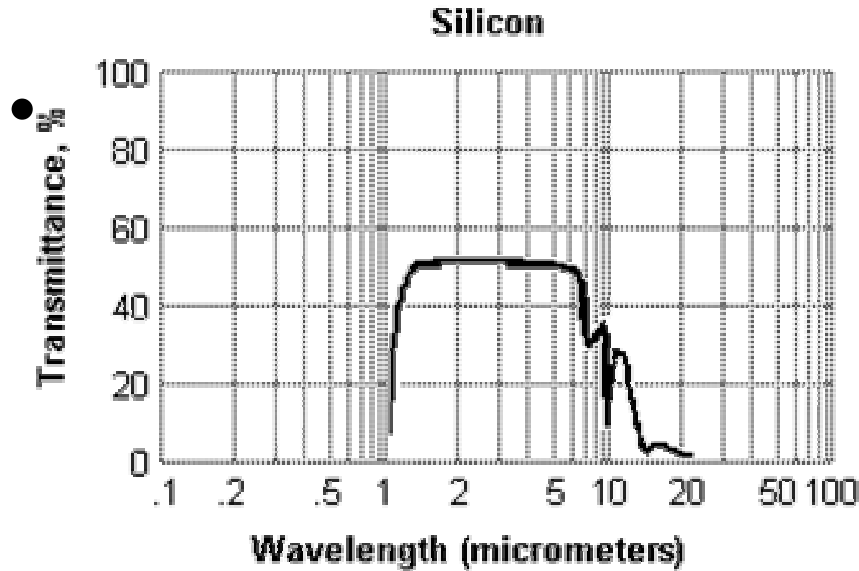
- Sun(6000K) warms a car with all wavelengths, but the interior of the car (300K-400K) emits IR (mostly around 10 μm) that can not pass through the glass.

So How Does a Space Suit Work in the Sun?

- By a “Secondary Mirror” Surface!

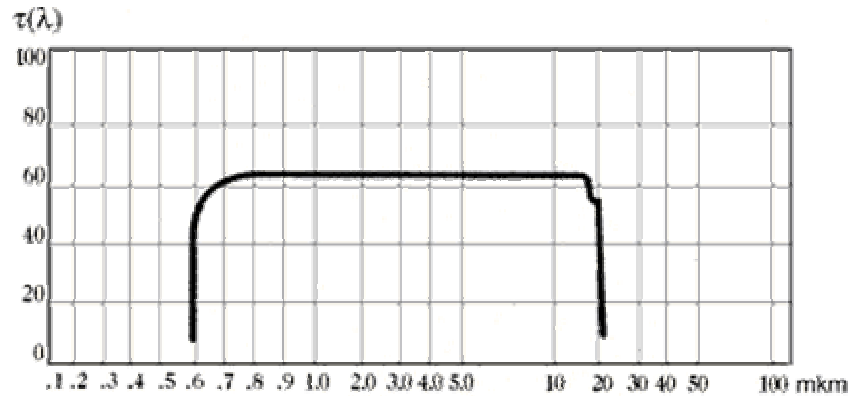


Si and Ge IR Transmissions



ZnSe Transmission

- <http://www.almazoptics.com/ZnSe.html>



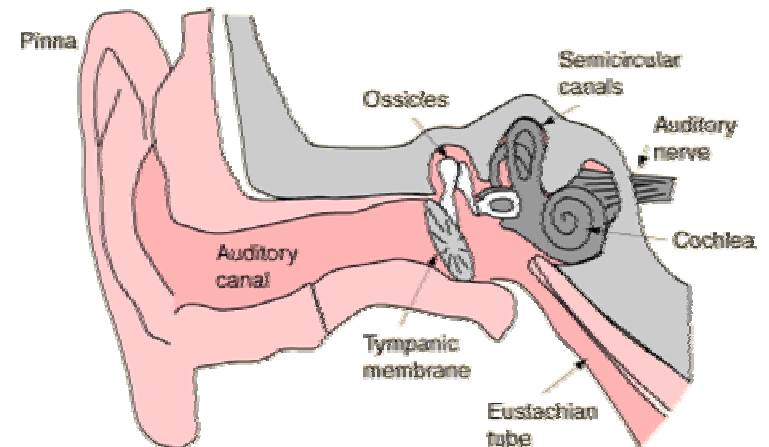
Regardless the “skin tone” difference,
“all men are equal” in Infrared

- Yes, from 0.95-- 0.98; **almost black!**

What is an Aural Thermometer", or Infrared Aural sensor

- Tympanic cavity as a blackbody cavity
- Emissivity~1.00
- Readily calibrated

- ****Must be in a cavity!!**



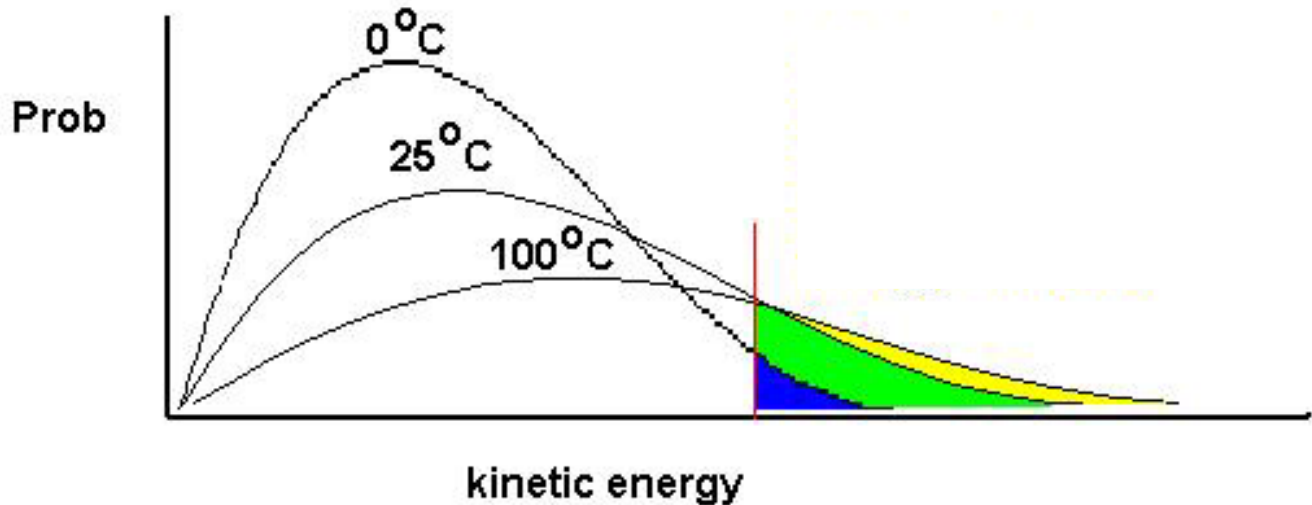
The Infamous SAR Fighter: Ear Cavity Thermometer

- a clinically reliable indicator of **body core** temperature
- Pyro-Electric Transducer



Electron Thermal Energy: Why IR Detectors Must be Cooled!

- $KE_{avg} = \left[\frac{1}{2} m v^2 \right] = \frac{3}{2} kT$



NEP Concept

- If we use the entire spectrum, then to detect 38°C (vs. 37 °C), the difference is $[(38 + 273)/(37 + 273)]^4 = 1.013\%$

So to resolve 1°C the “system” must be able to resolve 1.3% difference

=>Noise Equivalent Power or NEP

How good is my System Stacking Against the Others?

$$D^* [\text{cm}\cdot\sqrt{\text{Hz/W}}] = (A_{\text{det}} \Delta f)^{1/2} / \text{NEP}$$

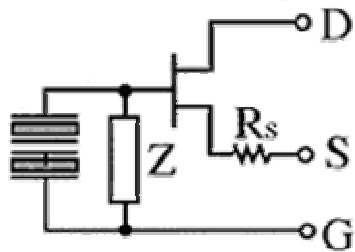
Pyro-electric Detectors

- Pyro: Gk “Fire”
- Pyro-electric: electrical output caused by heat
- Sometimes used for “fiery sparks” display for stage effects
- Low sensitivity, low cost
- Usually for intrusion detection only

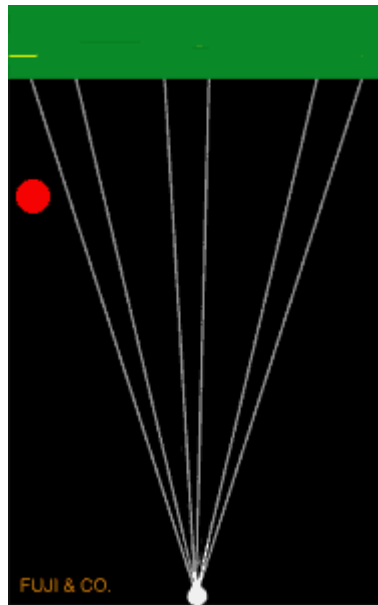
Pyro-electric Detector

polyethylene Fresnel lens are typically used for their low costs

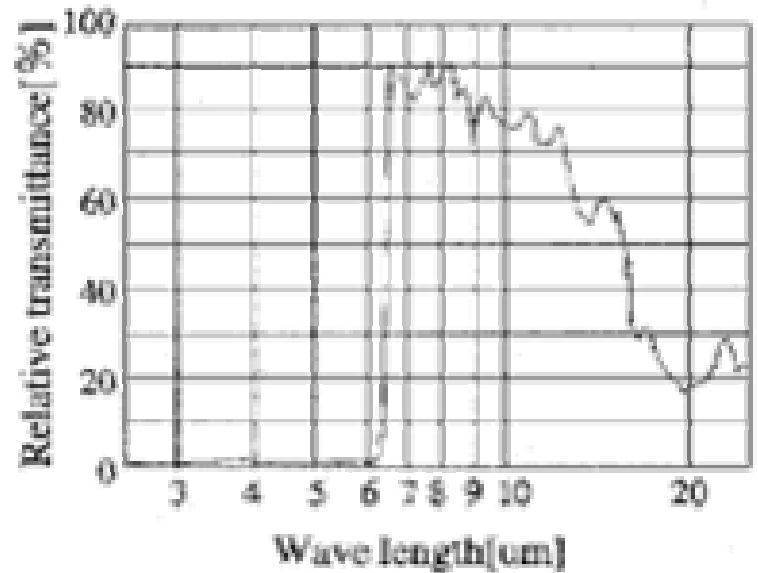
TGS (Tri-glicine-sulfate)



dual-1



typeB(7 μ m cut-on)



PV $\text{Hg}_x\text{C}_{1-x}\text{Te}$

- Short for “photo-voltaic Mer-Cad-Telluride”, or, “Mer-Cad”
- Chemical compound of HgTe and CdTe
- Response ranging from $1\mu\text{m}$ to $5.5\mu\text{m}$, and $8\mu\text{m}$ up to $13\mu\text{m}$, depending on the Hg to Cd ratio
- “Sprite” Detector: **S**ignal-**P**rocessing-in-the-**E**lement
- Most versatile IR detector

PC HgCTe

- Response to 18 microns
- Intrinsic Detectors
- Need “chopping” because of large $1/f$ noise
- Response varying with temperature
- Operative in higher temperature than PV

Thermal Transducer is “Export Control” Items

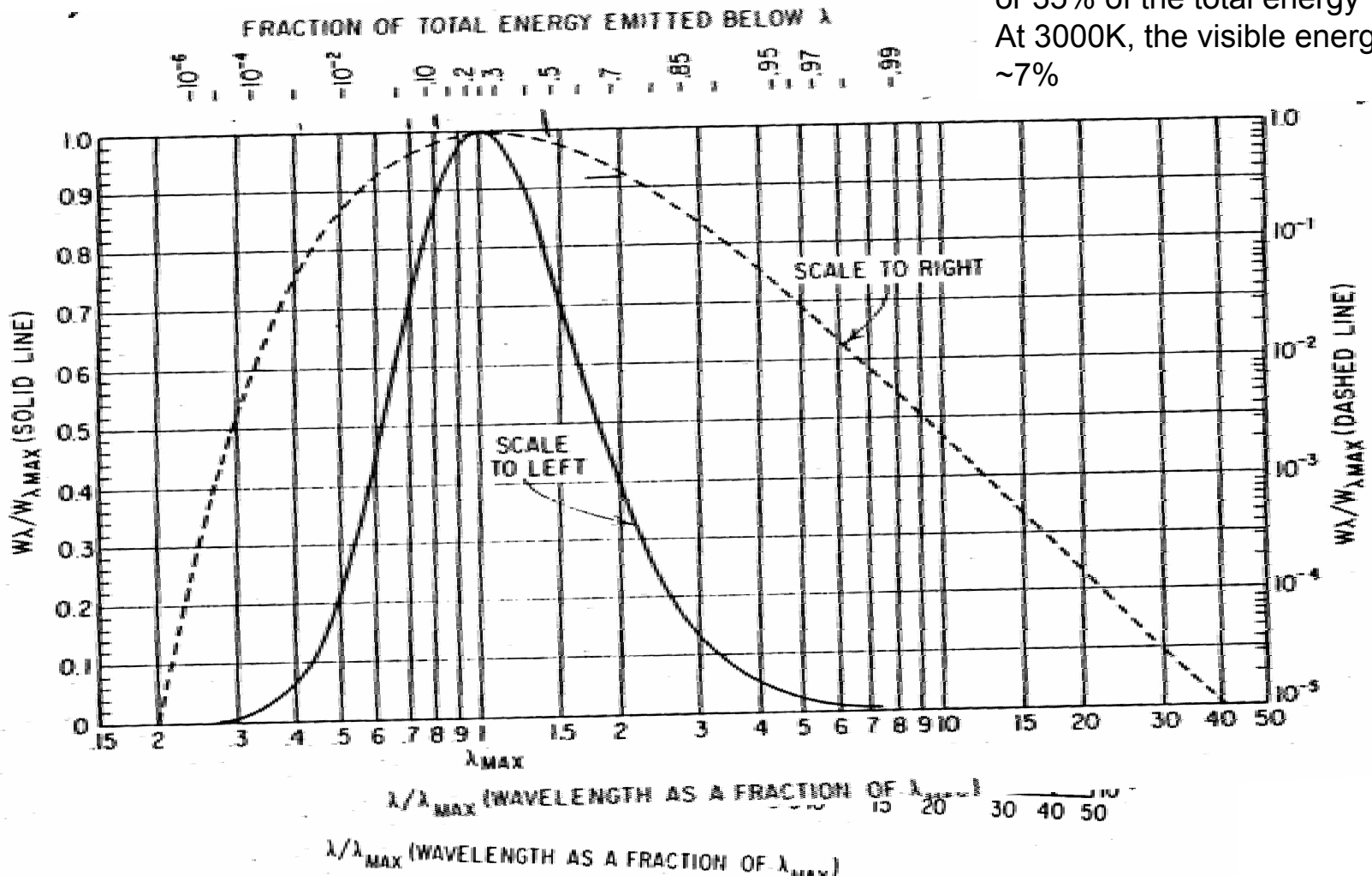
- InSb, HgCdTe, and room-temperature Thermal-pile Focal Plane Arrays (FPA) are all “Strategically sensitive” items

References

- *Electro-Optics* by Lewis J Pinson, John Wiley & Sons, Inc., (1985)
- *Modern Physics* by Serway, Moses, and Moyer, Saunders College Publishing, 1997
- *Optical Radiation Detectors* by Dereniak and Crowe, John Wiley and Sons
- *Infrared Handbook* by Wolfe etc., Environmental Research Institute of Michigan

Normalized Blackbody Equation

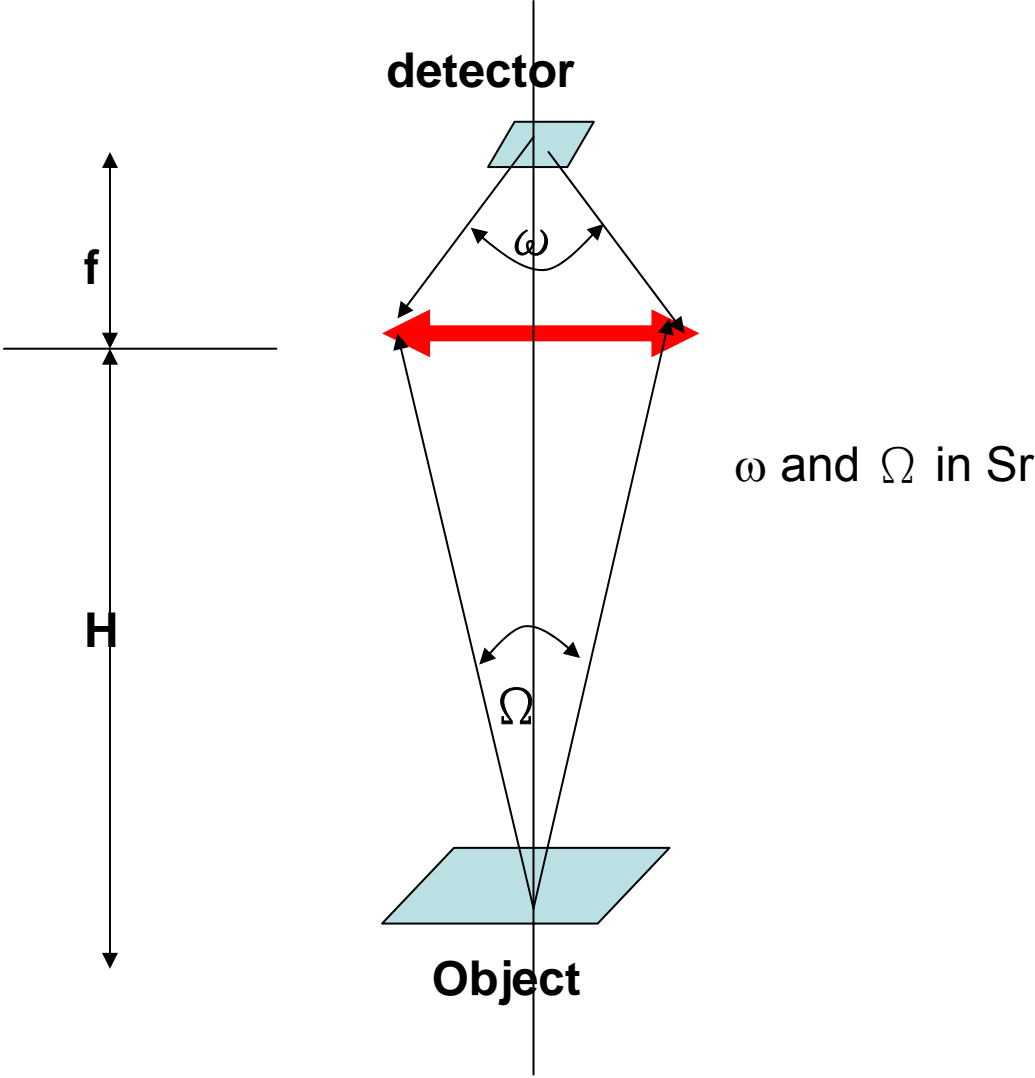
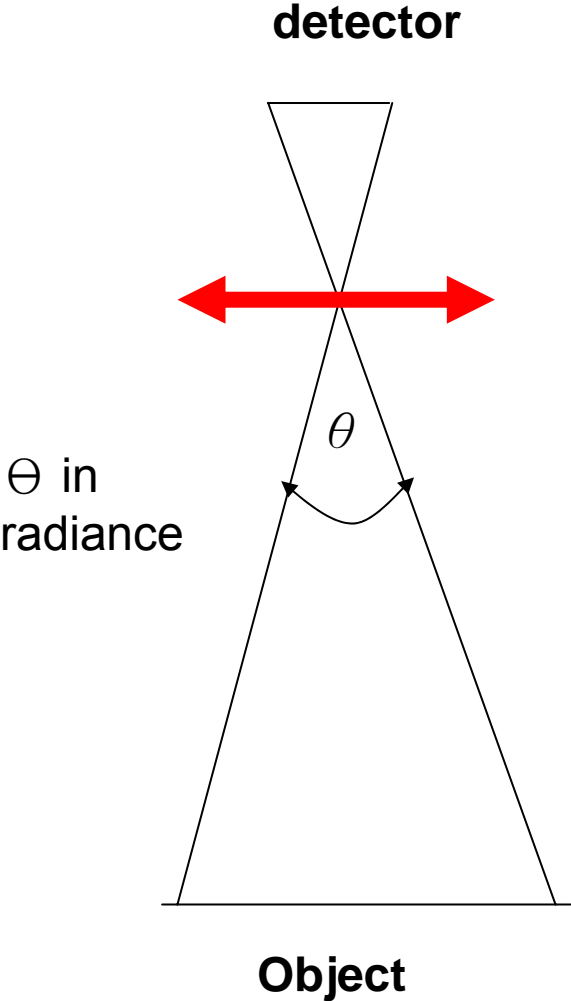
At $T=6000\text{K}$, $\lambda_{\text{max}}=0.5\ \mu\text{m}$
 Since $0.4 < \lambda_{\text{vis}}/\mu\text{m} < 0.7$ or
 $0.8 < \lambda / \lambda_{\text{max}} < 1.4$
 So % visible energy is
 $0.45 - 0.1 = 0.35$
 or 35% of the total energy
 At 3000K , the visible energy
 $\sim 7\%$



Homework(1)

- At the Daper point (800K), a blackbody begins to be visible, computer the visible exitance
- A 1-cm thick, 1m square plate has one side perpendicular to the sun and a conductivity of 0.01 W/m/K. If the emissivities of both surfaces are all 1/5.67, and the shodow side of the surface temperature is 300K, compute the solar absorptivity of the surface facing the Sun
- Prove the blackbody equation can be normalized as $M_{\lambda} / M_{\lambda, \max}$ vs. λ / λ_{\max}
- Compute the percentage increase in visible energy for a 3000K blackbody to 3400K (incandescent tungsten to halogen)

IFOV and Solid Angles



Radiometry Identity $a_d \omega = A \Omega$

$$\frac{a_d}{f^2} = \frac{A}{H^2}$$

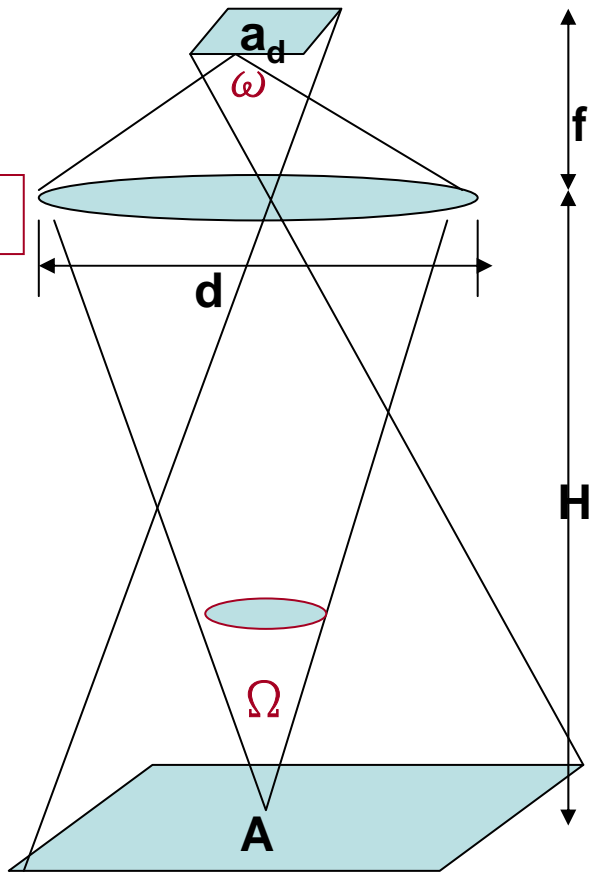
Multiplying both sides
by $\pi d^2/4$ yields

$$\frac{\pi d^2}{4} \frac{a_d}{f^2} = \frac{\pi d^2}{4} \frac{A}{H^2}$$

Since $\omega = \pi d^2/4f^2$ and $\Omega = \pi d^2/4H^2$

Thus

$$a_d \omega = A \Omega$$



Optical Power on a Detector

- The Optical Power Falling on a Detector is:

$$P[W]$$

$$= T_{\text{opt}} \cdot L[W \text{ Sr}^{-1} \text{ m}^2 \mu\text{m}] \cdot \Omega A \cdot \Delta \lambda$$

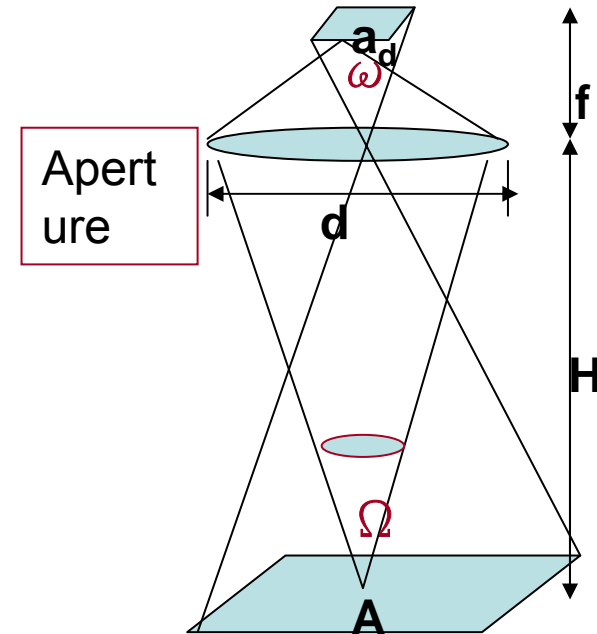
Substituting the Radiometry Identity yields:

$$= T_{\text{opt}} \cdot L \cdot \omega a_{\text{det}} \cdot \Delta \lambda$$

$$= T_{\text{opt}} \cdot L \cdot \frac{\pi}{4 \bullet (F/\#)^2} a_{\text{det}} \cdot \Delta \lambda$$

Since

$$\omega = \frac{\pi d^2}{4 f^2} = \frac{\pi}{4 (f/d)^2} = \frac{\pi}{4 \bullet (F/\#)^2}$$



Detector Responsivity

- An Ideal Detector Generates one e- for every Photon absorbed:

$$R_{ideal} = \frac{q}{h\nu} = \frac{q\lambda}{hc} \approx 0.8 \cdot \lambda [A/W]$$

An Actual Detector Responsivity is:

$$R[A/W] = \eta R_{ideal} = 0.8 \eta \lambda$$

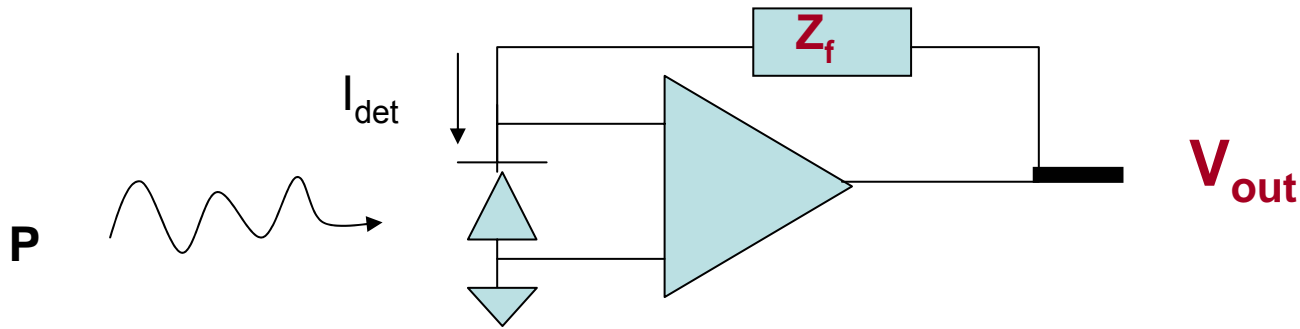
$q = 1.6 \times 10^{-19}$ Amp-sec

Note: λ in μm

Theoretical Detector Output (TIA)

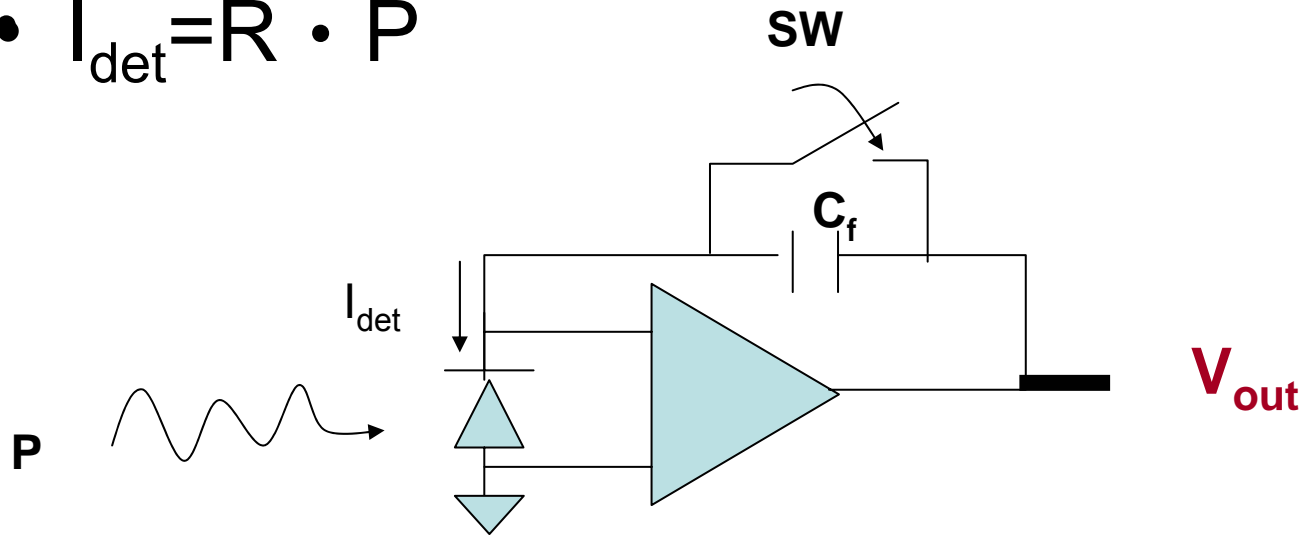
- $I_{\text{det}} = R \cdot P$

- $V_{\text{out}} = I_{\text{det}} \cdot Z_f$



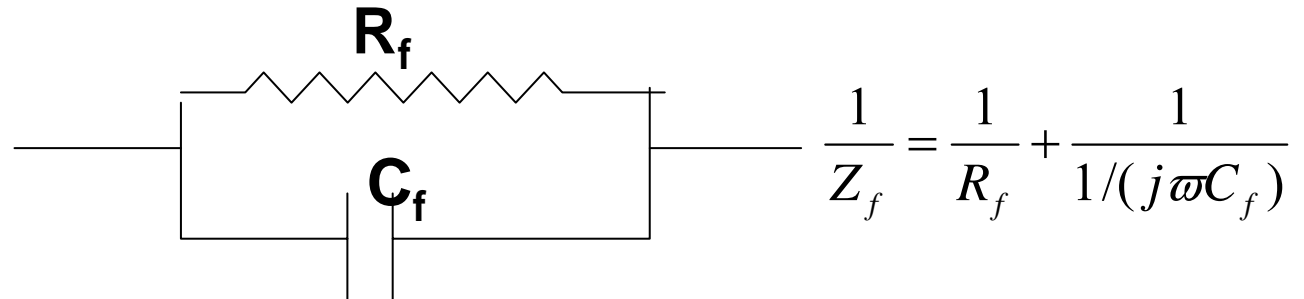
Theoretical Detector Output (CTIA)

- $I_{\text{det}} = R \cdot P$



$$V_{\text{out}} = \frac{I_{\text{det}} \cdot \tau_{\text{int}}}{C_f}$$

TIA Impedance Transfer Function



The Complex Impedance is

$$Z_f = \frac{R_f - j\omega R_f^2 C_f}{1 + \omega^2 R_f^2 C_f^2}$$

The Transfer Function is:

$$[Z_f] = \frac{R_f}{\sqrt{1 + \omega^2 R_f^2 C_f^2}}$$

When $\omega = 1/RC$

$$|Z_f| = R_f / \sqrt{2}$$

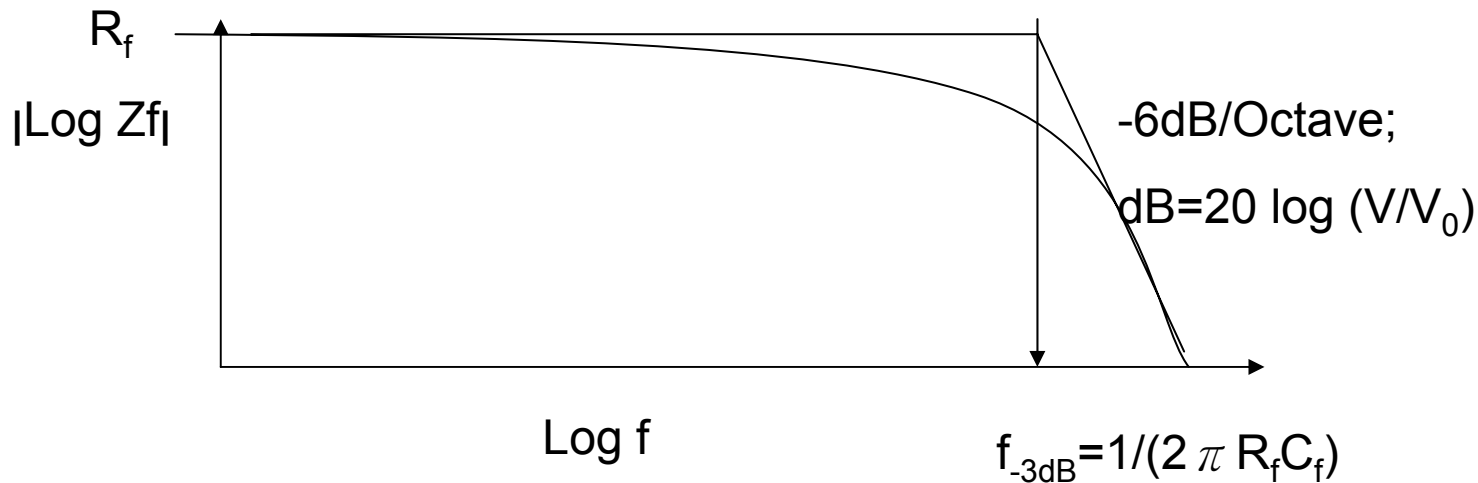
Frequency Response of Z_f

Since $\omega = 2\pi f$, and $f_{3dB} = 1/(2\pi R_f C_f)$

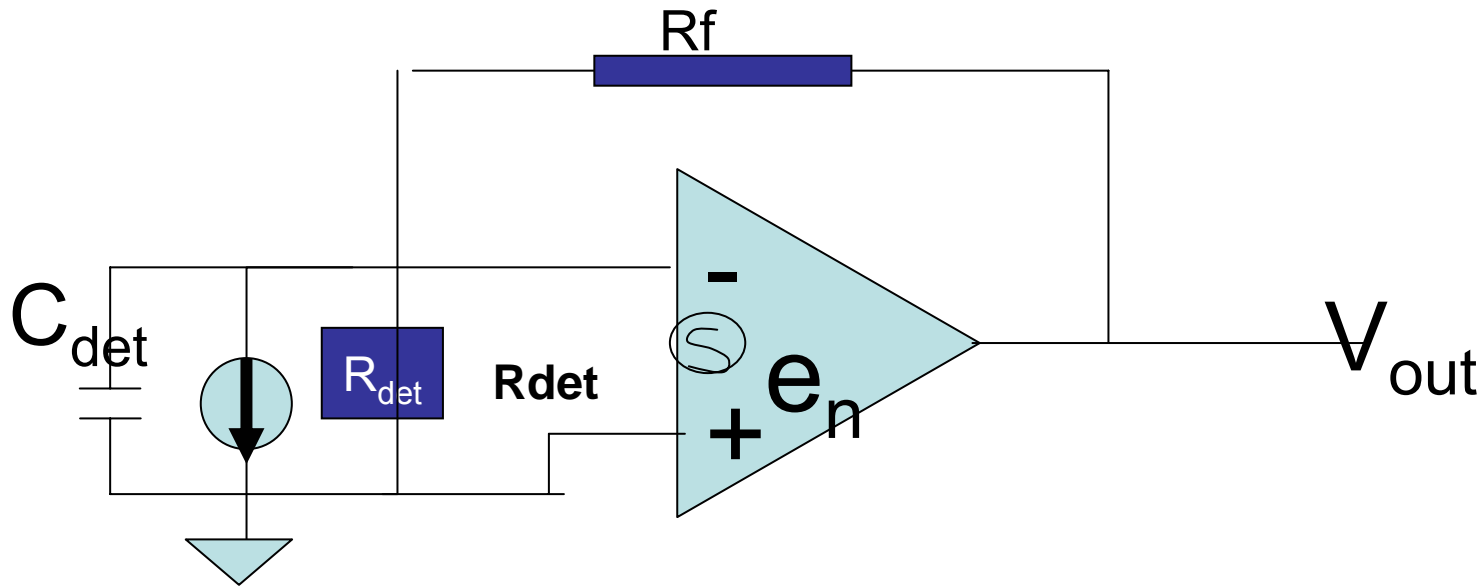
$|Z_f| = R_f / \sqrt{2} = 0.707 R_f$ @ $f_{-3dB} = 1/(2\pi R_f C_f)$

$\text{dB} = 20 \log(0.707) = -3 \text{ dB}$

$$[Z_f] = \frac{R_f}{\sqrt{1 + \omega^2 R_f^2 C_f^2}}$$



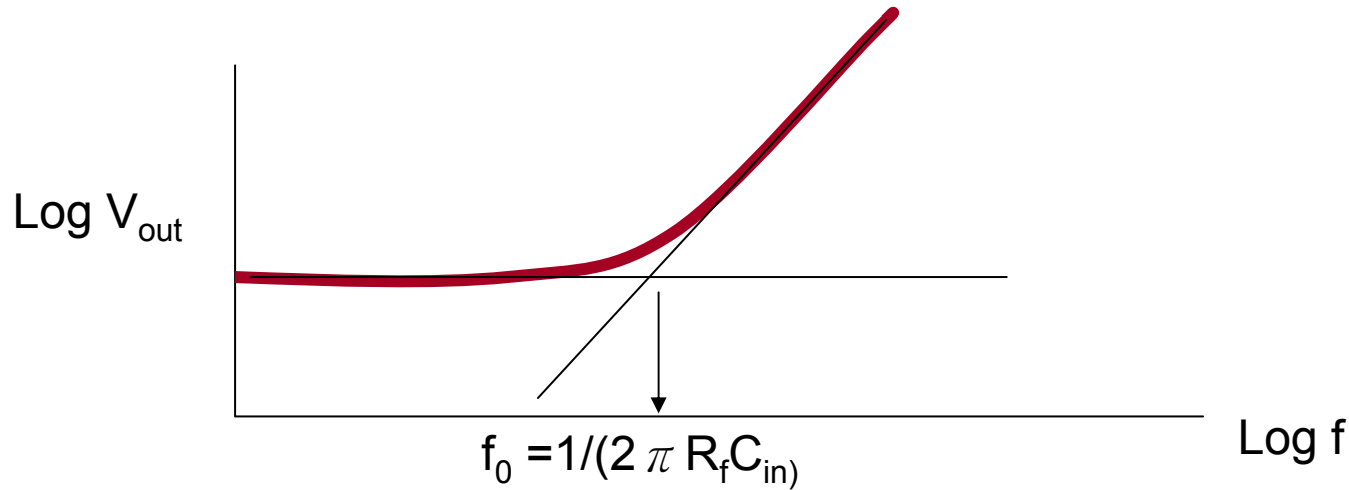
Why is that Detector Impedance needs to be High



If $R_{det} \gg R_f$

$$V_{out} = e_n \left(1 + \frac{R_f}{R_{det}} \right) \approx e_n$$

“Boosted Input Noise from C_{in} ”



$$V_{out} = e_n \left(1 + \frac{R_f}{1/j\omega C_{in}} \right) = e_n (1 - j\omega R_f C_{in})$$

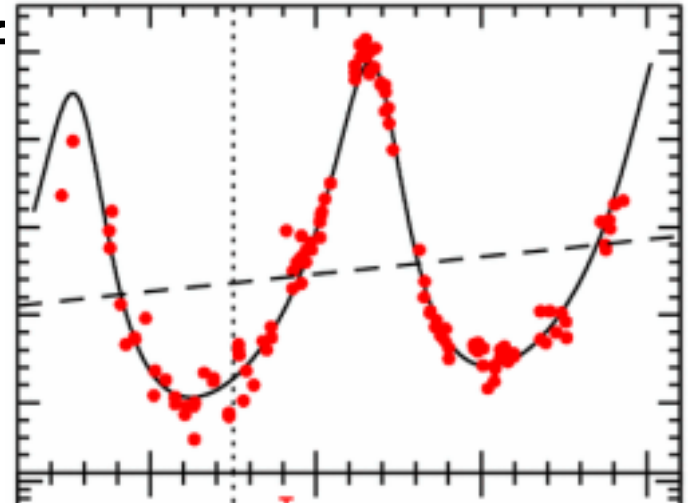
$$|V_{out}| = e_n \sqrt{(1 + \omega^2 R_f^2 C_{in}^2)}$$

When $f_0 = 1/(2\pi RC)$

$$|V_{out}| = \sqrt{2} e_n$$

Chopping is Essential for “Drifting Signal”

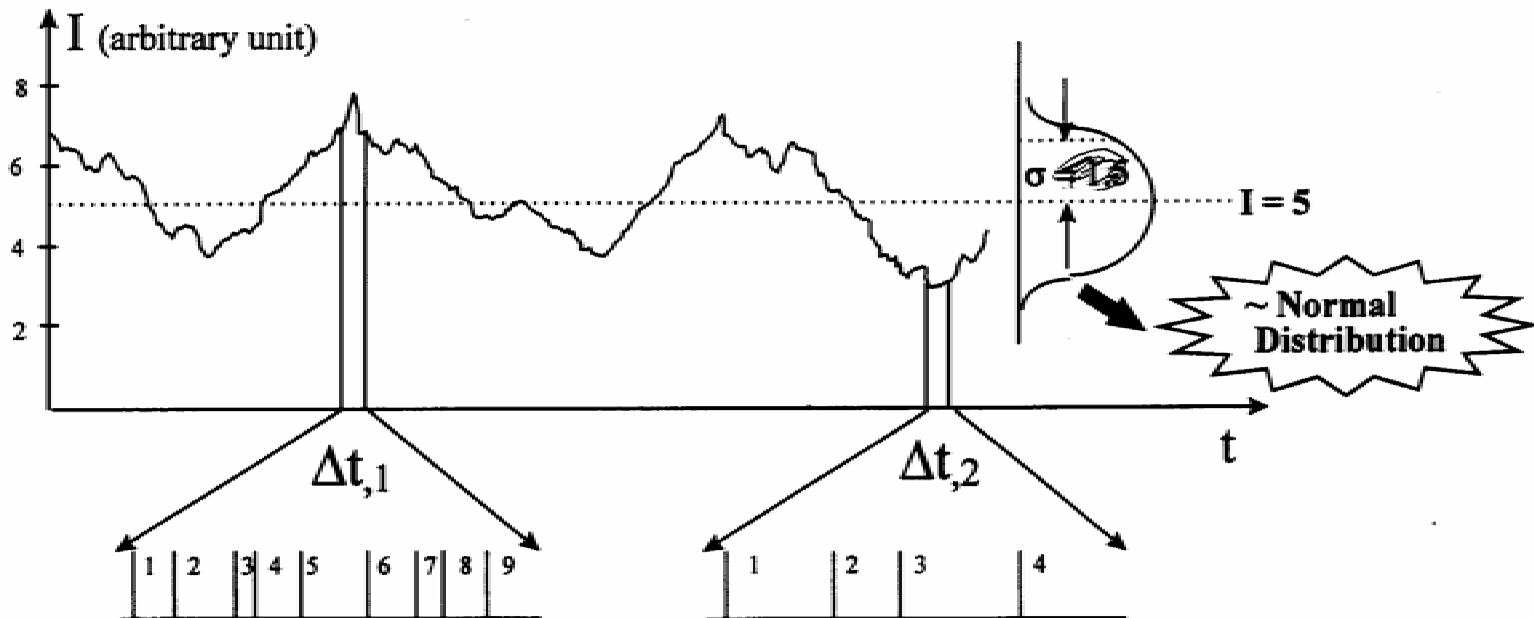
- Chopping Effectively “De-couples” the Slow Drifting “1/f” Noise (including D.C. Level)
- In CCD, A “Correlated Double Sampling” is Used to Eliminate Drif



Poisson and Gaussian

The arrival of Photons follows discrete Poisson process; when large number of photons arrive, the distribution pattern is a continuous Gaussian distribution.

Thermally generated electron-hole pairs (Dark current) generation essentially follows the same principle.



Shot Noise Current Density (Photon Noise)

Photon arrival is a "Poisson Distribution". $= N^k e^{-N} / k!$; $k=0,1,2,3$ (photon arrival sequence). If N is the average no. of photons, then the variance of a Poisson distribution is the same as the mean N .

$$I = \bar{i} = Nq / \Delta t$$

$$\overline{i_n^2} = \overline{(i - I)^2} = (q / \Delta t)^2 \overline{(n - N)^2} = (q / \Delta t)^2 \cdot N$$

Shot Noise

$$\overline{i_n^2} = qI / \Delta t = 2 \cdot q \cdot I \cdot \Delta f$$

Shot Current Noise is thus: $[2qI]^{1/2} / \sqrt{\Delta f}$

So Shot Noise Output Voltage in a TIA circuit is:

$$e_{\text{shot}} = [2 q I \Delta f]^{1/2} R_f$$

An ideal IT system is a Background Limited Infrared Photo-detector (BLIP) system.

Nyquist Frequency

- The minimum Digitization interval needed to represent a periodical signal is $\frac{1}{2}$ of its maximum frequency, or

$$\Delta t = 1/(2 f_{\max})$$

So the noise bandwidth Δf is defined as:

$$\Delta f = 1/(2\Delta t)$$

Total Noise $e_{n,\text{total}}$

$$e_{n,\text{total}}^2 = e_{\text{det}}^2 + e_{\text{dark}}^2 + e_{\text{amp}}^2 + e_{\text{Rf}}^2 + e_{\text{photon}}^2$$

Or

$$e_{n,\text{total}} = \{e_{\text{det}}^2 + e_{\text{dark}}^2 + e_{\text{amp}}^2 + e_{\text{Rf}}^2 + e_{\text{photon}}^2\}^{1/2}$$

$$e_{\text{det}}^2 = \int_{\Delta f} k f^{-\alpha} df$$

$$e_{\text{dark}}^2 = 2qI_{\text{dark}} R_f \Delta f$$

$$e_{\text{amp}}^2 = \int_{\Delta f} [e_{\text{input}} (1 + \omega^2 R_f^2 C_{\text{in}}^2)] df$$

$$e_{\text{Rf}}^2 = 4kR_f T \Delta f$$

$$e_{\text{photon}}^2 = 2qI_{\text{photon}} R_f \Delta f$$

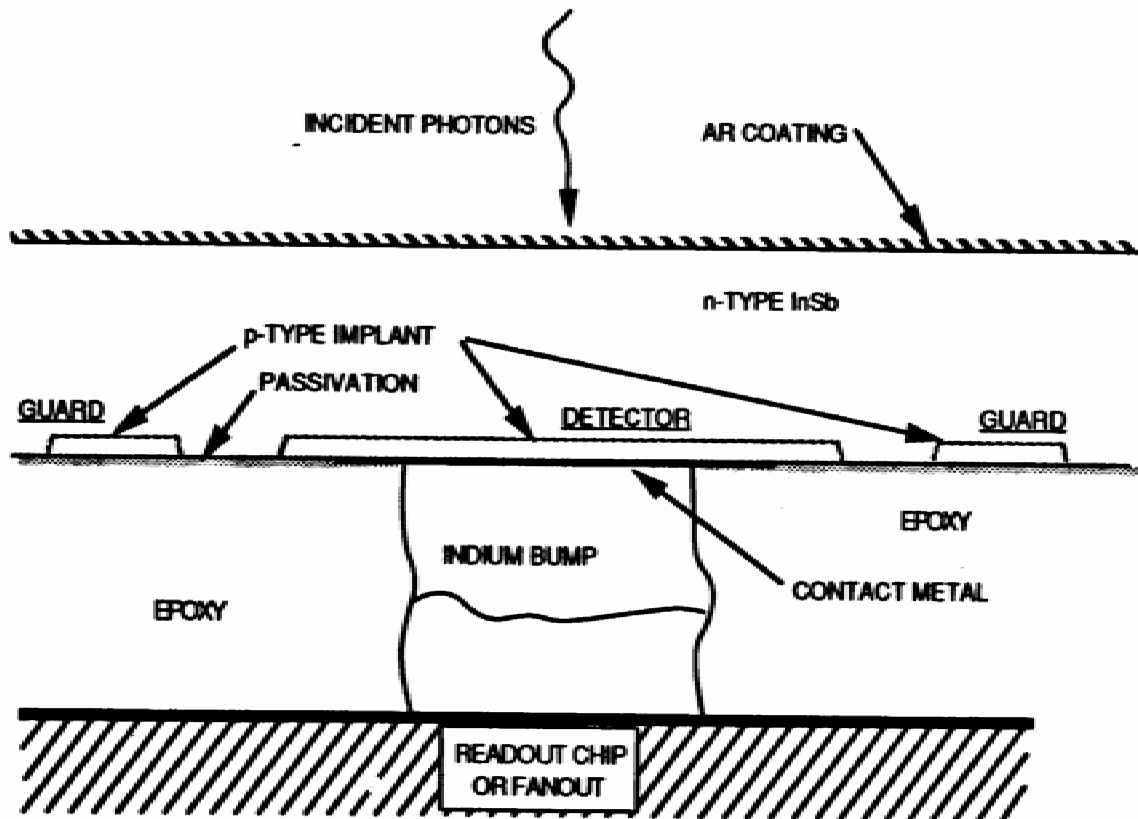
Two ways to Present “noises”

1. Input Referred: Noise current
2. Output Referred: Noise voltage

NEP, D, and D*

- $NEP = I_{\text{noise}} / \text{Resp} \text{ [W]}$
- $D = 1 / NEP \text{ [W}^{-1}\text{]}$
- $D^* = (a_{\text{det}} \cdot \Delta f) \cdot D \text{ [cm} \cdot \sqrt{\text{Hz}} \cdot \text{W}^{-1}\text{]}$

Planar Back-side Illuminated InSb SCA (System Chip Assembly)



Homework (II)

System Parameters:

A_{det} : $125 \mu\text{m} \times 125 \mu\text{m}$ InSb PV; Q_{sig} : 10^{16} Ph/cm² @ $5.5 \mu\text{m}$, @ 77K

$\eta=0.85$; unit area dark current I_{dark} : 8×10^{-5} A/cm²; $C_{\text{in}}=8\text{pF}$ $R_f=10\text{M}\Omega$; $C_f=2\text{pF}$;

$e_{n,\text{input}}=6 \text{ nV}/\sqrt{\text{Hz}}$

Physical Parameters:

h : 6.63×10^{-34} W-sec²; $c=3 \times 10^8$ m/sec; q : 1.6×10^{-19} A/sec;

k : 1.38×10^{-23} W-sec/K

Find

1 the “break frequency $f_{-3\text{dB}}$ ” of the feedback circuit, and use $f_{-3\text{dB}}$ for Δf

2a. Johnson noise voltage density; 2b: total Johnson noise voltage

3a. Total photon power falling on the detector 3b: detector responsibility

4a Photon Current, 4b: Signal voltage

5a. Photon Noise current density; 5b: total photon noise voltage

6a. Dark current; 6b: Dark current noise density 6c: dark current noise voltage

7a. Boosted noise voltage density; 7b: total boosted input noise voltage

8a. Total output-output referred noise voltage; 8b: SNR

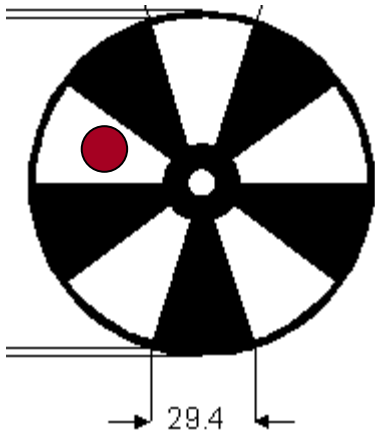
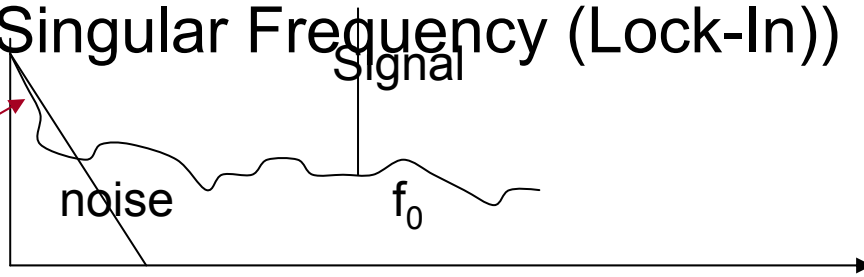
9a: NEP; 9b:D; 9c: system D^*

The Concept of Signal Chopping

- An Ideal way to “decouple” the $1/F$ noise is the use of a Sine Wave Chopper

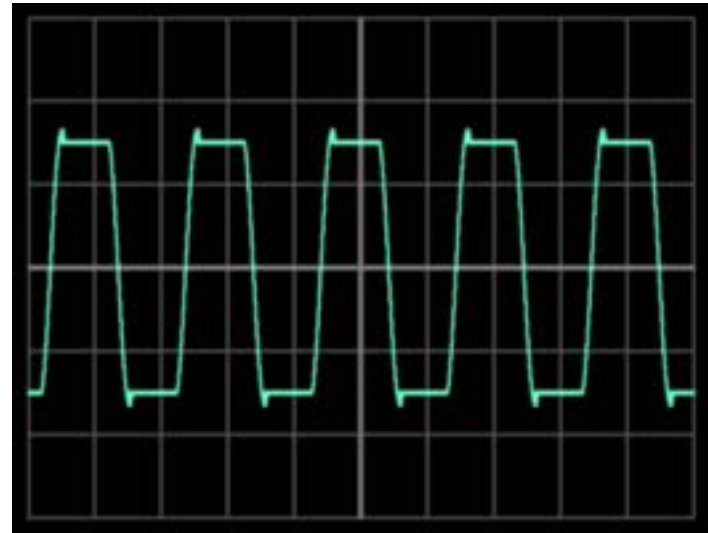
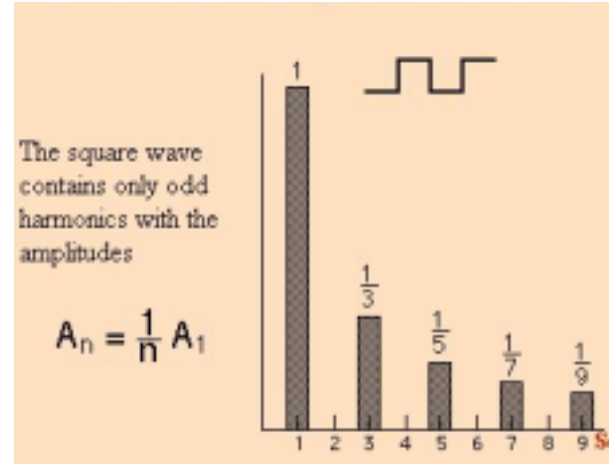
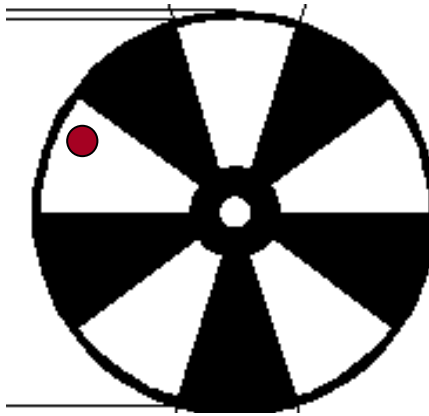
That Generates Singular Frequency (Lock-In)

“1/F”
noise



If the Aperture \gg Beam Size You Have Square Waves

- So we May Still Utilize the “Fundamental Frequency” for Signal Comparison



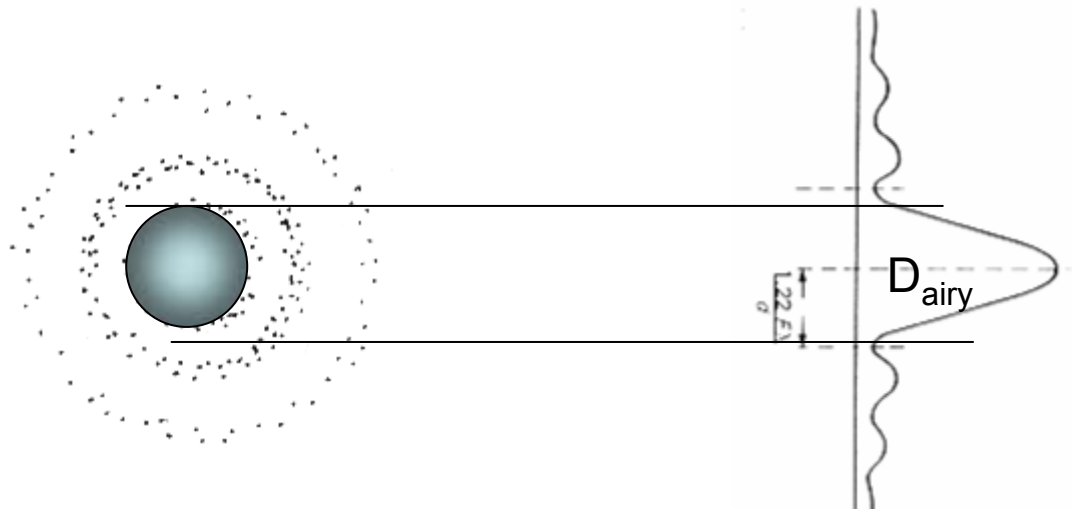
Airy Disc: Diffraction Limited Spot

$$D_{\text{airy}} = 2.44 \lambda (F/\#)$$

$F/\#$: F-Stop of an Optical System = $f.l./D_{\text{aperture}}$

Since Numerical Aperture $NA = D_{\text{aperture}} / (2 \cdot f.l.) = 1 / (2F/\#)$

So $D_{\text{airy}} = 1.22 \cdot \lambda \cdot NA$



How Many Pixels Do we need on a Digital Cameral?

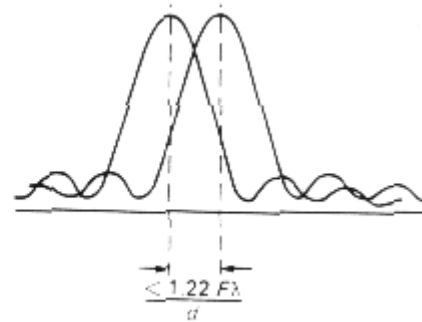
The More the Better?

- Suppose a CCD Chip is 1000x1000, with each pixel dimensions of $7\mu\text{m} \times 7\mu\text{m}$
- The $F/\ast=3.0$;
- at $0.7\mu\text{m}$, the Airy disc is
- $2.44 \times 0.7 \times 3 = 5\mu\text{m}$

- When $\lambda = 10\mu\text{m}$, then the Airy disc = $73\mu\text{m}$!
- In IR cameras, the pixels are “coarser”.

Rayleigh's Spatial Resolution

- The Resolution is Half of the Spot



Optical MTF

An Optical MTF is the Fourier Transform of its “Optical Spot” .If the system is Diffraction limited then its Optical MTF

Can be approximated by:

$$\text{MTF}_{\text{optical}} = (2/\pi) (\phi - \text{Cos}\phi \sin \phi)$$

Where $\phi = \text{Cos}^{-1}(\lambda f / 2\text{NA})$ because the “blur circle is wavelength dependent

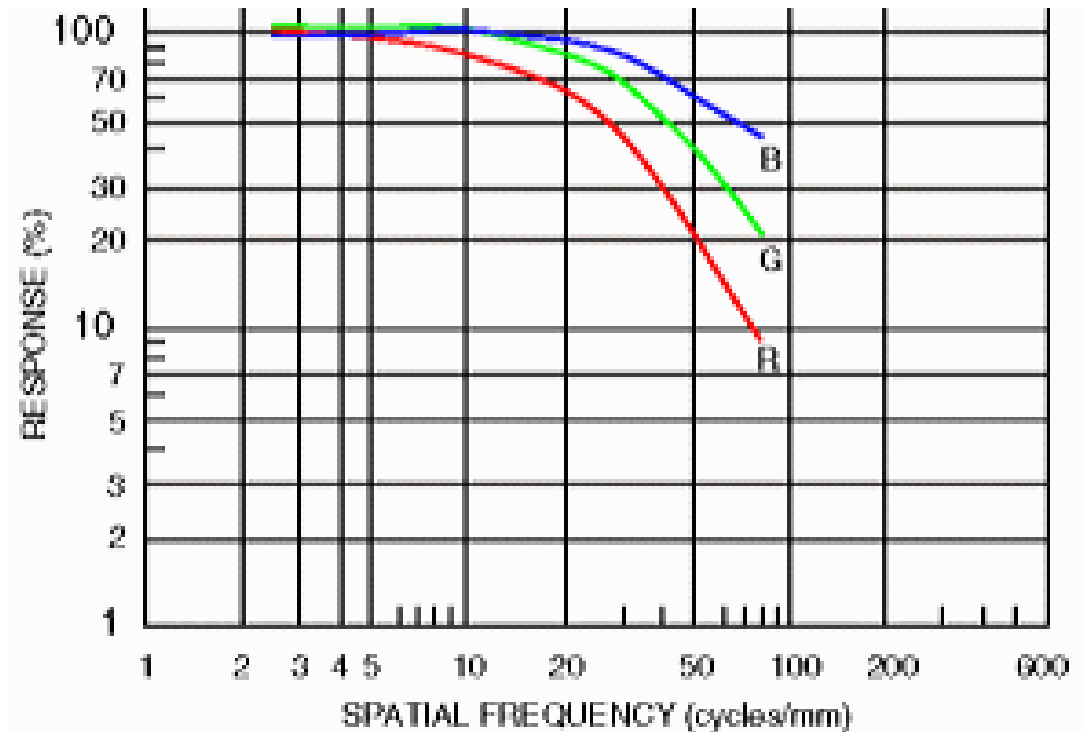
f: spatial frequency in “line pairs/ mm”

Practical Optical MTF Approximation

Most film MTF curves can be closely approximated by a Lorentzian function

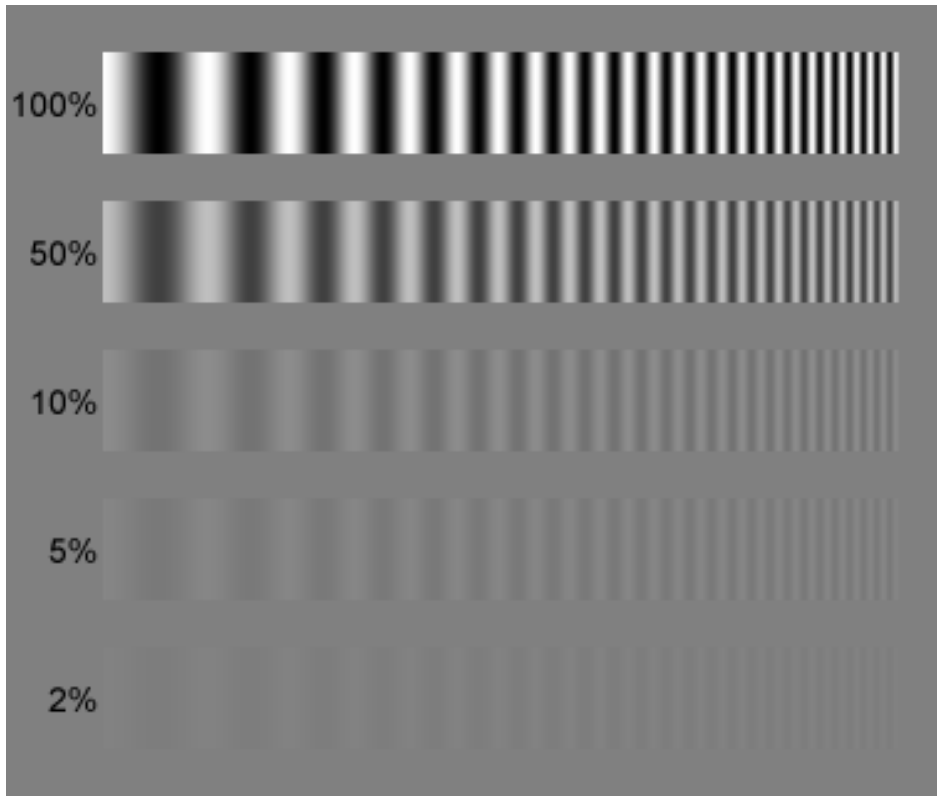
$$\text{MTF} (f) = 1 / (1 + (f / f_{50})^2)$$

Where the “Nyquist MTF”
 f_{50} is wavelength dependent

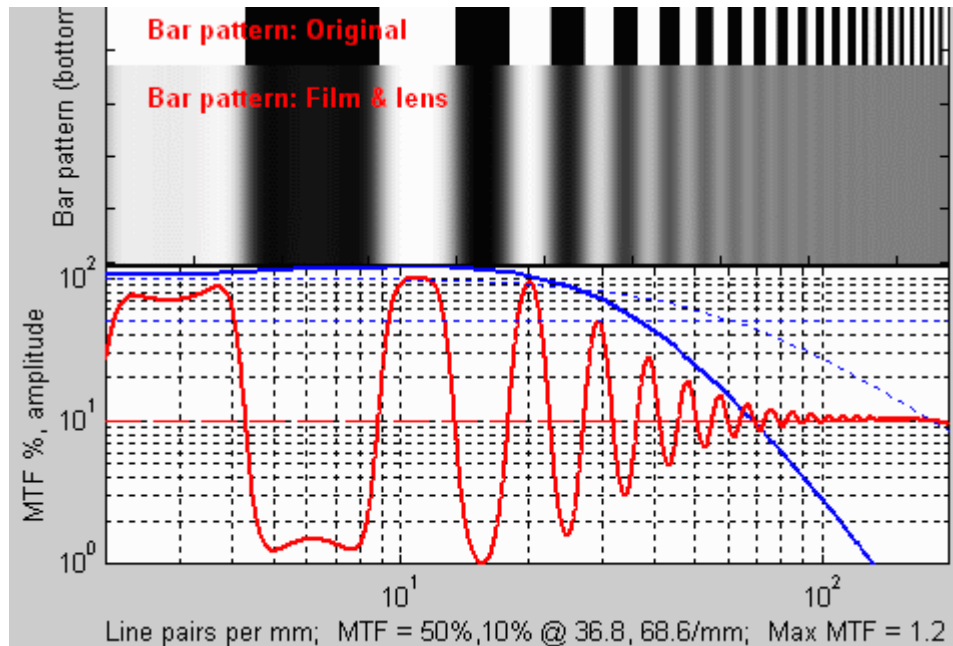


MTF Examples-1

- MTF for a “Pure Tone”: sine function

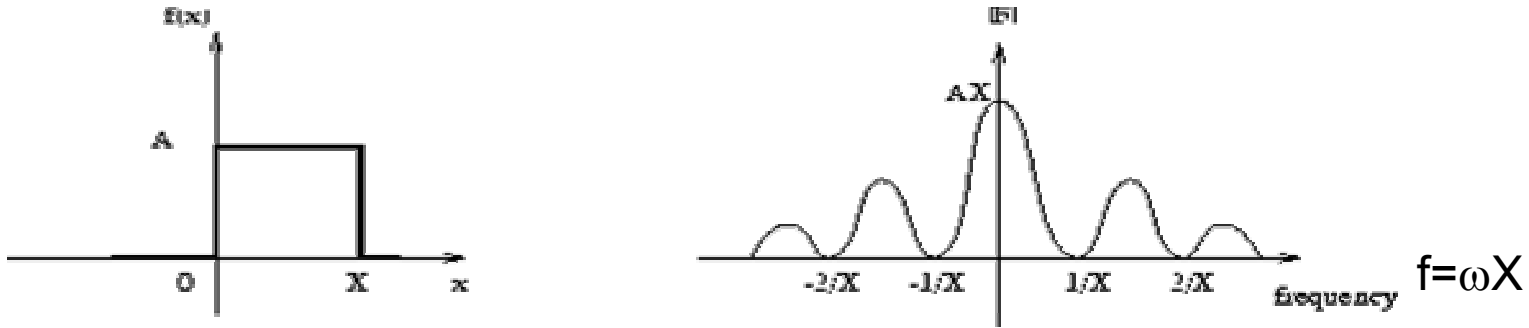


MTF Example-2



Detector MTF: Sinc Function

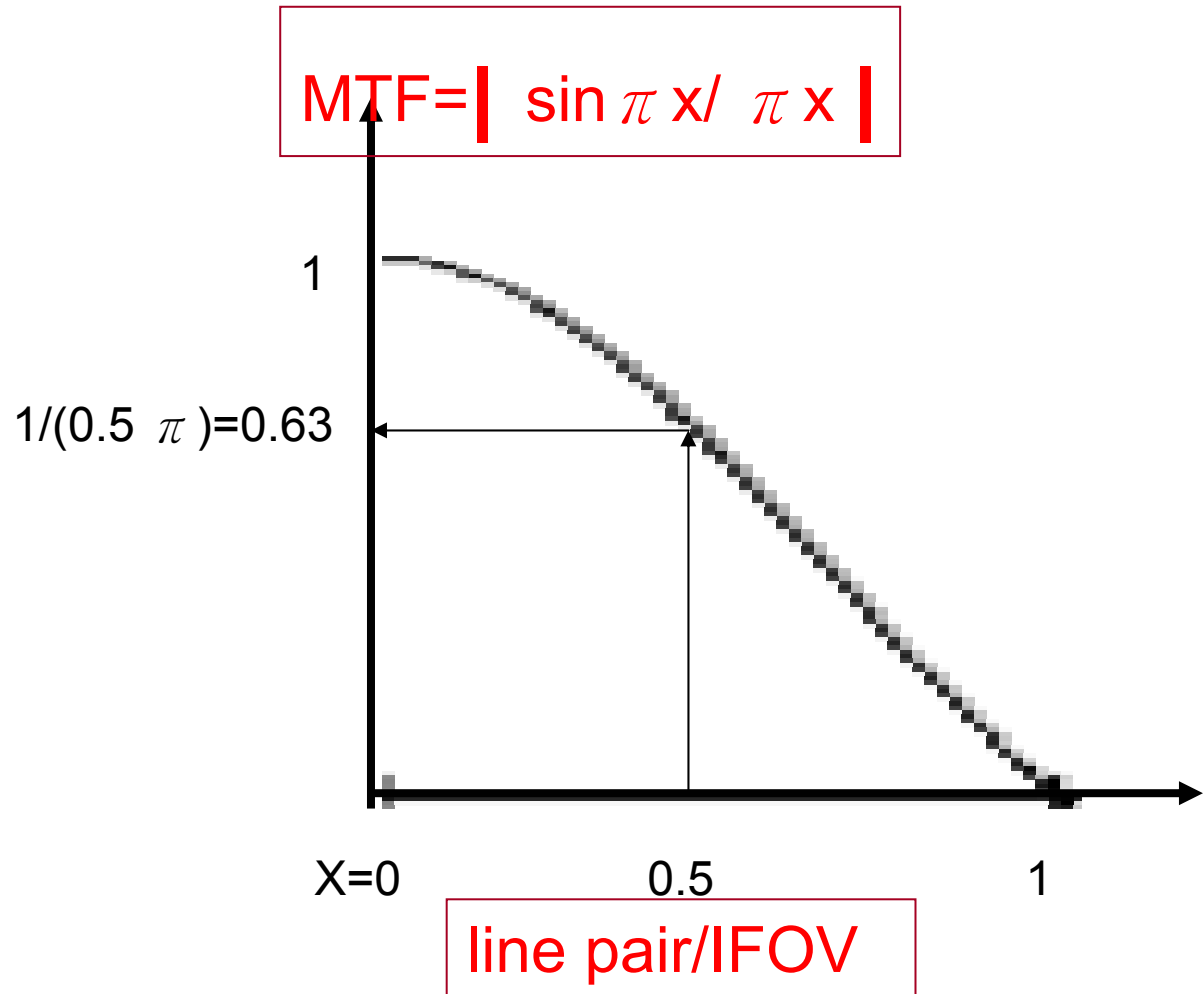
- Convolution of A Square Detector's and a Sine Scene is Detector's MTF



$$MTF = AX \frac{\sin(\pi\omega X)}{\pi\omega X}$$

X : I "IFOV"

Sinc Function: $\text{sinc } x = (\sin \pi x) / \pi x$



Physical Meaning of a Detector MTF

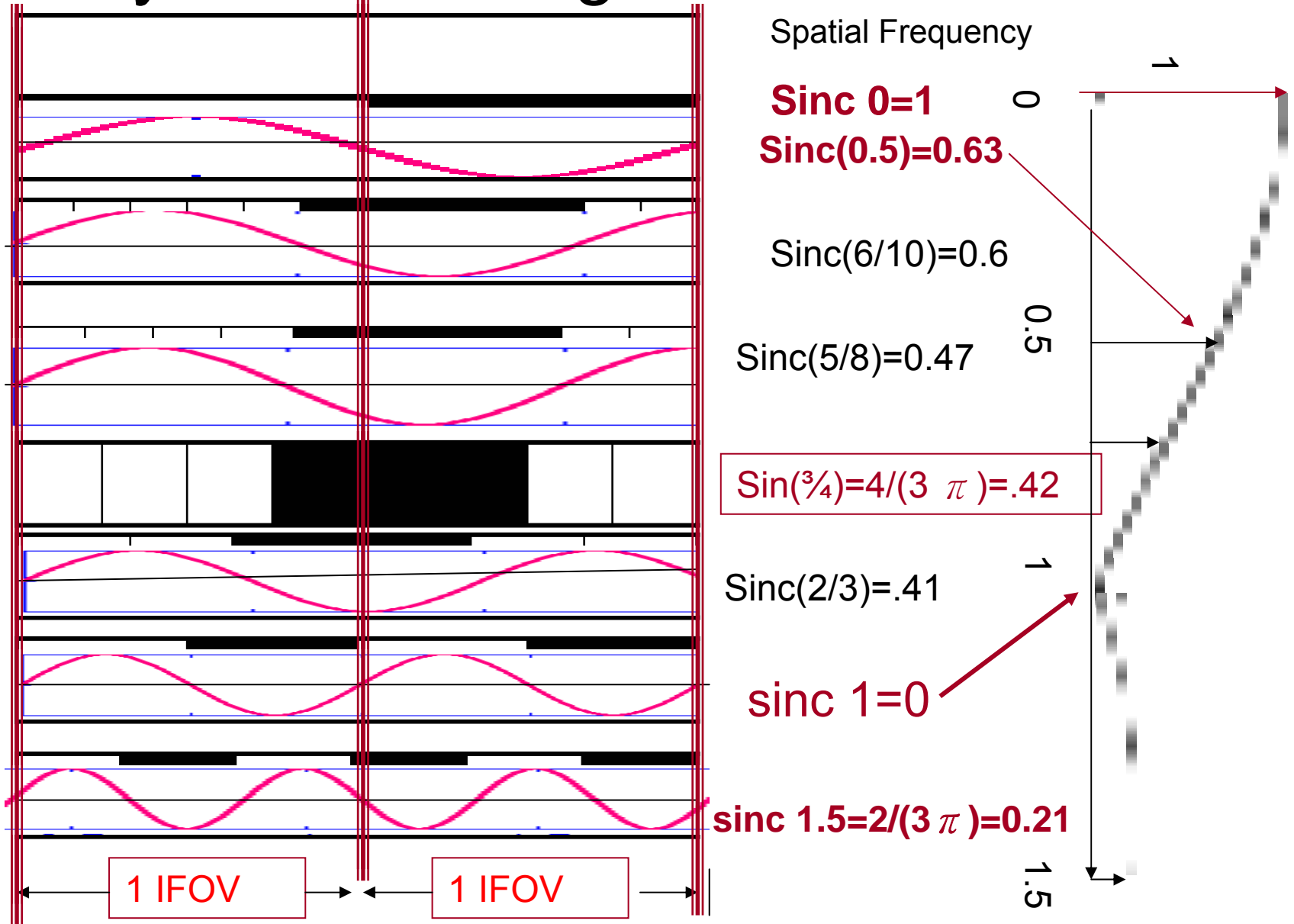


Image Convolution

Convolution of $f(x)$ and $h(x)$

$$f(x) * h(x) = \int_{-\infty}^{\infty} f(u)h(x - u)du$$

Then Total MTF= MTF(f) • MTF(h)

Since Both Optical MTF and Detector MTF never exceed unity, the total MTF is smaller than either MTF,

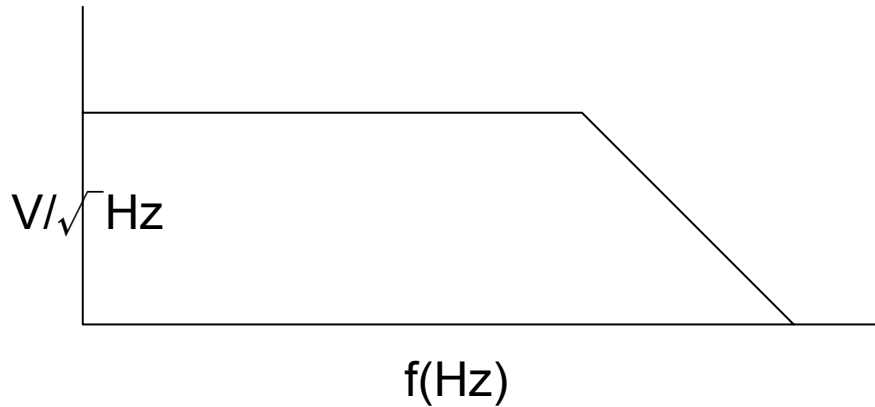
Or,

MTF only degrades as the system gets more complicated!

Total MTF

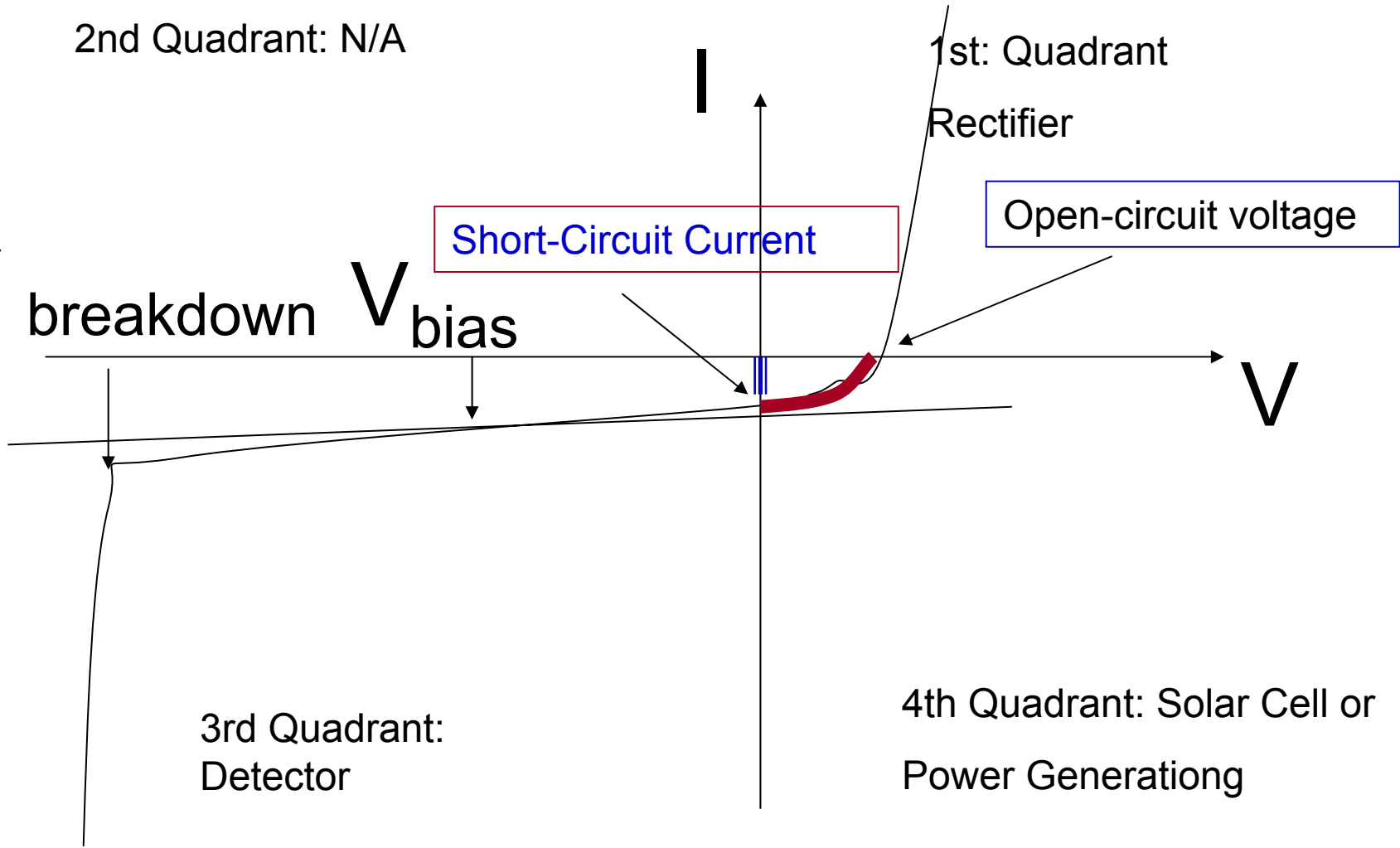
- The Total MTF is the Product of:
- $MTF_{total} = MTF_{optical} \cdot MTF_{det}$
- (If scanning is involved, another MTF electronics would be included as well)
- In the Actual Imaging Space, it means
- “Convolution” of both Optical Blur and the Detector with a Pure Tone Sine wave

Noise Density

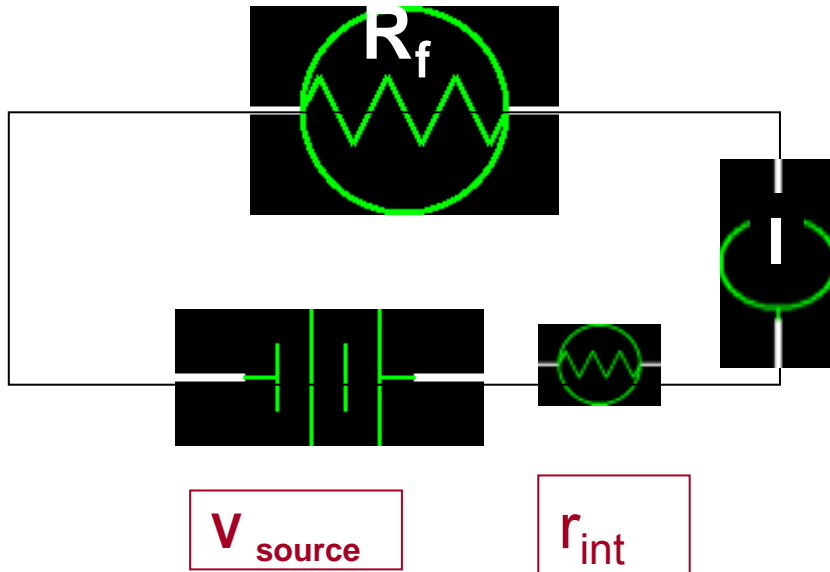


$$r.m.s.noise = \sqrt{\frac{\int_{\Delta f} v^2 df}{\Delta f}}$$

An Ideal Diode Curve



An Ideal Current Source has “Infinite Output Impedance”



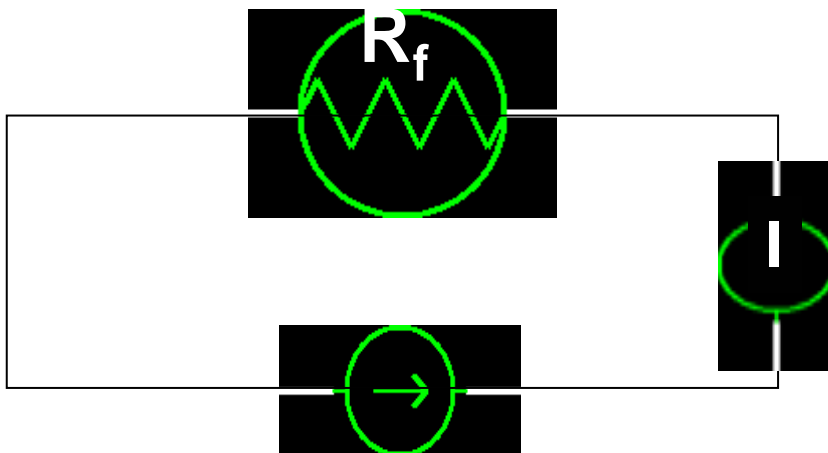
$$I = V_{\text{source}} / (R_f + r_{\text{int}})$$

When $r_{\text{int}} \gg R_f$

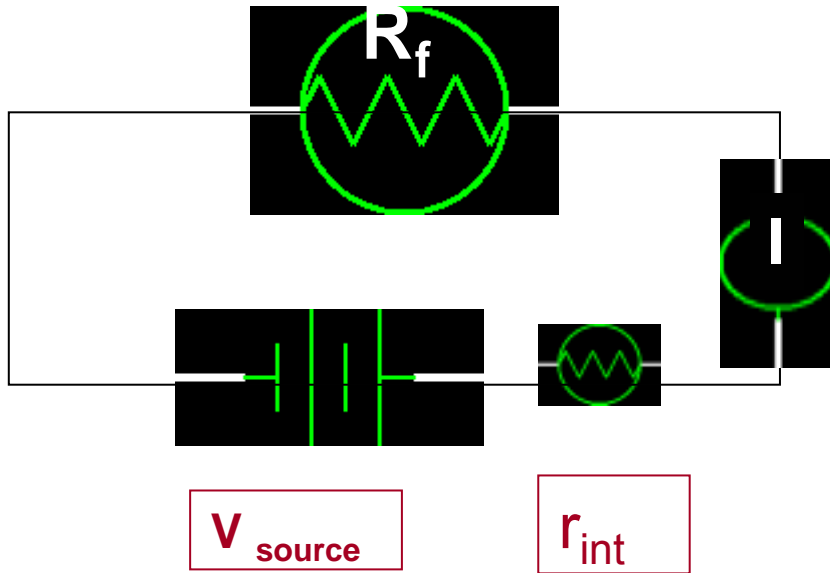
Then

$$I = V_{\text{source}} / r_{\text{int}}$$

Photo-Diodes are a good
Current Sources



An Ideal Voltage Source has “Infinitesimal Output Impedance”

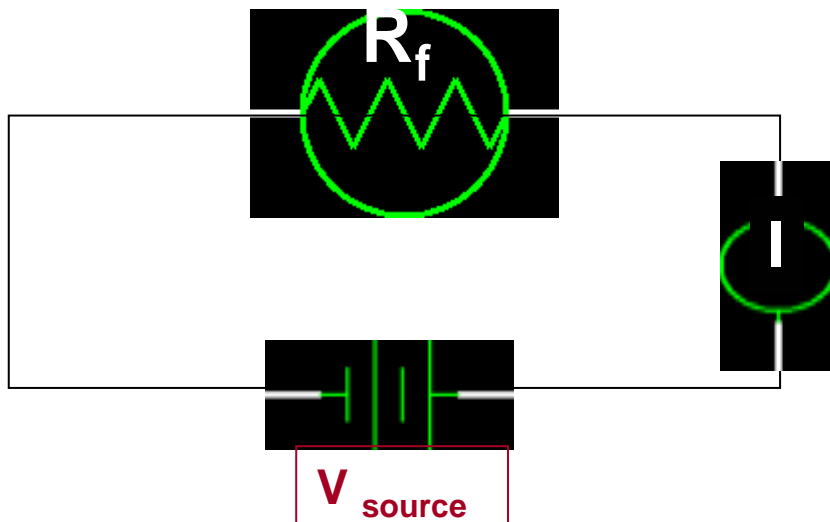


$$I = V_{\text{source}} / (R_f + r_{\text{int}})$$

When $r_{\text{int}} \ll R_f$

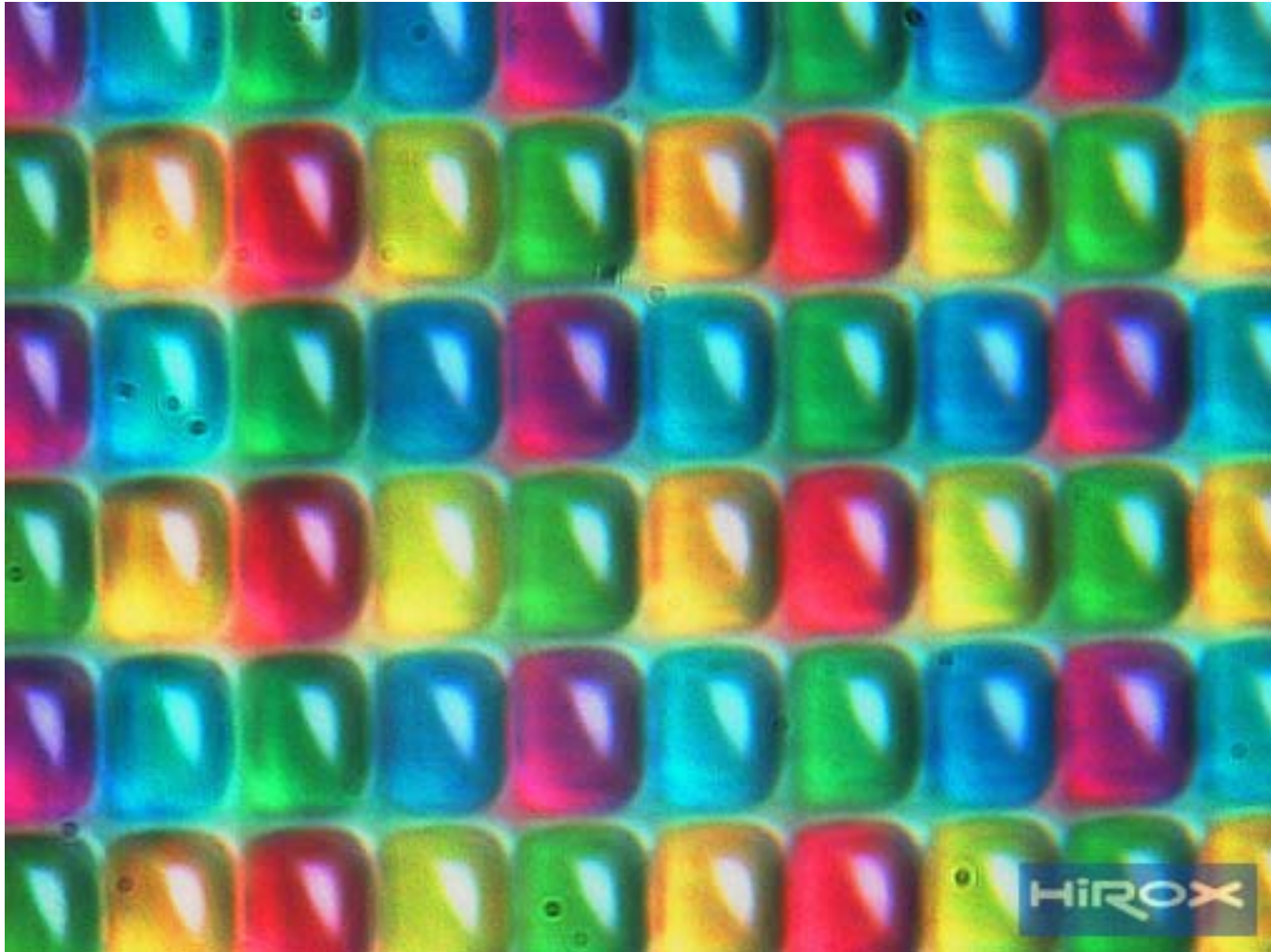
Then

$$I = V_{\text{source}} / R_f$$



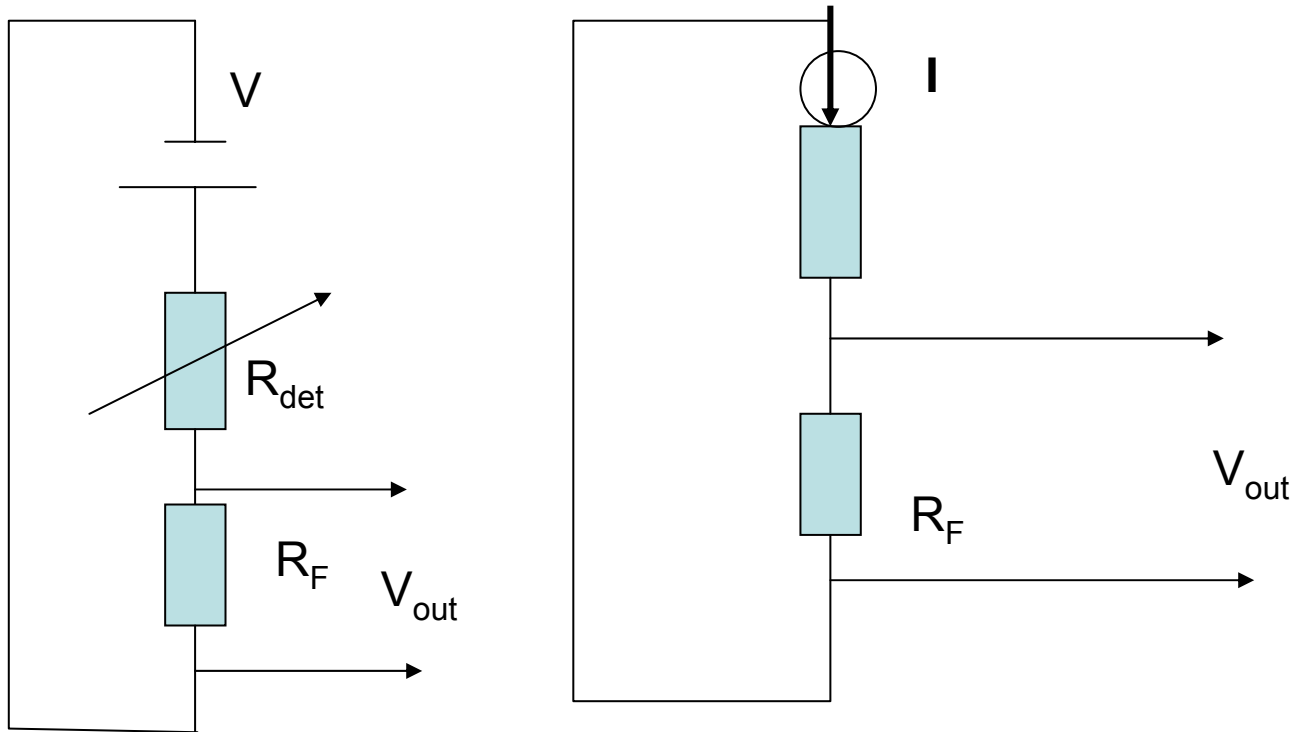
Power Supplies and Voltage Buffers are good Current Source

Video CCD Close-up

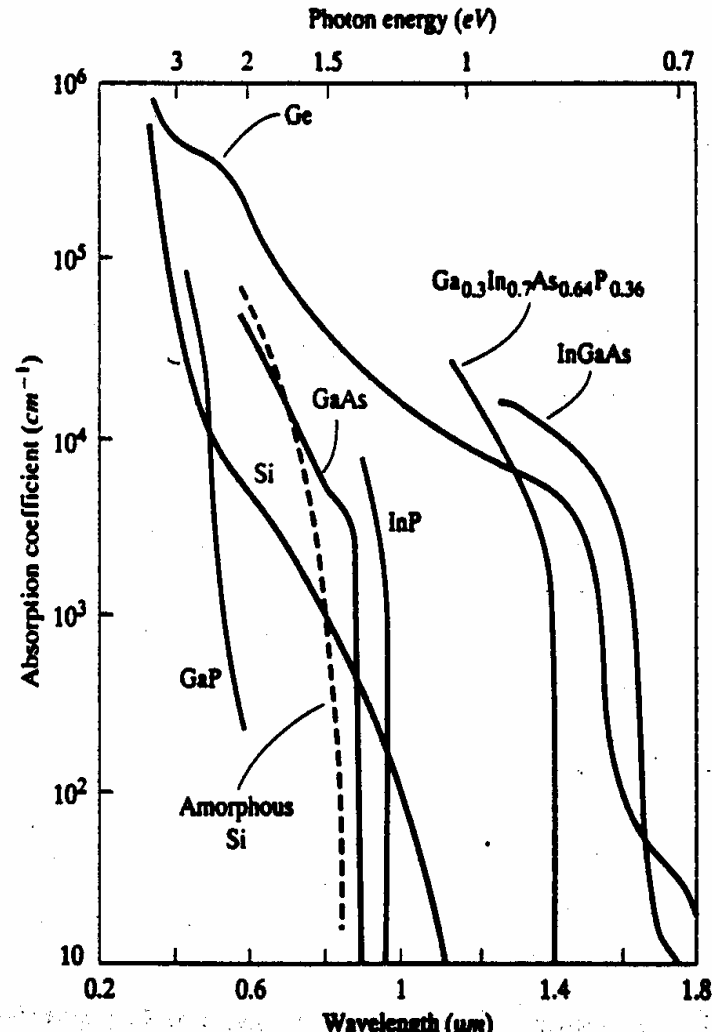


Simple PC Detector Biasing

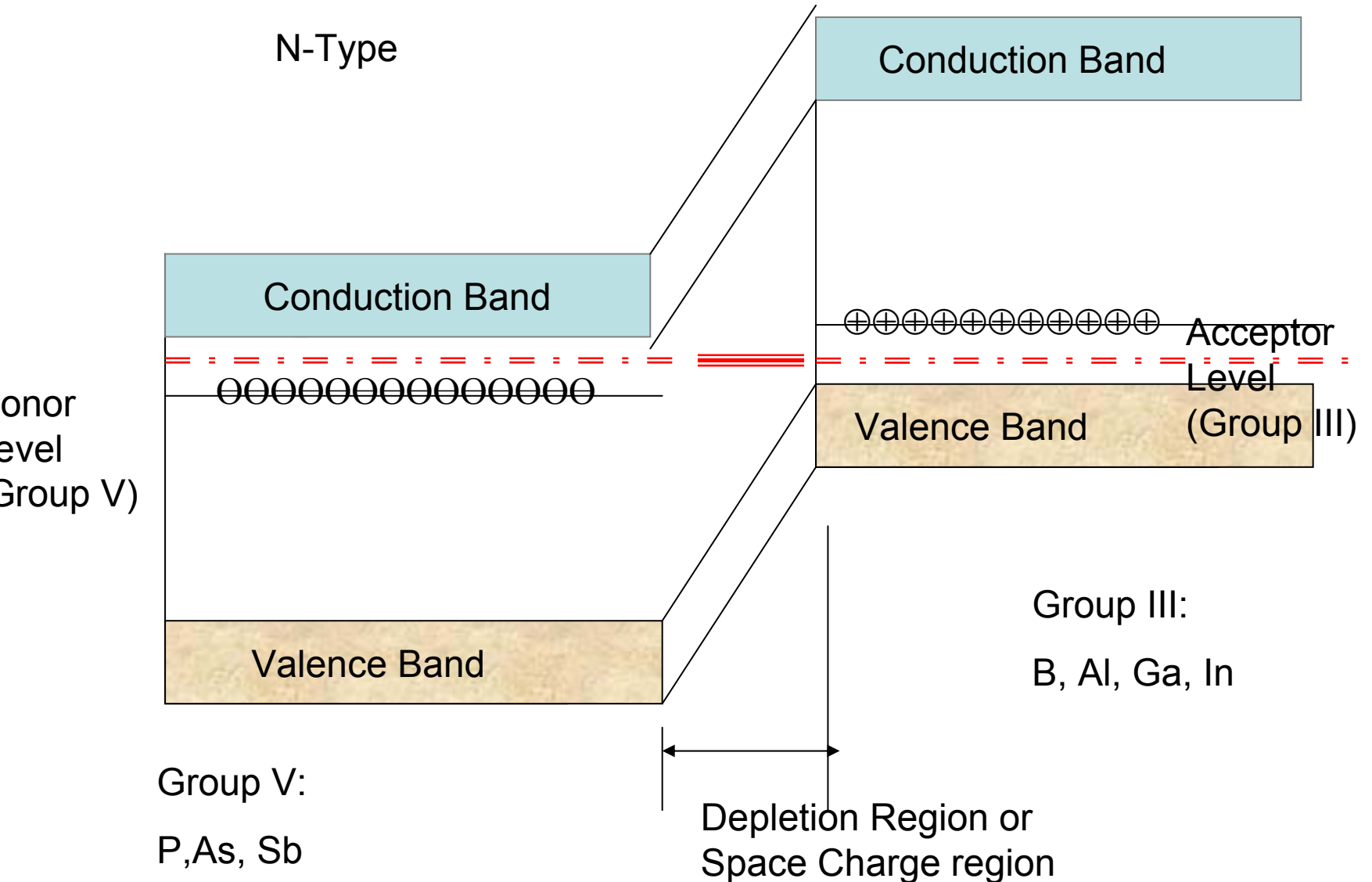
Voltage Biasing



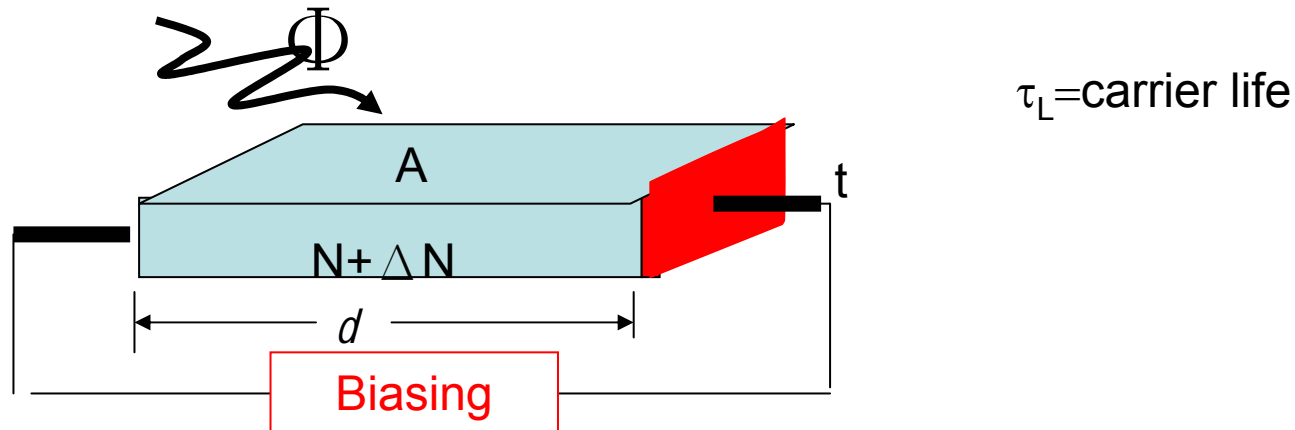
Absorption Coefficients vs. λ



Semi-Conductor P and N types



PC Detector Principle



Photon-induced Charges $\Delta N = \eta \cdot \Phi \tau_L / (A \cdot t)$

Conductivity $\Delta \sigma = q \cdot \Delta N \cdot (\mu_e + \mu_h) \sim \Delta N \sim \Phi$

$R_{\text{det}} = 1 / (\sigma \cdot A)$

$\Delta R_{\text{det}} = -1 / (\sigma^2 \cdot A) \cdot \Delta \sigma = -(R_{\text{det}} / \sigma) \cdot \Delta \sigma \sim \sim \Phi$

$R_{\text{PC}} (A/W) = \eta (q/h \nu) G$

PC Detector Responsibility

$$RPC(A/W) = \eta (q/h \nu) G = 0.8 \eta \cdot \lambda \cdot G$$

G: photo-conductive gain $= \tau \cdot \mu \cdot E/d$

Where $\mu = (\mu_e + \mu_h)$

d=inter-electrode spacing

G can be greater than unity; a blessing and a curse!

PV Detector Principle

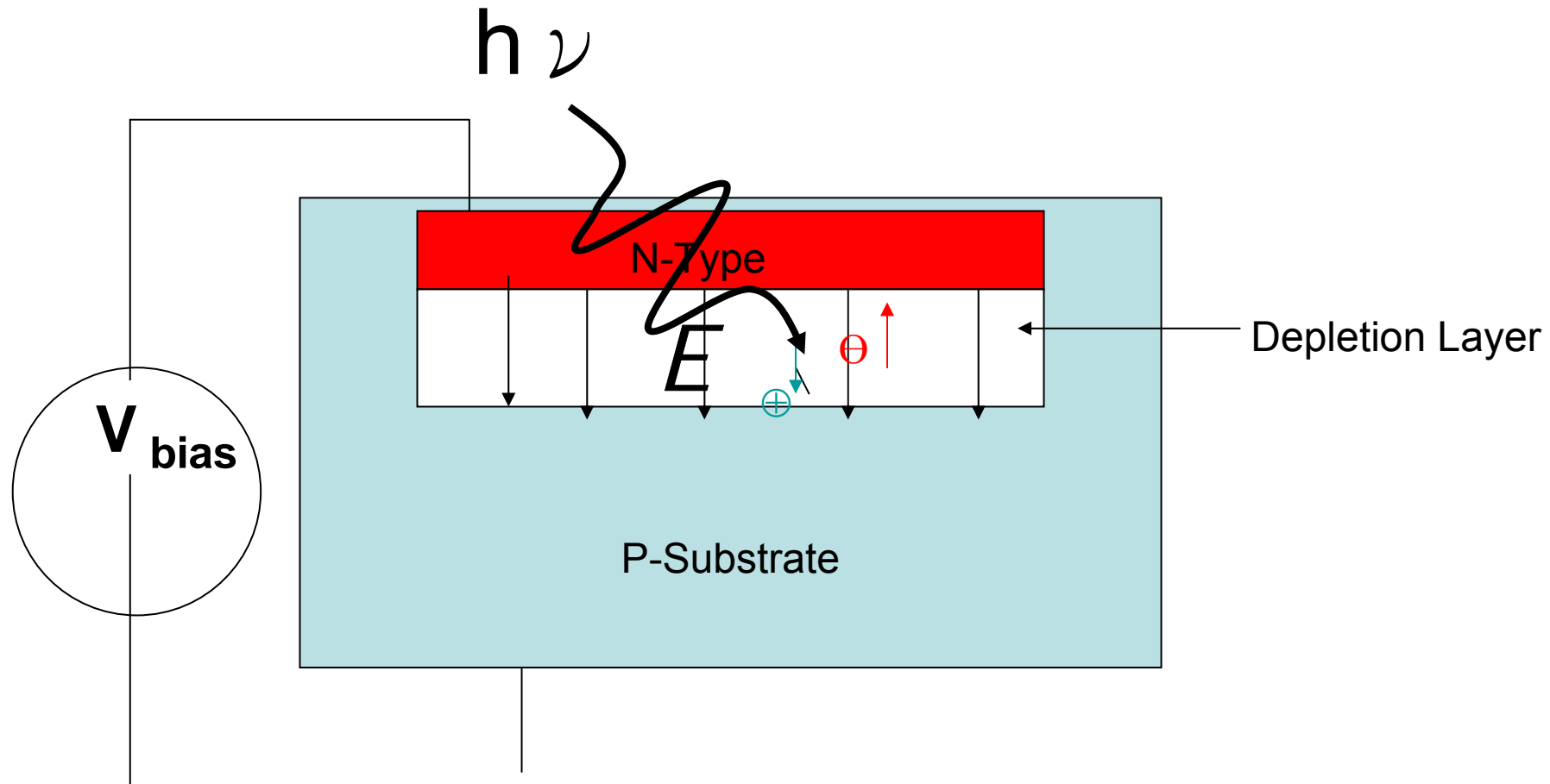
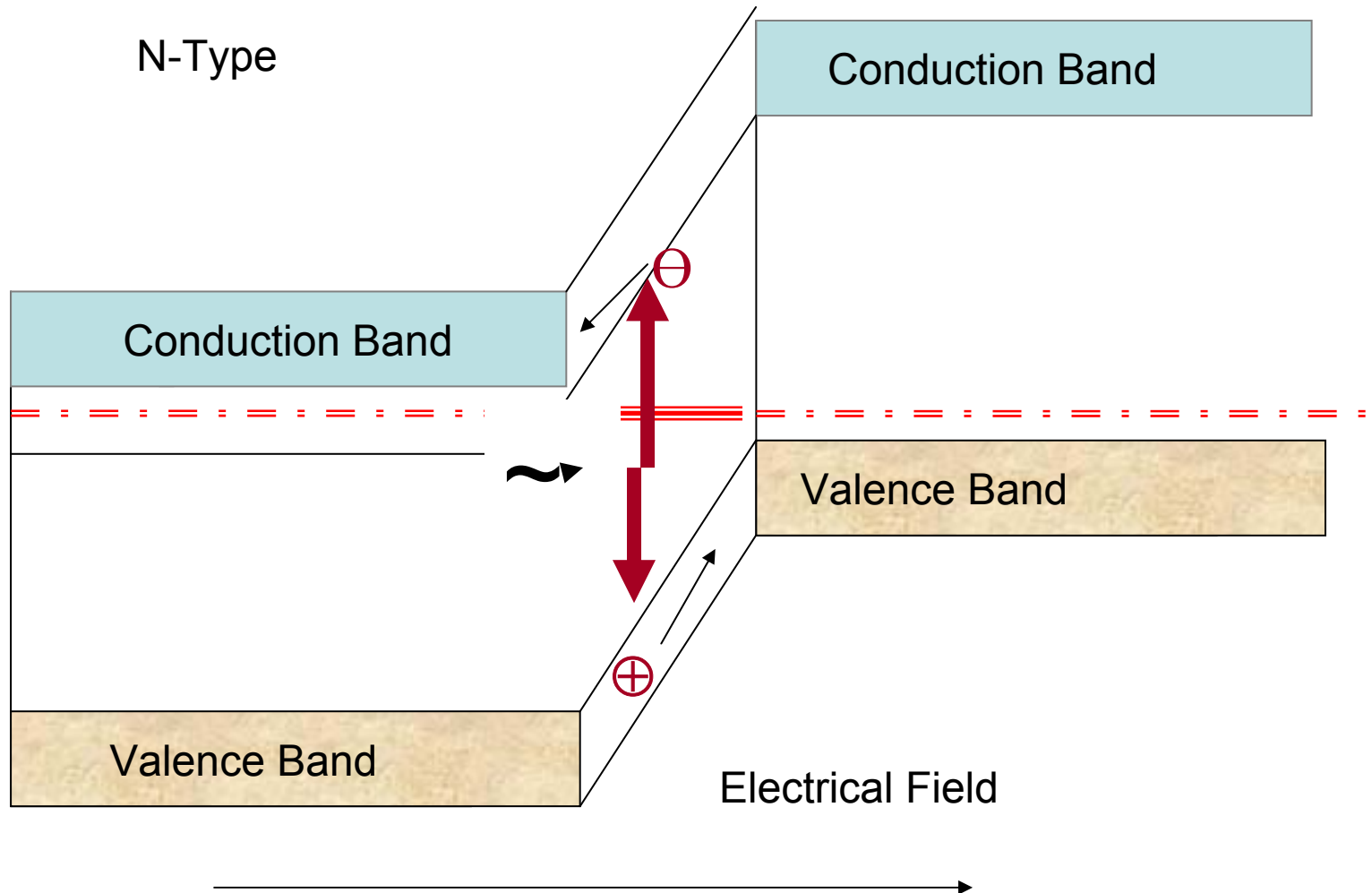
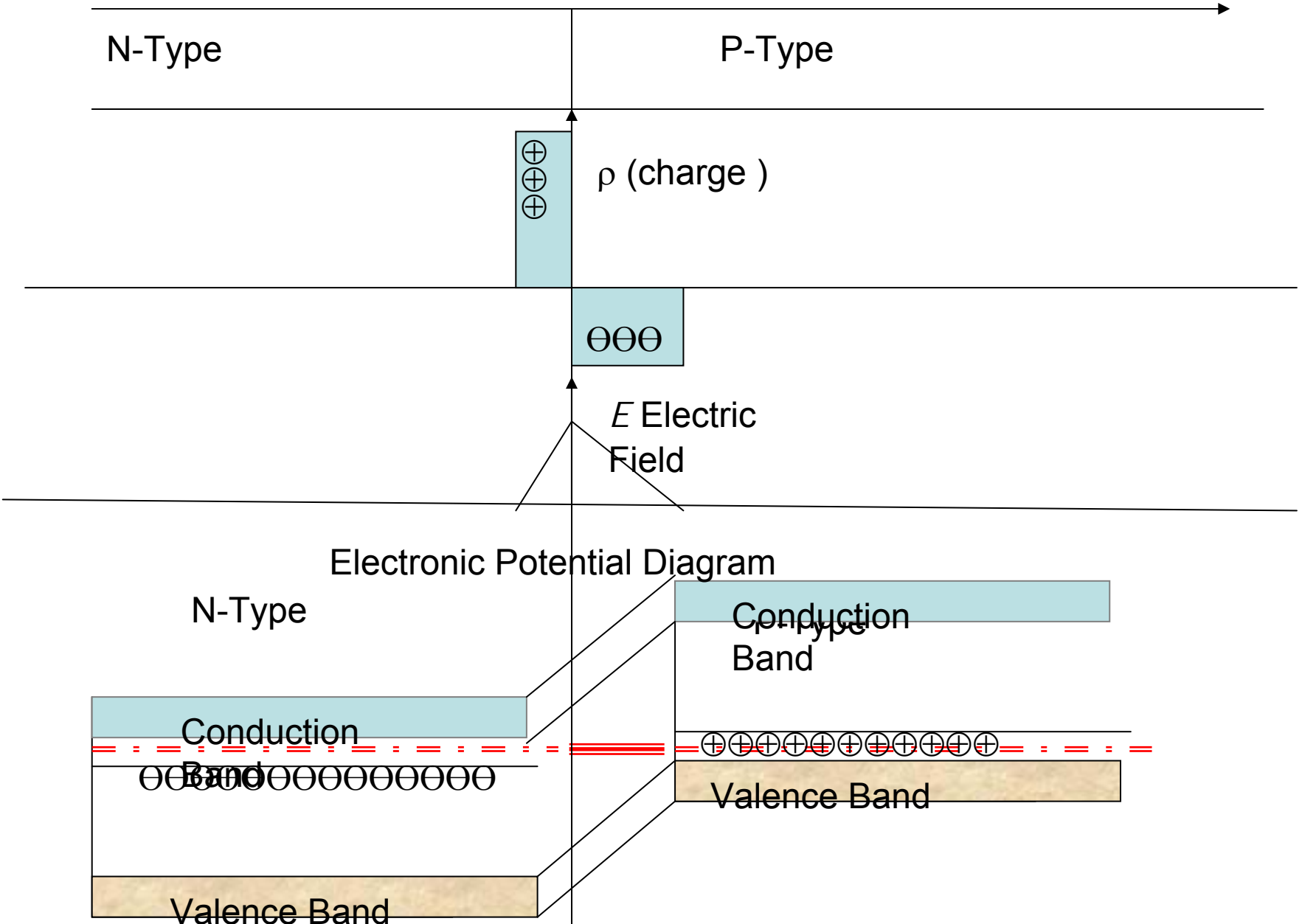


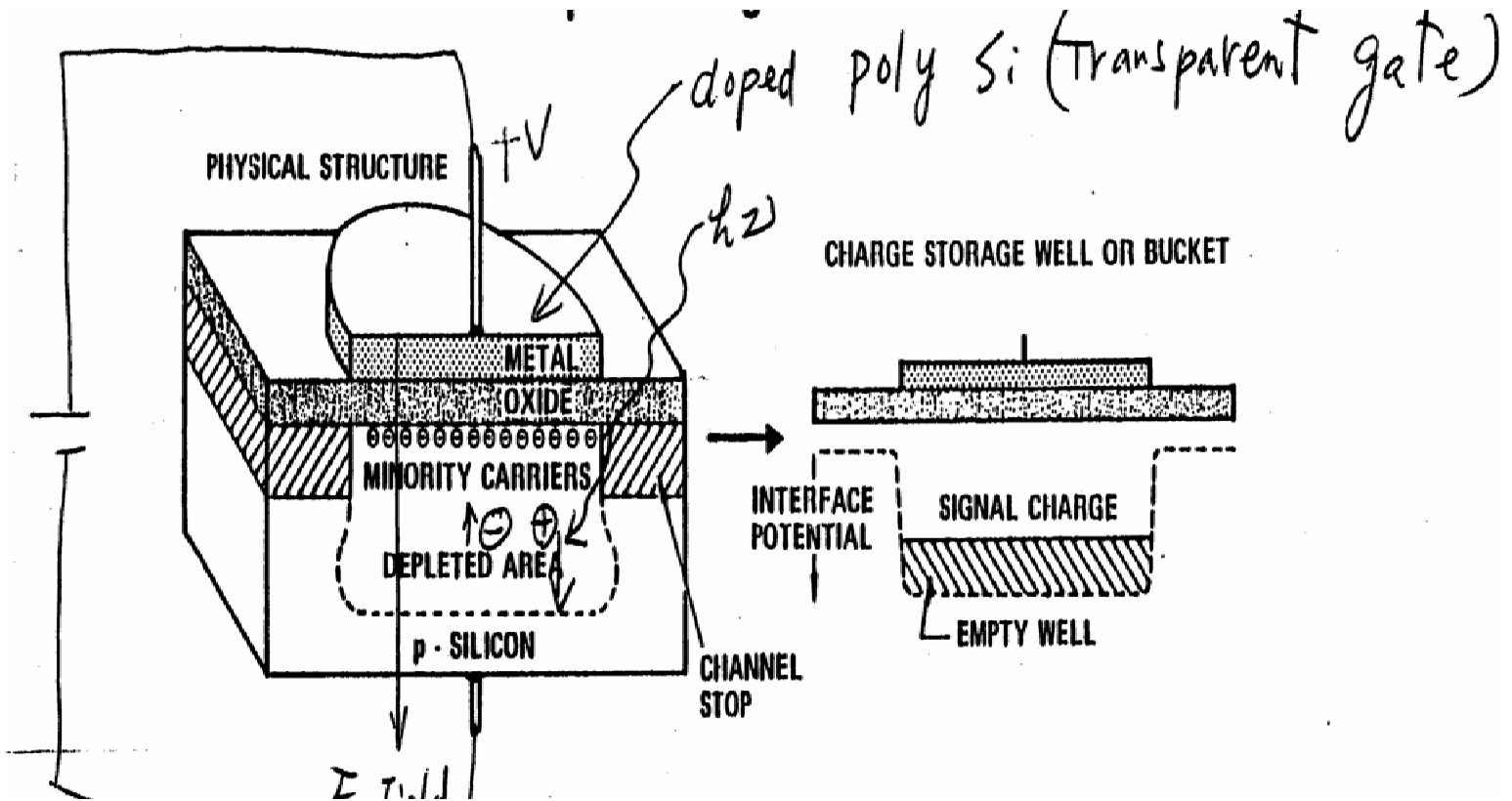
Photo-Voltaic Operation



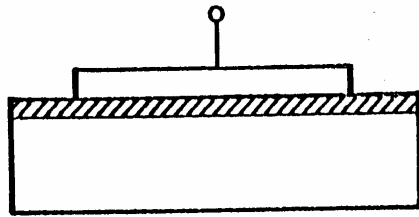
Diode Schematics



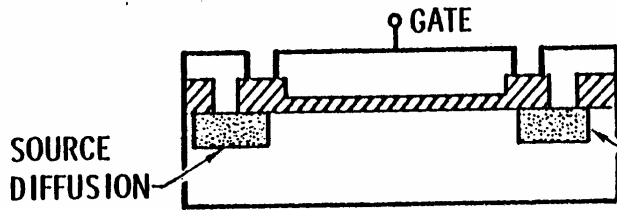
CCD Storage "Well"



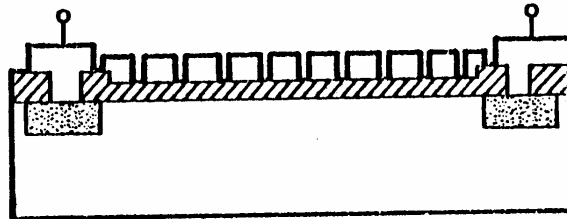
MOS and CCD Representations



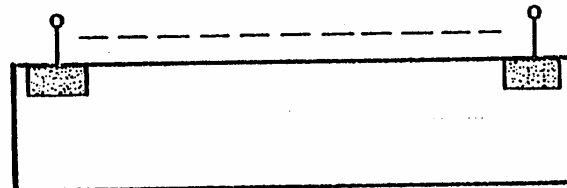
◀ MOS STRUCTURE
(metal oxide semiconductor, silicon)



◀ BASIC MOST (transistor)

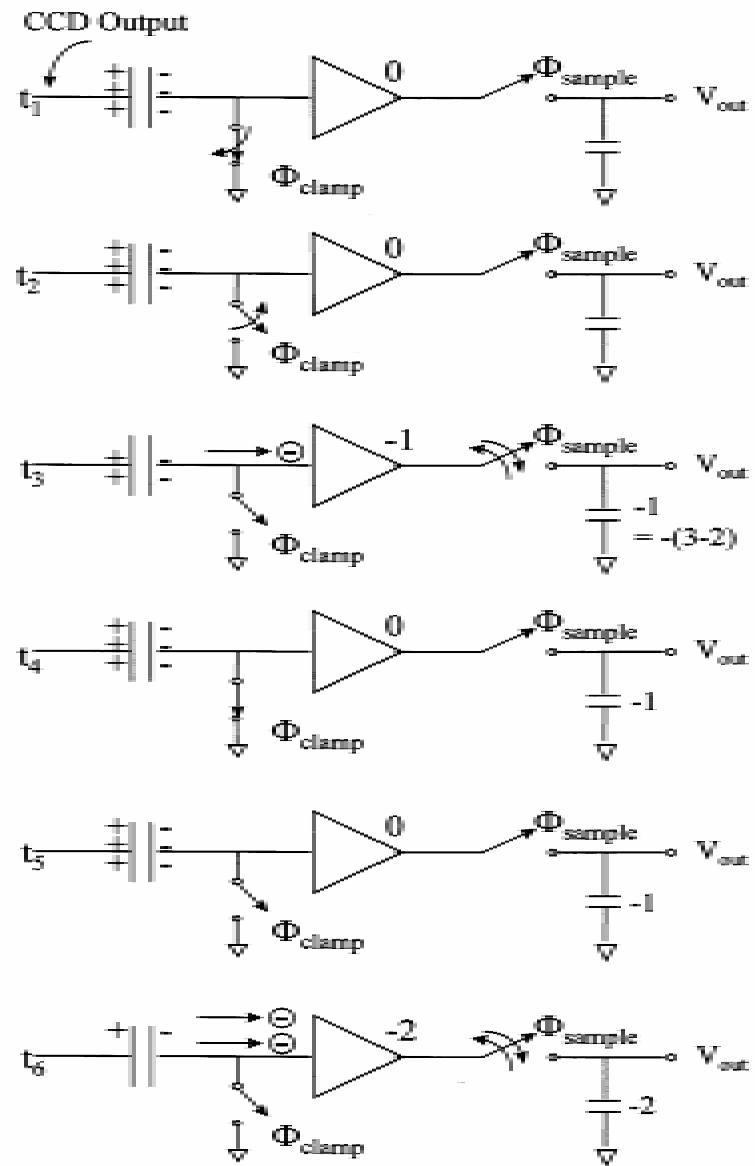


◀ BASIC CCD STRUCTURE

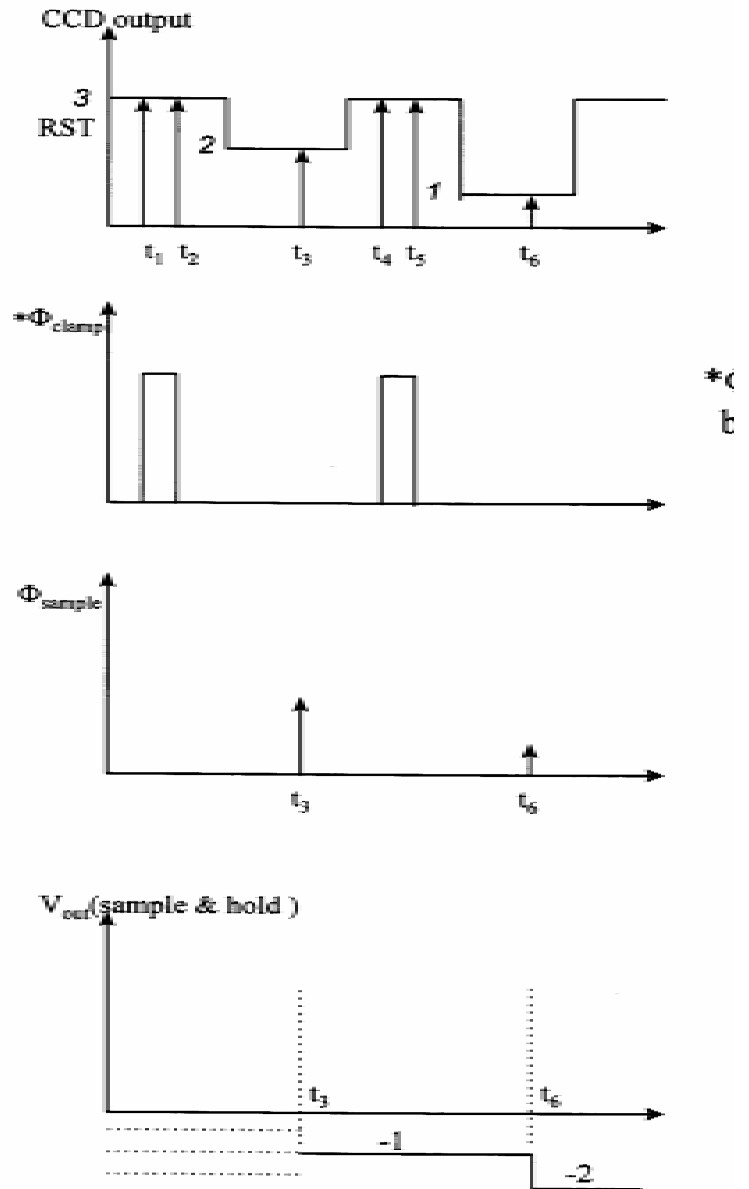


◀ REPRESENTATION OF CCD STRUCTURE

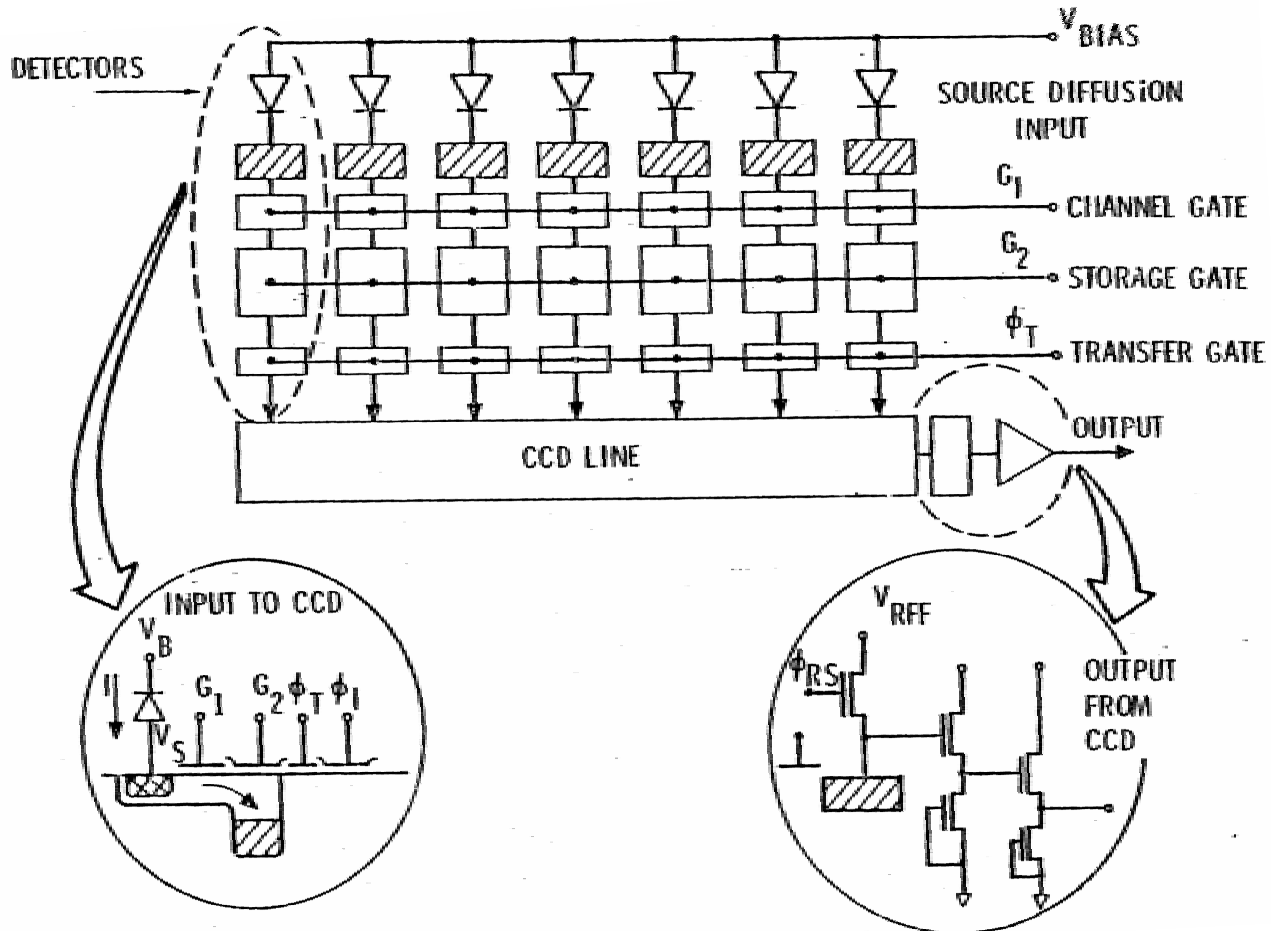
Correlated Double Sampling Sequence



Sample and Hold Timing Diagram



Linear CCD Array



Homework 3

- An IR system has an optical blur function of $\exp(-ax/2)$, where $a=125 \mu\text{m}$, and the detector dimension of $200 \mu\text{m}$ square.
- 1. Plot both blur functions and the convolution function
- Compute and plot the system MTF. What is the Nyquist MTF?

Hint: MTF of $\exp(-ax/2)$ is $[1+(af/f_0)^2]^{-1/2}$